

Original Article

Dynamic LQR control with an Extended State Observer for Flywheel Supported by Active Magnetic Bearings

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Abstract - Flywheel Energy Storage Systems (FESS) suspended by Active Magnetic Bearings (AMB) are fundamental to allowing high-velocity operations with negligible friction losses, making them excellent for modern energy grids. However, these systems operate at high speeds, which induce strong effects, such as couplings and disturbances, that affect the stability of the system. To address this challenge, this investigation proposes and designs a dynamic gain-scheduled Linear Quadratic Regulator (LQR) control scheme with an Extended State Observer (ESO) to stabilize a rotor magnetically levitated Flywheel Energy Storage System (FESS). To validate the proposed control system, a multi-physics model was evaluated using MATLAB. The results show a reduction of 36% in maximum radial displacement and 37% for angular peaks with a minimum change of 2.2% in energy consumption compared to the conventional LQR. Finally, the proposed controller ensures robust 5-DOF stabilization and mechanical safety for FESS in highly dynamic environments.

Keywords - Flywheel, Active Magnetic Bearing, Extended State Observer, LQR, FESS.

1. Introduction

Flywheels Energy Storage Systems (FESS) are highly efficient and reliable energy management solutions that outperform conventional chemical technologies due to their fast response and long service life [1-4]. To achieve high efficiency, Active Magnetic Bearings (AMB) are fundamental to enable working in vacuum and high-speed conditions [5] and controlling the dynamics of the rotor in various degrees of freedom to guarantee stability for continuous working [6, 7].

To maximize energy storage capacity, recent literature focuses on rotor design optimization and the use of composite materials and Hybrid multi-layer configurations [4, 8].

However, the dynamics of systems with active magnetic bearings are an inherently unstable system, strongly coupled and nonlinear, due to the gyroscopic effect and geometric dependencies [9-11]. Also, harmonic disturbances are generated by the residual mass imbalance, possible base movements in mobile applications, and structural resonances associated with the system's geometry [12, 13]. Furthermore, specific electromagnetic and geometric parameters of the actuators determine their stabilization capacity [14-16]. For these reasons, to ensure safe and continuous operation of the FESS system under these complex conditions, a robust and efficient control system is required. Artificial Neural Networks (ANNs) provide a powerful tool for system

identification when exact physical parameters are unknown or difficult to determine [10]. However, their application in AMB is generally restricted to dynamic modelling rather than acting as a controller [10]. Consequently, structured model-based control architecture remains indispensable to enforce closed loop stabilization [17]. Conventional Single-Input Single-Output (SISO) control strategies, such as PID, are widely employed due to their implementation simplicity [9, 12, 15].

To develop a multi-axis-coupling within a SISO framework, many decoupling methods are integrated into the PID loops: modal coordinate transformation matrices are applied to separate spatial geometric channels into translational and tilting modes [9, 18], while displacement and velocity cross-feedback compensation channels are introduced across orthogonal axes to counteract gyroscopic torques [7, 9]. However, modal transformations cannot eliminate the speed-dependent coupling between axes. And static cross-feedback gains present phase lags and power amplifier bandwidth constraints as rotational speed increases [7, 15].

Other alternatives like complex-coefficient filtering and inverse system decoupling remain highly sensitive to digital delays, sensor noise and parameter drift, such as thermal-induced coil resistance variations [19]. Consequently, PID controllers fail to provide sufficient modal damping at



elevated speeds, leading to nutation instability, unbalanced vibration amplification, actuator saturation and high energy consumption [12, 19].

To overcome the coupling limitations of SISO systems, multivariable state-space control strategies, such as Linear Quadratic Regulator (LQR controller), have been adopted due to their ability to achieve an optimal multi-axis regulation with good energy efficiency [9, 20]. However, conventional LQR controller presents critical disadvantages when applied to high-speed AMB systems [10, 15].

First, although heuristics methods such as Bryson's rule, used for obtaining Q and R , achieves optimal dynamic modal damping under strong gyroscopic cross couplings still heavily relies on trial-and-error tuning [20].

Also, fixed-gain LQR formulations fail to maintain closed-loop stability across variable speed ranges [10, 15]. Finally, because standard LQR operates purely as a proportional state-feedback regulator, it cannot completely suppress harmonic forces and it requires the complete state vector, making necessary the integration of a state observer due to the lack of speed sensors [17, 21].

To manage system non-linearities and parameter uncertainties without relying on the exact physical model, Fuzzy logic controllers have been applied in AMB systems [7, 17]. However, applying fuzzy logic to a 5-DOF multivariable system exponentially increases fuzzy rules, increasing processing delays and degrading static and dynamic positioning precision [9, 17].

To enforce rigorous stability margins and attenuate high-speed vibrations, robust H_∞ controller has been widely adopted in AMB [15]. Through weighting functions and notch filters, these controllers effectively suppress synchronous harmonic disturbances and prevent nutation instabilities [15]. Nevertheless, H_∞ controllers are inherently synthesized for a single nominal speed, leading to severe performance degradation or loss of levitation stability [10, 15].

On the other hand, Model Predictive Control (MPC) has been used [13, 17]. Where this controller enforces explicit limits on state variables and actuator constraints, such as power amplifier current and voltage saturation [10, 13]. However, MPC relies on precise dynamic models and demands a high computational load for fast sampling rates (5-10 kHz), frequently exceeding the processing of embedded hardware [10, 17].

