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Selecting an Optimal Alternative by Constructing a Regression Model for Each Option

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Abstract - *Optimization for identifying the best solution is a pivotal technique that has garnered significant attention from the scientific community. It plays a decisive role in enhancing system efficiency, particularly across multidisciplinary engineering sectors such as mechanical, medical, environmental, food, and chemical engineering... This study introduces a novel approach to selecting the optimal alternative from a set of available options. For each alternative, a unique regression model is constructed to represent the relationship between its value and the evaluation criteria. The Design of Experiments (DOE) methodology is employed to build an experimental matrix, where each evaluation criterion for the alternatives is treated as an input parameter for the experimental process. The score for each alternative is calculated using the PSI method, a multi-objective optimization technique that does not require weighting the criteria. The use of DOE allows the proposed method to quickly rank alternatives when the number of options changes (increases or decreases). A notable advantage is that the proposed method can swiftly identify the optimal alternative even when a new option is added whose criteria values fall outside the range of existing alternatives' criteria values. This is a significant superiority over all current methods. The proposed method is applied to three numerical examples and subsequently to a typical example in the field of chemical extraction. All tests were rigorously evaluated by comparing the proposed method with other existing methods, thereby confirming the validity of the proposed approach.*

Keywords - *Multi-objective optimization, MCDM, DOE, PSI, Extraction.*

1. Introduction

Selecting the optimal alternative from a list of options based on multiple criteria is a complex task. This process is widely known as Multi-Criteria Decision-Making (MCDM) [1-3]. Today, a multitude of MCDM methods, each with distinct algorithms, have been applied across various fields, including economics, construction, medicine, education, energy, and management [4, 5]. However, a common characteristic of all existing MCDM methods is their limitation to ranking only the available alternatives to identify the optimal one. They do not account for scenarios where new alternatives might be added to the list during the ranking process.

A simple example can clarify this limitation: Suppose a person uses an MCDM technique to rank and choose the best car from ten available models on a dealership's website. This ranking is based on multiple criteria for each car. However, upon visiting the dealership, they might find 11 or more car models on display, beyond the ten initially considered. To determine the best car in this new scenario, the user would need to re-apply the MCDM technique to re-rank all the options. This is a tedious and inefficient process, especially when urgent decisions are required. Recognizing this drawback, a few recent studies have combined the Design of Experiments (DOE) method with MCDM techniques to facilitate rapid ranking when new alternatives are introduced. Examples of such research include the combination of DOE with the RAM method [6], PIV [7], MARCOS [8], and EDAS [9]. The integration of DOE and MCDM methods involves constructing a regression model for each alternative's score based on its evaluation parameters.

When a new alternative is added, the regression equation can be used to quickly calculate its score, allowing for a swift re-ranking of all options. Nevertheless, all MCDM methods used in the aforementioned studies (RAM, PIV, MARCOS, EDAS) require the user to assign weights to the criteria. This means that if the number of alternatives changes, the process of weighting the criteria must be performed again from the beginning. This significantly narrows the scope of application for the combination of DOE with these specific MCDM methods. This is a gap that needs to be filled. The aforementioned limitations from the literature review have opened up a new direction: if an MCDM method that does not require weight assignment to criteria is used, the applicability of the combination of DOE and MCDM could be significantly expanded.

The PSI method is one such MCDM approach that eliminates the need for criteria weighting [10]. Due to this advantage, PSI has been extensively used to rank alternatives and select the best option in many different fields [11-14]. The research problem here focuses on leveraging the advantages of the PIS method over all other methods.

A key benefit of PSI is that because it does not require weighting the criteria, changes in the number of alternatives to be ranked, and even the addition of an alternative whose criteria values fall outside the range of existing options-do not necessitate a re-calculation of criteria weights.

Leveraging this unique advantage of the PSI method, this study combines DOE with PSI to create a novel approach for selecting the optimal alternative, even when the number of options changes and even when a newly added option's criteria values fall outside the range of the corresponding criteria for existing alternatives. This is a completely new aspect compared to all published literature.

2. Proposed Method

The proposed method is based on a combination of Design of Experiments (DOE) and the Preference Selection Index (PSI) method, a multi-objective optimization technique that does not require criteria weighting. Therefore, we will first introduce the multi-objective optimization process using the PSI method.

Assume we have m alternatives from which we need to identify the optimal one. Each alternative is described by n parameters (or criteria). Criteria where a larger value is better are denoted by the letter 'B', and criteria where a smaller value is better are denoted by 'C'.

