Physical Significance of Wave Function

Bhushan Poojary
1(NIMS University, India)

Abstract

Wave function is a mathematical tool used in quantum mechanics to describe any physical system. Currently there is no physical explanation about wave function. This paper describes wave function as function of space time fluctuation.

Keywords – Wave function, space time interval, space time curvature

I. INTRODUCTION

The physical interpretation of the wave function is context dependent shown below.

1) One particle in one spatial dimension
2) One particle in two spatial dimensions
3) One particle in three spatial dimensions
4) One particle in one dimensional momentum space

II. DERIVATION OF WAVE FUNCTION IN TERMS OF MIX SPACE TIME MATRIX COMPONENTS

From paper “Electroweak Field Equations” 1, we know that electromagnetic equations are derived from mix matrix shown below.

\[ s^2 = \eta_{\mu\nu} x_\mu x_\nu \rightarrow \text{Eq} 1 \]

Schrödinger equation is a partial differential equation that describes how the quantum state of a physical system changes with time.

Schrödinger equation if considered as similar case of electromagnetic waves we can say that wave function is function of space time interval components.

\[ \psi(r, t) = f(\eta_{\mu\nu} x_\mu x_\nu) \rightarrow \text{Eq} 2 \]

III. ONE PARTICLE IN ONE SPATIAL DIMENSION

Let us consider particle in box scenario as shown below in Fig 1.

\[ V(x) = \infty \text{ (barrier)} \quad V(x) = 0 \text{ (well)} \quad V(x) = \infty \text{ (barrier)} \]

Fig 1: The barriers outside a one-dimensional box have infinitely large potential, while the interior of the box has a constant, zero potential.

Wave equation of particle in one dimension is given as \(^3\).

\[ \psi(x, t) = |\Psi_0| e^{-i(kx - \omega t)} \rightarrow \text{Eq} 3 \]

Units \(^3\) of the above wave function is 1/(meter)\(^1/2\). If we solve Schrödinger equation for particle inside a box we get following equation \(^4\).

\[ \psi_n(x, t) = \begin{cases} \frac{2}{L} \sin\left(\frac{n\pi}{L} x\right) & 0 \leq x \leq L \\ 0 & 0 > x > L \end{cases} \rightarrow \text{Eq} 4 \]

From Eq4 we know that

\[ |\Psi_0| = \frac{2}{L} \rightarrow \text{Eq} 5 \]

From Eq2 if we consider only one dimension we would get following equation

\[ \psi(x, t) = f(x, x) \rightarrow \text{Eq} 6 \]

Where \( x, x_i \) is the first component of mix matrix \( \eta_{\mu\nu} \). From paper “Electroweak Field Equations” \(^5\), we know that we can expand the matrix by assuming

\[ x_i x_i = (x_1^2) + (x_2^2) \rightarrow \text{Eq} 7 \]

We can express \( x_r \) and \( x_i \) as

\[ X = (x_r) + i(x_i) \rightarrow \text{Eq} 8 \]

We know that these dimensions in Eq8 should be waves and should be function of kx-wt. Hence we can write it as.

\[ (x_r) = X \left| \sin(kx - \omega t) \right| \rightarrow \text{Eq} 9 \]
\[ (x_i) = \frac{1}{X} \cos(kx - \omega t) \rightarrow \text{Eq10} \]

If we put Eq9 and Eq10 in Eq8 we get following expression.
\[ \frac{1}{X} = \frac{1}{X} e^{-i(kx - \omega t)} \rightarrow \text{Eq11} \]

We know from Eq8 that
\[ \frac{1}{X} = \sqrt{x_i x_i} \rightarrow \text{Eq12} \]

Putting Eq12 in Eq11 we get
\[ \frac{1}{X} = \sqrt{x_i x_i} e^{-i(kx - \omega t)} \rightarrow \text{Eq13} \]

Eq 13 is very close to Eq3 but still units do not match, to match units one will have to divide Eq13 by \( \lambda^2 \) (where \( \lambda \) is wavelength) we will get
\[ \frac{1}{\lambda^2} x_i x_i e^{-i(kx - \omega t)} \rightarrow \text{Eq14} \]

Eq 14 and Eq 3 are now same and units also match, so now we can say that.
\[ \Psi(x, t) = \frac{\sqrt{x_i x_i}}{\lambda^2} e^{-i(kx - \omega t)} \rightarrow \text{Eq15} \]

Where
\[ |\Psi_0\rangle = \frac{\sqrt{x_i x_i}}{\lambda^2} \rightarrow \text{Eq16} \]

The figure below shows wave functions of first 2 energy levels

If we put the graph of \( x_i x_i \) for \( n = 1, 2 \) we get following output see Fig 4 and Fig 5.
Fig 4: plot of \((x, x_1)\) for \(n=1\)

Fig 5: plot of \((x, x_1)\) for \(n=2\)

From Fig 3, 4 and 5 we can say that Probability of finding a particle is correlated to \((x, x_1)\) in space, which is evident from Eq17 as well.