To increase the operational range at different conditions, such as changing rotational speed conditions, researchers use linear control laws into Linear Parameter-Varying (LPV) frameworks, typically synthesized via Linear Matrix

Inequalities (LMIs) [10]. Parameterizing system matrices with respect to speed allows LPV controllers to adapt stability bounds across variable operating points [10]. However, this approach presents substantial practical implementation constraints: it yields high-order, overly conservative controllers that require heavy online matrix interpolation or complex real-time optimization, placing a severe computational burden on embedded processors and increasing high-frequency sensor noise [10, 15].

To handle the non-linear effects, such as electromagnetic forces and unmodeled parametric uncertainties, non-linear control strategies, most prominently Sliding Mode Control (SMC) have been used for AMB systems [10, 17]. Sliding mode control offers exceptional insensitivity to parametric variations and external disturbances without requiring an exact mathematical plant model [17]. However, SMC introduces a severe physical limitation: high frequency switching oscillations, known as chattering [17]. In electromagnetic actuators, chattering severely degrades power amplifier performance, induces current and voltage saturation and can excite high-frequency structural resonances in the rotor, causing system instability or hardware degradation [10, 17].

Due to the high-speed external vibrations and unmodeled forces, state and disturbances estimation techniques have been widely integrated into AMB control loops [21-23]. While classical Kalman filters effectively filter noise measurement, Kalman filters depend heavily on accurate noise covariance matrices and dynamic models [22]. In contrast, Extended State Observers (ESOs) provide an alternative by lumping parametric uncertainties, unmodeled dynamics and external forces into an observable variable, forming the basis of Active Disturbance Rejection Control (ADRC) [17, 21]. Although ESOs have been combined with Sliding mode control, H_∞ controller or conventional fixed-gain LQR strategies in magnetic bearings, significant operational trade-offs persist [21, 22]. SMC-ESO configuration induces high frequency chattering [17], whereas fixed Gain LQR-ESO scheme loses stability across variable speed ranges [15, 21]. Consequently, a clear gap persists balancing real-time adaptability under severe gyroscopic effects and high-speed disturbances while respecting actuator constraints and avoiding heavy computational burdens.

To overcome these limitations, this study proposes a novel dynamic gain-scheduled LQR controller with an Extended State Observer (ESO) applied to flywheels suspended by AMBs, along with a comprehensive methodology for its development and validation. This methodology begins by designing and performing a modal analysis of the rotor. Subsequently, the dynamic model of the system is developed, followed by the development of a novel dynamic gain LQR controller with an Extended State Observer (ESO).

This dynamic gain LQR controller allows recalculation and updates the optimal feedback matrix in real time, guaranteeing efficient and robust performance according to the operating point. Integration of ESO is justified by its ability to estimate system state variables and estimate unmodeled uncertainties, and external excitations such as mass imbalance. This system consolidates an Active Disturbance Rejection Control (ADRC) scheme, substantially improving the model's robustness [21, 23]. Finally, this proposal will be validated through numerical simulations in MATLAB/Simulink, showing its superior ability to reject disturbances under various critical operating conditions and creating the way for future experimental implementation.

2. Mathematical Model and Rotor Analysis

2.1. Design and Parameters of the Flywheel Rotor

The physical configuration and design parameters of the flywheel rotor were taken from previous literature [7, 8]. Figure 1 presents the rotor dimensions used in the 3D model developed in Autodesk Inventor to determine moments of inertia, center of mass location, volume and mass. The chosen material was AISI 4340, which is the standard material for FESS applications [1, 8]. Figure 2 shows the view of the architecture of Active Magnetic Bearings (AMBs). The nominal electromagnetic parameters for the radial and axial AMBs, extracted from the reference literature, are summarized in Table 1 [5, 11, 19]. These values were selected to provide a realistic and physically feasible baseline that ensures that the simulated operating conditions accurately represent standard FESS requirements and serve as a reliable basis for future experimental implementation.

Table 1. AMB parameters

Parameters	Axial	Radial
Number of turns (N)	1000	200
Nominal Airgap (s_0)	1mm	
Cross sectional area (A)	$4\pi \times 10^{-4} \text{ m}^2$	
Bias Current (i_{bias})	0.75 A	
Magnetic permeability of vacuum (μ_0)	$4\pi \times 10^{-7} \text{ H/m}$	

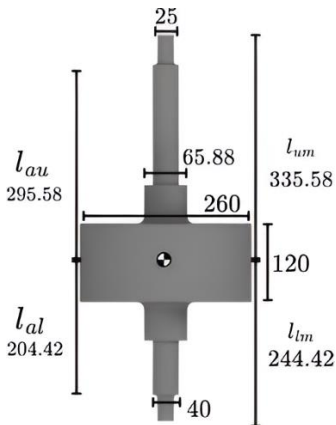


Fig. 1 Geometric labels and dimensions in mm of the flywheel

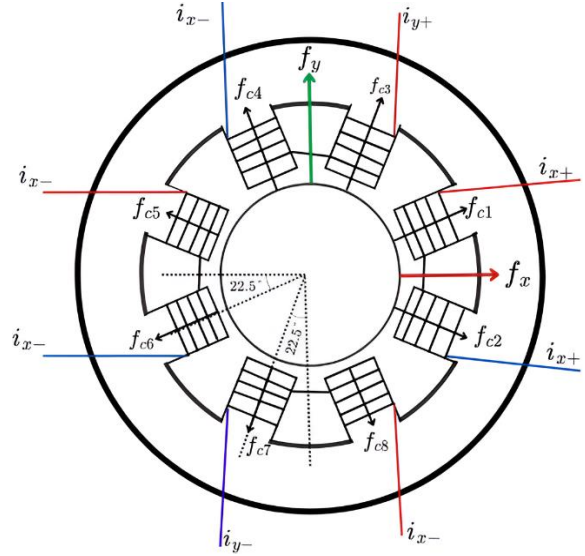


Fig. 2 Schematic of the 8-pole heteropolar radial AMB: Poles distribution, electrical interconnections, and force vectors