Let xij be the value of criterion j for alternative i, with j=1,…,n and i=1,…,m. The sequence for determining the optimal alternative using the PSI method is as follows [10]:

Calculate the normalized values using Equations (1) and (2).

$$n_{ij} = \frac{x_{ij}}{max x_{ij}} \; if \quad j \in B \tag{1}$$

$$n_{ij} = \frac{min\, x_{ij}}{x_{ij}} \; if \quad j \in C \tag{2}$$

Calculate the average value of the normalized data for each criterion using Equation (3).

$$n_{ij} = \frac{1}{m}\sum_{i=1}^{m} n_{ij} \tag{3}$$

Determine the preference value from the average value for each criterion using Equation (4).

$$\varphi_j = \sum_{i=1}^{m}\left[n_{ij} - n_j\right]^2 \tag{4}$$

Determine the deviation in the preference value for each criterion using Equation (5).

$$\phi_j = \left[1 - \varphi_j\right] \tag{5}$$

Determine the overall preference value for each criterion using Equation (6).

$$\beta_j = \frac{\phi_j}{\sum_{j=1}^{n}\phi_j} \tag{6}$$

Calculate the score for each alternative using Equation (7), then rank the alternatives based on the principle that the best alternative is the one with the highest score.

$$y_j = \sum_{j=1}^{n} n_{ij} \cdot \beta_j \tag{7}$$

The method proposed in this study is designed not only to quickly rank existing alternatives but also to handle the scenario where new alternatives are added to the list.

A crucial aspect is that the criteria values for these new alternatives can fall outside the range of the corresponding criteria for the existing options. Therefore, the proposed method is implemented in the following sequence:

- Step 1: Define the limiting values (maximum and minimum) for the criteria of the qualified alternatives under consideration for ranking.
- Step 2: Construct an experimental matrix where the input variables for the experimental process are all the parameters used to evaluate each alternative, based on the limiting values defined in Step 1.
- Step 3: Calculate the score for each experimental run using the PSI method.
- Step 4: Establish the relationship between the experimental score and the input parameters (criteria). This relationship is defined as a regression equation.
- Step 5: Use the regression equation to calculate the score for each alternative that needs to be ranked, including any new alternatives added to the list.
- Step 6: Rank the alternatives according to their scores.

3. Evaluation of the Proposed Method

To assess the effectiveness of the proposed method, we will test it through four different case studies in this section. The first three cases are based on numerical examples where the number of alternatives, the number of evaluation parameters, and the type of parameters (B-type criteria and C-type criteria) are intentionally varied.

This is done to create distinct scenarios for each example, with the goal of obtaining the most objective evaluation of the proposed method. In the fourth example, the proposed method is evaluated in a real-world problem involving the selection of an optimal solution for a chemical extraction process. The details of each case are clarified below.

3.1. Example 1

In this example, assume there are four alternatives, from A1 to A4, each described by four parameters, from C1 to C4. All four parameters are B-type criteria (larger is better), as summarized in Table 1.

Table 1. Data for example 1

Alt.	C1	C2	C3	C4
	B	B	B	B
A1	12	6.5	25	0.08
A2	13	6	25	0.12
A3	14	6	20	0.95
A4	12.5	5.8	26.5	0.10

As previously stated, the proposed method not only aims to identify the optimal alternative among a pre-existing set quickly, but also has a key feature, which is its ability to find the optimal solution when a new alternative is added, even if its criteria values fall outside the range of the existing alternatives' criteria values. In this example, this feature is further clarified as follows: For the four existing alternatives, the minimum value for criterion C1 is 12, for C2 is 5.8, for C3 is 20, and for C4 is 0.08. Suppose a new alternative is added to the list with C1, C2, C3, and C4 values that are all smaller than the minimum values of the existing alternatives. The task of identifying the optimal alternative must still be performed. However, it is crucial to determine which new alternative is qualified to be considered for comparison and which is not.

In other words, what are the minimum acceptable values for C1, C2, C3, and C4 that would allow a new alternative to be considered? Answering this question corresponds to Step 1 of the proposed method. In this case, let us assume a new alternative with C1, C2, C3, and C4 values that are smaller than the minimum values of the existing alternatives by 1, 0.5, 1.5, and 0.01, respectively, would still be considered.