Quantum mechanics successfully predicted the probability without knowing the reason behind it; the main reason is fluctuation in mix matrix space time components.

IV. ONE PARTICLE IN TWO SPATIAL DIMENSION

If a particle is trapped in a two-dimensional box, it may freely move in the x and y-directions, between barriers separated by lengths \(L_x\) and \(L_y\) respectively. Using a similar approach to that of the one-dimensional box, it can be shown that the wave functions is

\[
\psi_{n_x, n_y} = \frac{4}{\sqrt{L_x L_y}} \sin(k_{n_x} x) \sin(k_{n_y} y) \rightarrow \text{Eq}24
\]

Where the two-dimensional wave vector is given by

\[
k_{n_x, n_y} = k_{n_x} \hat{x} + k_{n_y} \hat{y} \rightarrow \text{Eq}25
\]

Equivalent of wave function of Eq24 in terms of mix matrix can be written as

\[
\psi_{n_x, n_y} = \frac{1}{\lambda_x^{\frac{1}{2}} \lambda_y^{\frac{1}{2}}} \frac{X Y}{\frac{1}{2} \lambda_x^{\frac{3}{2}} \lambda_y^{\frac{1}{2}}} \sqrt{(x, y)(y, y)} \sin(k_{n_x} x) \sin(k_{n_y} y) \\
\rightarrow \text{Eq}26
\]

Because we know that

\[
\frac{1}{\lambda_x^{\frac{1}{2}} \lambda_y^{\frac{1}{2}}} \sqrt{(x, y)(y, y)} = \frac{4}{\sqrt{L_x L_y}} \rightarrow \text{Eq}28
\]

If we plot Eq6 we get following output (see Fig 6) in terms of wave function.
From Eq26 and Eq27 we can see the wave function is function of \((x_i, x_j)\) and \((y_i, y_j)\).

V. ONE PARTICLE IN 3 SPATIAL DIMENSION

If particle is trapped in 3 dimensional box, its wave function is given as:

\[
\psi_{n_x, n_y, n_z} = \frac{8}{L_x L_y L_z} \sin(k_{n_x} x) \sin(k_{n_y} y) \sin(k_{n_z} y) \\
\rightarrow \text{Eq29}
\]

Where the 3-dimensional wave vector is given by

\[
k_{n_x, n_y, n_z} = k_{n_x} \hat{x} + k_{n_y} \hat{y} + k_{n_z} \hat{z} \rightarrow \text{Eq30}
\]

Similarly if we follow the approach done for particle in one or two dimensions we can write wave function in terms of mix matrix in Eq31

\[
\psi_{n_x, n_y, n_z} = \frac{1}{(\lambda_x \lambda_y \lambda_z)^{\frac{1}{2}}} \sqrt{\frac{(x_i x_j)(y_i y_j)}{L_x L_y L_z}} \sin(k_{n_x} x) \sin(k_{n_y} y) \sin(k_{n_z} z) \\
\rightarrow \text{Eq30}
\]

VI. ONE PARTICLE IN ONE DIMENSIONAL MOMENTUM SPACE

Momentum-space wave functions frequently are most easily obtained by the Fourier transform of the already available position-space wave function. For the particle in a one-dimensional box the Fourier transform is given by the following equation:

\[
\phi(n, p, L) = \frac{1}{\sqrt{2\pi}} \int_0^L e^{-ipx} \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) dx \\
\rightarrow \text{Eq31}
\]

If we put Eq16 value in Eq31 we get following equation

\[
\phi(n, p, L) = \frac{1}{\sqrt{2\pi}} \int_0^L e^{-ipx} \sqrt{\frac{x_i x_j}{\lambda_x \lambda_y \lambda_z}} \sin\left(\frac{n\pi x}{L}\right) dx \\
\rightarrow \text{Eq32}
\]

From above we can say that momentum space function is also function of \(x_i, x_j\) (mix matrix component's).

\[
\phi(n, p, L) = f(x_i, x_j, n, p, L) \rightarrow \text{Eq33}
\]

VII. CONCLUSION

Wave function and Momentum wave function are due to fluctuations in mix space time matrix components, as we move towards the particle these disturbance will be higher compared to when away from the particle. These disturbances give rise to probability in Quantum mechanics.

ACKNOWLEDGEMENTS

I would like to thank my friend Marco Magagnini for encouraging me and guiding me to write this manuscript.

REFERENCES