2.2. Structural Analysis of the Flywheel Rotor

Once the rotor design was defined, a modal analysis was made through the Finite Element Method (FEM) in Ansys Mechanical. To ensure reliability and precision a grid-convergence study was made [5]. Three natural frequencies (modes 7, 13 and 20) were evaluated as a function of element size, obtaining that from 18490 elements onwards the mesh begins to converge.

Based on this criterion, the meshing was refined, obtaining a meshing quality between 0.1542 and 0.997. As described in Table 2, the first vibration mode occurs at 800.73 Hz. It demonstrates that the first critical speed is approximately 48,000 RPM, which is much higher than the previously established maximum operating speed. Identifying these frequencies to avoid in the control system design ensures that mechanical resonances and deformations remain negligible, successfully validating the rigid rotor assumption [18].

Table 2. Frequency vibration of every rotor's mode

Mode	Frequency (Hz)
1	800.73
2	801.01
3	2600.3
4	2601.5
5	3665
6	3665.7
7	4155.1

2.3. Dynamic Model of the Rotor

To model the flywheel, the rotor was established as a rigid body with five degrees of freedom. The coordinates vector was defined as $[x, y, z, \alpha, \beta]$. The rotation around the z-axis (Ω) is independently controlled by the motor.

To formulate the equations of motion, the free-body diagram shown in Figure 3 was made, where f represents the forces generated by AMBs.

Based on Figure 3 and 1, Newton's second law and Euler's equations, the dynamic behavior of the system is formulated to obtain the equation of motion [5, 7].

Also, during high-speed operation, centrifugal expansion is given by Equation (1), where this deformation is directly proportional to the square of rotational speed and depends on the material's mechanical properties, such as density, Young's modulus, and Poisson's ratio. Incorporating this effect is important because reducing the air gap at high speeds affects the forces generated by the AMB [5, 24].

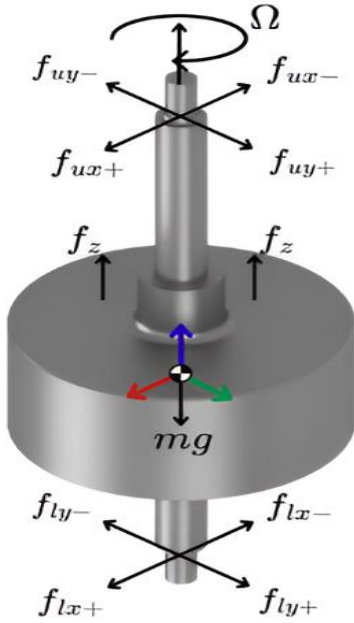


Fig. 3 Free Body diagram of rotor's flywheel

$$x_{ce} = \frac{\rho \Omega^2 r^3}{4E_s} (1 - \nu_s) \quad (1)$$

Where x_{ce} is the centrifugal radial expansion, ρ is the material density, Ω is the rotational speed, r is the rotor radius, E_s is the Young's modulus and ν_s is Poisson's ratio.

2.4. Mathematical Model of AMB

As shown in Figure 2, the AMBs operate in a differential configuration with a bias current to generate a controllable net force [5, 17]. Based on the virtual working principle, this force is calculated as:

$$F_{amb} = \frac{\mu_0 N^2 A}{4} \left[\left(\frac{i_b + i_x}{s_0 - x_{ce} - x} \right)^2 - \left(\frac{i_b - i_x}{s_0 - x_{ce} + x} \right)^2 \right] \quad (2)$$

Where x_{ce} is the centrifugal expansion and x is the radial displacement on an extreme, s_0 is the nominal Airgap, μ_0 is the

permeability of the air, N is the number of turns, A is the cross-sectional area, i_b is the bias current and i_x is the current control.

Based on [18], the position for the upper (x_u) and lower (x_l) extremes of the rotor is expressed as follows:

$$x_u = (x - l_{au}\beta) \cos(\theta_a) + (y + l_{au}\alpha) \sin(\theta_a) \quad (3)$$

$$x_l = (x + l_{al}\beta) \cos(\theta_a) + (y - l_{al}\alpha) \sin(\theta_a) \quad (4)$$

Where θ_a is the pole location angle, as shown in Figure 2, for every pole. x is the rotor's center mass position, l_{au} and l_{al} are the distances from the upper and lower extremes to the center of mass, respectively, while α and β are respectively the rotation angles about x and y axes.

As shown in Figure 2, radial AMB has an 8-pole heteropolar configuration. While linearized models typically simplify this arrange by grouping adjacent coils under the assumption of equal air gaps, the proposed non-linear model avoids this geometric simplification to ensure high-fidelity validation.