Thus, the value ranges for each criterion for all alternatives (including the new one) would be: C1 $\in$[12-1,14], C2 $\in$[5.8-0.5,6.5], C3 $\in$[20-1.5,26.5], and C4 $\in$[0.08-0.01,0.12]. This can be simplified to: C1 $\in$[11,14], C2 $\in$[5.3,6.5], C3 $\in$[18.5,26.5], C4 $\in$[0.07,0.12].

Table 2. Experimental matrix for case 1

No.	C1	C2	C3	C4	y
#1	14	5.9	18.5	0.095	0.86444
#2	12.5	5.9	22.5	0.095	0.87027
#3	14	5.3	22.5	0.095	0.87348
#4	12.5	5.9	22.5	0.095	0.87027
#5	12.5	5.9	18.5	0.07	0.79963
#6	11	5.9	22.5	0.07	0.80546
#7	12.5	5.9	22.5	0.095	0.87027
#8	14	5.9	22.5	0.12	0.93508
#9	12.5	5.9	18.5	0.12	0.86695
#10	11	6.5	22.5	0.095	0.86706
#11	12.5	5.3	22.5	0.07	0.80867
#12	12.5	6.5	22.5	0.12	0.93187
#13	12.5	6.5	22.5	0.07	0.86455
#14	11	5.9	26.5	0.095	0.87610
#15	12.5	5.3	18.5	0.095	0.80535
#16	12.5	5.9	26.5	0.12	0.94091
#17	14	6.5	22.5	0.095	0.92936
#18	14	5.9	22.5	0.07	0.86776
#19	12.5	5.3	26.5	0.095	0.87931

#20	14	5.9	26.5	0.095	0.93840
#21	11	5.9	18.5	0.095	0.80214
#22	11	5.3	22.5	0.095	0.81118
#23	12.5	6.5	26.5	0.095	0.93519
#24	12.5	5.3	22.5	0.12	0.87599
#25	12.5	6.5	18.5	0.095	0.86123
#26	11	5.9	22.5	0.12	0.87278
#27	12.5	5.9	26.5	0.07	0.87359

This set of value ranges will be used to construct a Box-Behnken experimental matrix consisting of 27 experiments, as summarized in Table 2. The construction of a Box-Behnken matrix is a standard procedure easily found in experimental design literature [15, 16].

Applying formulas (1) through (7), the score for each experiment was calculated and is summarized in the last column of Table 2. From these scores, the regression equation was built as shown in formula (8).

$$y = -8.66367 \cdot 10^{-15} + 0.02076C_1 + 0.04657C_2 + 0.00924C_3 + 1.34639C_4 \quad (8)$$

The ranking of alternatives when a new one is added is a form of sensitivity analysis, which will be performed in Section 4 of this paper. For now, the alternatives in Table 1 are ranked using three methods: first, by the proposed method in this study; second, by the PSI method alone; and third, by using several other well-known ranking methods.

For the first method, we use formula (8) to calculate the scores and then rank the alternatives. For the second method, we apply formulas (1) through (7) to calculate the scores for each alternative using the PSI method and then rank them. For the third method, five other popular methods, TOPSIS, SAW, MOORA, RAM, and COPRAS, were also used to rank the alternatives. These five methods are widely used, and their documentation can be easily found in recent studies [17, 18]. When applying these five methods, we assume the weights of the criteria are equal. Table 3 summarizes all the calculated results for this case.

It is observed that the rankings are not entirely consistent across the different methods. This is a common occurrence reported in the literature because the algorithms within each method are different [19]. However, it is noteworthy that alternative A3 is identified as the optimal one by the proposed method, and this is perfectly consistent with the results from all other methods. Thus, in terms of effectiveness in identifying the optimal alternative, the proposed method demonstrates performance similar to that of existing methods.

Table 3. Ranking of alternatives in example 1 using different methods

Alt.	Proposed Method	PSI	TOPSIS	SAW	MOORA	RAM	COPRAS
A1	4	3	4	4	4	4	3
A2	2	2	3	2	2	2	2
A3	1	1	1	1	1	1	1
A4	3	4	2	3	3	3	4
Spearman	-	0.8	0.8	1	1	1	0.8

To evaluate the stability of the rankings across different methods, the Spearman's Rank Correlation Coefficient (hereafter referred to as the Spearman coefficient) was calculated using formula (9), where Di is the difference in rank for alternative i when ranked by different methods [20, 21].

$$S = 1 - \frac{6\sum_{i=1}^{m} D_i^2}{m(m^2-1)} \quad (9)$$

The last row of Table 3 summarizes the Spearman coefficient between the proposed method and each of the other methods. We observe that the Spearman coefficients between the proposed method and the existing methods are all very high, with the lowest being 0.8. This confirms a high degree of similarity in the rankings when using the proposed method compared to the PSI, TOPSIS, and COPRAS methods [22]. It is particularly noteworthy that the rankings produced by the proposed method are perfectly consistent with those from the SAW, MOORA, and RAM methods.