Instead, it individually calculates the exact air gap for each pole pair based on instantaneous rotor displacements and specific pole location angles, as formulated in Equation (3) and (4).

Finally, the total current in the actuator is governed by Faraday's, Ampere's, and Ohm's laws [5], where the voltage in the coils is established as:

$$u(t) = R i_t(t) + \frac{\mu_0 N^2 A}{2} \frac{d}{dt} \left(\frac{i_t(t)}{s_0} \right) \quad (5)$$

Where $u(t)$ is the coil voltage, R is the electrical resistance, $i_t(t)$ is the current, μ_0 is the vacuum permeability, N is the number of turns, A is the cross-sectional area, and s_0 is the nominal airgap.

2.5. System Linearization and State-Space Representation

Designing an LQR controller requires a linear state-space model. Assuming $\approx s_0$, $i_t(t) \approx i_{bias}$, where s is the instant airgap and there is no centrifugal expansion, the model reveals that the electromagnetic force consists of two parts [5, 18], as shown in Equation (6), a displacement constant K_x and a current constant K_i [10, 21].

$$F_{amb} = K_x x(t) + K_i i(t) \quad (6)$$

Where $i(t)$ is the current control, K_i is current stiffness, K_x is displacement stiffness and $x(t)$ is the actual position. These constants were defined as:

$$k_i = \frac{\mu_0 N^2 A r i_{bias}}{s^2}, k_x = \frac{\mu_0 N^2 A r i_{bias}^2}{s^3} \quad (7)$$

Based on this, the force generated by the AMBs is replaced, and the space, input and output vector is defined as:

$$X = [x, y, z, \alpha, \beta, \dot{x}, \dot{y}, \dot{z}, \dot{\alpha}, \dot{\beta}] \quad (8)$$

$$U = [i_{lx}, i_{ly}, i_{ux}, i_{uy}, i_z], Y = [x, y, z, \alpha, \beta] \quad (9)$$

Where X is the state vector comprising the translational and rotational displacements (x, y, z, α, β) and their respective velocities ($\dot{x}, \dot{y}, \dot{z}, \dot{\alpha}, \dot{\beta}$); U represents the control currents for the lower and upper radial AMBs ($i_{lx}, i_{ly}, i_{ux}, i_{uy}$) and the axial AMB (i_z); and Y is the output displacement vector.

With these vectors established, matrices A , B , C and D were found, where A depends on the rotational speed (Ω). Consequently, open-loop system poles will continuously change. For this reason, a fixed-gain controller would be ineffective, requiring the implementation of an LQR controller with dynamic gain scheduling to guarantee system stability [10].

2.6. Scope and Model Assumptions

For development of the dynamic model, control system design, and validation, fundamental assumptions have been established. For the modal analysis, damping effects and external forces are not considered [10, 18]. The system is assumed to operate within a vacuum chamber. Therefore, windage losses are negligible in the rotor dynamics. Furthermore, the system is evaluated under isothermal conditions [5, 19, 25]. For the formulation of the forces generated by the active magnetic bearings, it assumes no magnetic flux leakage or edge effects, and it is considered that all the magnetic flux ideally circulates through the core cross-section and the air gap [5, 19]. Regarding the centrifugal deformation analysis, the rotor was modeled as a long solid cylinder where the length significantly exceeds the radius ($L \gg r$). The model assumes finite displacement at the core ($r=0$) and a free outer surface [24]. The scope of this research is framed within the numerical validation applied in MATLAB/Simulink without reaching a physical hardware implementation.

3. Control Strategy Design

This section describes the development of control systems, including the Extended Observer State and the disturbance model.

3.1. Dynamic LQR Controller Formulation

The LQR controller design is based on state feedback control, where the input is given by Equation (10).

$$U = -KX \quad (10)$$

Where U is the signal control, X is the state vector and K is the feedback gain. To find K , the problem uses the Riccati algebraic equation [20], which is expressed as:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad (11)$$

Where P is a positive semi-definite matrix, representing the solution of Equation (11), A and B are the matrices for the state space representation. Subsequently, the optimal feedback gain (K) is found by:

$$K = R^{-1}B^T P \quad (12)$$

The Q matrix in Equation (11) was determined using Bryson's rule, with maximum limits of 0.1 mm for position, 1×10^{-3} rad for rotation and 0.1 m/s for velocity to keep the rotor safely within the AMB air gap. To avoid saturation in actuators and account for varying gyroscopic effects, the R matrix was tuned iteratively using 10 different angular velocities. Resulting in a scalar array ranging from 0.7 to 1.4 multiplied by a 5×5 identity matrix. An initial value was selected to provide a balance between control effort and robust disturbance rejection at the initial point in the simulation. To reduce the computational cost, a gain-scheduling algorithm was used to precompute and store 10 gain matrices for the predefined R values, choosing only the active gain in real time.

$$f_{\Omega} = 1 + \left(\frac{\Omega_{actual}}{\Omega_{max}} \right)^2 \quad (13)$$

$$f_I = \frac{1}{1 - \left(\frac{I_{actual}}{I_{max}} \right)^2 + 10^{-6}} \quad (14)$$

$$f_{err} = 1 + e^{(-10000 \cdot \|e_{pos}\|)} \quad (15)$$

Where f_{Ω} , f_I , and f_{err} are the dynamic scaling factors; Ω_{actual} is the current rotational speed; Ω_{max} is the maximum operational speed; I_{actual} is the actual actuator current; I_{max} is the maximum allowable current limit; and e_{pos} is the position error vector.

Table 3. Pole Location and Natural frequencies of LQR control c values for a matrix

Closed Loop Pole analysis for both LQR gain limits			
POLE LOCATION ANALYSIS FOR K=0.7		POLE LOCATION ANALYSIS FOR K=1.4	
Pole Location	Natural Frequency (Hz)	Pole Location	Natural Frequency (Hz)
$-536.7 \pm 2216.7i$	362.99	$-268.4 \pm 2216.5i$	355.35
$-90.0 \pm 84.2i$	19.62	$-65.4 \pm 58.0i$	13.91
$-90.0 \pm 84.2i$	19.61	$-65.4 \pm 58.0i$	13.91
$-23.2 \pm 0.2i$	3.68	$-11.8 \pm 0.1i$	1.88
$-415.2 \pm 343.9i$	85.80	$-290.0 \pm 248.2i$	60.75

The dynamic operating conditions are evaluated using normalized factors governed by Equations (13)-(15), which quantify velocity, current constraints, and position error to compute a target scalar index. Based on R_{target} value governed by the Equation (16), the algorithm selects the precomputed gain matrix corresponding to the closest predefined R value, updating the active controller gain in real time with minimal computational overhead.

$$R_{target} = R_{base} \cdot sat_{0.5}^2 \left(\frac{f \cdot f_{err}}{f_{\Omega}} \right) \quad (16)$$

According to the data in Table 3, the natural frequencies of the closed-loop system are safely located in the low-frequency spectrum, maintaining a wide margin of separation from the rotor's first flexible vibration mode. Therefore, it is established that the dynamic response of the magnetic actuators will not excite high-frequency structural resonances in the mechanical system [15, 18].

3.2. Extended State Observer (ESO) Design

The Extended State Observer (ESO) is designed by considering the system's external disturbances and model uncertainties as an additional state variable [21]. Starting from the linearized model the system is augmented as follows:

$$\begin{cases} \dot{X} = AX + BU + E\dot{f} \\ Y = CX \end{cases} \quad (17)$$

Where f is the total disturbance X is the augmented state vector, U is the augmented input vector, A , B and C are the augmented matrices of the system, Y is the output of the system and E is the disturbance vector which relates disturbances with accelerations. Given that the disturbance dynamics $\dot{f}$ are unknown and can be estimated by the correction term, it can be omitted [21].

This simplification allows the ESO to be formulated as a standard Luenberger observer structure. Based on [21] the bandwidth parameterization technique is adopted, where L is defined as: $[3\omega_0, 3\omega_0^2, \omega_0^3]$.

Following this method, all the poles of observer are located at $(-\omega_0, 0)$.

In this study, the observer bandwidth was set to $\omega_0 = 3500$ rad/s. This specific value was strategically selected to encompass the highest natural frequency of the relevant dynamic system, ensuring fast and accurate real-time tracking of concentrated disturbance and gyration modes [19].

3.3. Disturbances Formulation

To rigorously evaluate the robustness and active disturbance rejection capabilities of the proposed LQR-ESO control scheme. The most critical and common disturbance in FESS is residual mass imbalance. This disturbance, applied to the model, is defined as:

$$\begin{bmatrix} F_{unb_x} \\ F_{unb_y} \\ M_{unb_x} \\ M_{unb_y} \end{bmatrix} = me\Omega^2 \begin{bmatrix} \cos(\Omega t + \alpha) \\ \sin(\Omega t + \alpha) \\ \frac{(I_x - I_z)\delta_e}{me} \cos(\Omega t + \alpha) \\ \frac{(I_y - I_z)\delta_e}{me} \sin(\Omega t + \alpha) \end{bmatrix} \quad (18)$$

Where e , which value was established in 5.5×10^{-7} m, is mass eccentricity and δ_e , with the value of 5×10^{-5} rads, represents the eccentric tilt angle [8, 22]; F_{unb_x} and F_{unb_y} are the unbalanced forces along the x and y axes, respectively; M_{unb_x} and M_{unb_y} are the unbalanced moments; m is the rotor mass Ω is the rotational speed; α is the initial phase angle and I_x , I_y , and I_z are the principal moments of inertia.

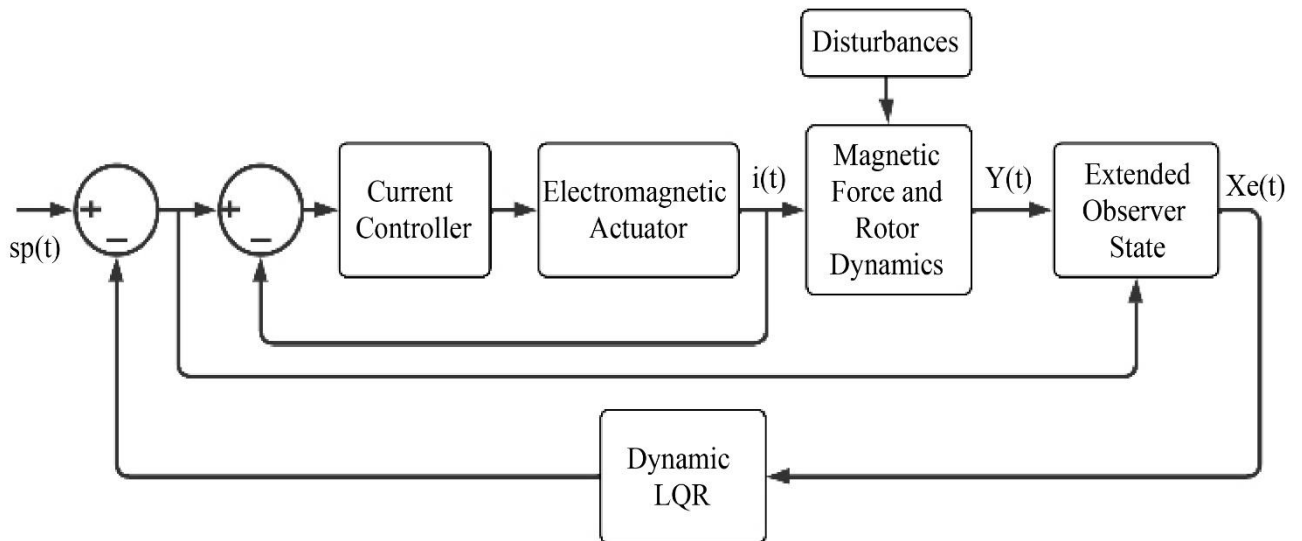


Fig. 4 Block diagram of the proposed control architecture, integrating the augmented ESO, and the LQR stabilization

Furthermore, external disturbances were introduced. These external disturbances are modeled as sinusoidal waves with frequencies of 10 Hz and an amplitude of 10 N. For other case external translational forces with amplitudes of 20N intermittently injected into the system's dynamics, along with external perturbing torques ranging from 5 N · m per axis [3]. Finally, the complete control architecture is depicted in Figure 4, where $sp(t)$ is the setpoint, $Y(t)$ is the distance between the rotor and the AMB and $Xe(t)$ is the estimated state vector.

Furthermore, two saturation limits were added to the voltage and control current of 24v and 3A for radial AMB's [5, 12], while for axial AMB the limit voltage increases to 250v [3]. These values were selected in accordance with state of art. Additionally, to simulate parameter variations and evaluate the worst-case scenario, the nominal system parameters were decreased by 10% [10].

3.4. Simulation Setup

To validate the proposed control strategy, the complete model of AMB-FESS, shown in Figure 4, was developed in MATLAB/Simulink. The governing differential equations were solved using the Runge-Kutta method with a step-size of 0.0002 seconds, to achieve a balance that satisfies the Nyquist-Shannon theorem without computational over cost. Also, three scenarios were evaluated: unbalanced disturbances alone, and unbalance combined with either external square pulses or sinusoidal excitations.

4. Results and Discussion

4.1. Response Time and Disturbance Rejection

The maximum deviations were obtained in second and third cases. Figure 5 and 6 show the response of the system with square pulses as disturbances, in these figures the dynamic LQR+ESO shows a faster reaction and a minor deviation compared to conventional LQR controller.

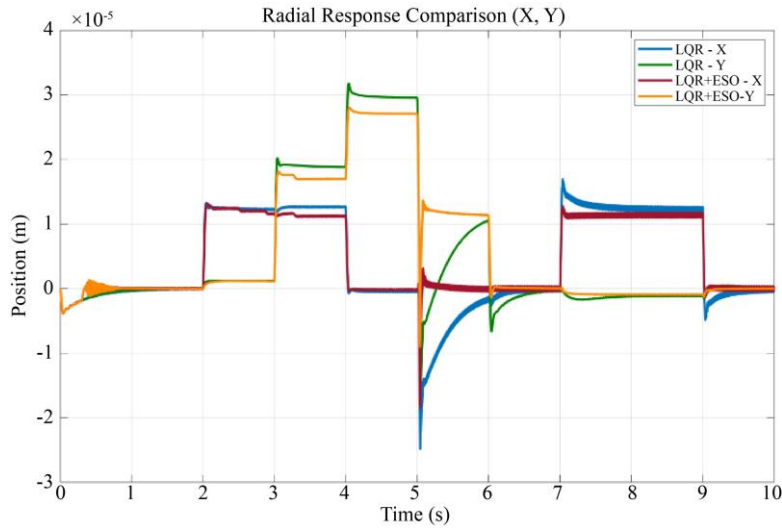


Fig. 5 Radial response in meters for square pulses and unbalanced disturbances

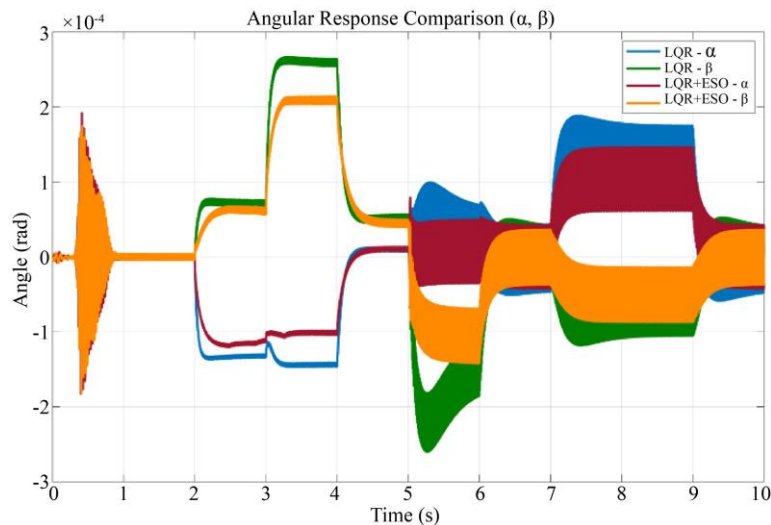


Fig. 6 Angular response in radians for square pulses and unbalanced disturbances

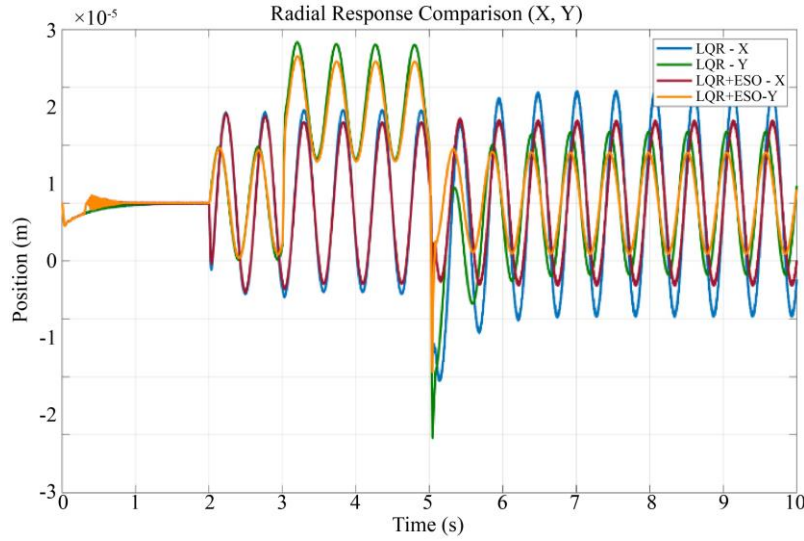


Fig. 7 Radial response in meters for sinusoidal and unbalanced disturbances

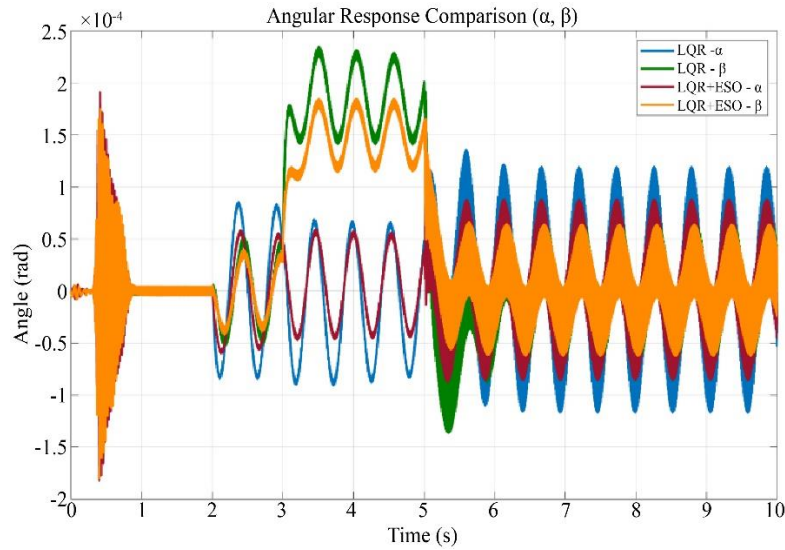


Fig. 8 Angular response in radians for sinusoidal and unbalanced disturbances

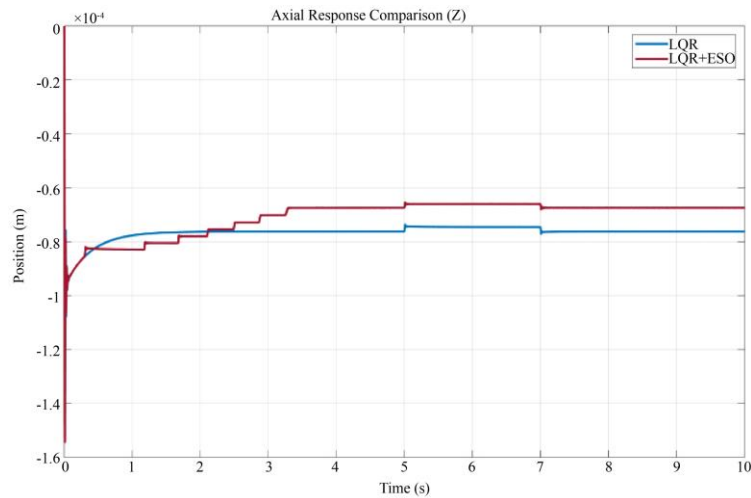


Fig. 9 Axial response in meters for sinusoidal and unbalanced disturbances

Figure 7 and 8 demonstrate the superiority of the proposed controller, with a better disturbance rejection, achieving minor deviations. For axial response, illustrated in Figure 9, the response was similar for three cases. Demonstrating that the proposed controller helps to reduce steady state error.

To provide a quantitative evaluation, Table 4 presents the global Mean Absolute Error (MAE) for each axis. The values shown represent the average performance across all evaluated test cases confirming that the observer-based strategy consistently enhances precision throughout different operating conditions. The proposed controller achieves a reduction of 36% in maximum radial displacement and 37% for angular peaks. While advanced controllers, in the literature report maximum radial deviations in the range of $40\ \mu\text{m}$ to $50\ \mu\text{m}$ under severe unbalance [17, 18], the proposed Dynamic LQR-ESO controller achieves a maximum deviation of $28\ \mu\text{m}$, reducing in a 30% to other controllers, while simultaneously regulating the rotor's tilt. Also, under the transient square waves disturbances of 20N, the system achieves its steady-state in 0.15 seconds, compared to other controllers, with a settling time of 0.1 and 0.6 seconds [17], this controller is slower due to saturation limits and the bandwidth of the ESO. Furthermore, the system smoothly recovers its steady-state equilibrium under severe 20 N transient impacts, matching the standards of 5-DoF (Degrees of Freedom) shock tests [3].

Table 4. Quantitative assessment of the control precision for the five degrees of freedom using MAE performance index

Axis	LQR	Dynamic LQR+ESO	Improvement %
X-Axis	6.2670e-06	4.8589e-06	22.47
Y-Axis	1.2285e-05	1.1338e-05	7.71
Z-Axis	7.5982e-05	7.1182e-05	6.32
Alpha	6.5170e-05	4.9967e-05	23.33
Beta	1.1210e-04	8.2732e-05	26.20

Table 5. Quantitative assessment of the control precision for the five degrees of freedom using IAE performance index

Axis	LQR	Dynamic LQR+ESO	Improvement %
X-Axis	3.1336e-05	2.4295e-05	22.45
Y-Axis	6.1386e-05	5.6688e-05	7.65
Z-Axis	3.7991e-04	3.5591e-04	6.32
Alpha	3.2586e-04	2.4984e-04	23.33
Beta	5.6050e-04	4.1367e-04	26.19

Table 6. Quantitative assessment of the control precision for the five degrees of freedom using ITSE performance index

Axis	LQR	Dynamic LQR+ESO	Improvement %
X-Axis	8.4418e-10	5.5331e-10	34.46
Y-Axis	4.2610e-09	3.9152e-09	8.12
Z-Axis	7.1465e-08	5.8427e-08	18.24
Alpha	8.8608e-08	4.9475e-08	44.16
Beta	3.2803e-07	1.6763e-07	48.90

On the other hand, the average Integral Absolute Error (IAE) and Integral Time Squared Error (ITSE) indicators, shown in Table 5 and 6 respectively, demonstrate an improvement for every axis compared to LQR controller. Ultimately, this ensures safe FESS levitation in highly demanding dynamic environments.

4.2. Analysis of Energy Consumption

Finally, the energy efficiency was 0.642 Wh and 0.628 Wh on average for both cases, for Dynamic LQR-ESO and conventional LQR controllers respectively. This is a highly favorable outcome, demonstrating that the integration of the Extended State Observer significantly improves the transient performance, yielding faster stabilization and tighter trajectory bounds, without imposing any additional energy penalty on the power amplifiers. Thus, the proposed LQR-ESO not only guarantees mechanical safety but also maximizes the overall energy retention of the FESS.

5. Conclusion

This study successfully developed and validated a Dynamic LQR-ESO controller for a 5-DOF AMB-FESS, effectively mitigating strong gyroscopic couplings and intense external excitations at high speeds.

The Dynamic LQR-ESO controller reduced the maximum radial deviation and the steady state error compared to conventional LQR controller.

Furthermore, the quantitative evaluation demonstrated a substantial improvement in overall tracking accuracy, reducing the Mean Absolute Error (MAE) by over 5% on the radial axes and by up to 6% on the axial steady-state error, being the X-axis the most noticeable improvement in a linear axis with an improvement over 22% for MAE and IAE indexes and over 34% for ITSE index. For angular axis, Alpha gets the best improvement over 26% for MAE and IAE indexes and an improvement by up to 48% for ITSE index.

Although there was a brief initial oscillation, the controller reduced the angular tilt over 23%. This kept the rotor safely within the narrow clearance and avoided any contact with the backup bearings, all without increasing energy consumption.

The total energy expenditure during the simulation remained with a negligible increase of 2.2% compared to LQR. This demonstrates that fast feed forward compensation prevents the continuous injection of high peak currents, preventing power amplifier saturation and maximizing the total net energy retention of the FESS.

Future research will focus on experimental implementation of the proposed controller in a physical testbed. The implementation of a prototype will require addressing some specific engineering challenges, such as the

sensor runout [13], actuator time delays [11], and the integration of the control loop with advanced diagnostics or adaptive health-monitoring features to avoid errors during its working [19]. In addition, the digital controller implementation must strictly satisfy the Nyquist-Shannon sampling criterion to prevent the aliasing of high-frequency rotor dynamics and harmonic vibrations [14], which consequently demands a high-performance real-time embedded system operating with a demanding execution duty cycle. Additionally, while the numerical results demonstrate high efficacy, further investigations are needed to analyze the system's robustness boundaries under complex, real-world disturbance scenarios.

Finally, this investigation establishes the basis for an advanced controller in FESS applications, guaranteeing stable, efficient, and robust levitation across highly demanding environments.

Conflicts of Interest

The author declares no conflict of interest.

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