All these findings affirm that the proposed method demonstrates accuracy in this example.

3.2. Example 2

In this example, we assume there are five alternatives, A1 to A5, that need to be ranked. Each alternative is described by three criteria, C1 to C3, all of which are C-type parameters (smaller is better), as shown in Table 4.

In this case, it is also assumed that, in addition to the five existing alternatives in Table 4, new alternatives may be added whose criteria values could fall outside the corresponding ranges of the criteria listed in Table 4.

The question then arises: what are the maximum acceptable values for C1, C2, and C3 for a new alternative to be considered? In this scenario, we assume a new alternative will be considered if its C1, C2, and C3 values are greater than the maximum values of the existing alternatives by 3, 1, and 0.5, respectively.

Thus, the value range for each criterion across all alternatives, including any new ones, would be: C1 ∈[51,57+3], C2 ∈[12.5,14.2+1], and C3 ∈[6.2,7+0.5]. This can be simplified to: C1 ∈[51,60], C2 ∈[12.5,15.2], and C3 ∈[6.2,7.5]. This set of parameter ranges was used to construct an experimental matrix comprising 15 experiments, as detailed in Table 5.

Table 4. Data for example 2

Alt.	**C1**	**C2**	**C3**
	C	**C**	**C**
A1	57	12.5	6.8
A2	55	14	6.2
A3	51	13.8	6.5
A4	57	14.2	7
A5	54	14	7

Following a procedure similar to Example 1, the score for each experiment was calculated and is summarized in the last column of Table 5, from which the regression equation was developed as shown in formula (10).

Following the same process as in Example 1, the alternatives in Table 4 were ranked, and the results are summarized in Table 6.

Table 5. Experimental matrix for case 2

No.	**C1**	**C2**	**C3**	**y**
#1	55.5	12.5	7.5	0.91512
#2	60	13.85	6.2	0.91716
#3	55.5	15.2	6.2	0.91389
…	…	…	…	…
#15	51	15.2	6.85	0.90974

$$y = 2.74203 - 0.01686C_1 - 0.06529C_2 - 0.13276C_3 + 0.00010C_1^2 + 0.00157C_2^2 + 0.00646C_3^2 \quad (10)$$

Table 6. Ranking of alternatives in example 2 using different methods

Alt.	**Proposed Method**	**PSI**	**TOPSIS**	**SAW**	**MOORA**	**RAM**	**COPRAS**
A1	3	3	3	3	3	3	3
A2	2	2	2	2	2	2	2
A3	1	1	1	1	1	1	1
A4	5	5	5	5	5	5	5
A5	4	4	4	4	4	4	4
Spearman	-	1	1	1	1	1	1

In this example, the rankings of all alternatives are perfectly consistent across all methods used, including the one proposed in this study. Naturally, the Spearman’s Rank Correlation Coefficient between the proposed method and each of the other methods is equal to 1. In summary, this example further confirms the accuracy and validity of the proposed method.

3.3. Example 3

In this example, we assume there are four alternatives, A1 to A4, with each alternative described by four parameters: two B-type criteria (C1 and C2) and two C-type criteria (C3 and C4), as summarized in Table 7.

Once again, in this case, we assume that in addition to the four existing alternatives in Table 7, a new alternative might be added to the ranking list. Let us assume the new alternative has C1 and C2 values that are lower than the minimum existing values in Table 7, and C3 and C4 values that are higher than the maximum existing values. Specifically, we assume the new alternative has C1 and C2 values that are lower than the minimum existing values by 2 and 1 units, respectively, and C3 and C4 values that are higher than the maximum existing values by 2.5 and 0.15 units, respectively.

Table 7. Data for example 3

Alt.	**C1**	**C2**	**C3**	**C4**
	B	**B**	**C**	**C**
A1	29	17	55	0.8
A2	32	16	53	0.75
A3	30	16	56	0.7
A4	34	15	57	0.9

Thus, the value ranges for each criterion across all alternatives, including the new one, would be: C1 ∈[29-2,34], C2 ∈[15-17], C3 ∈[53,57+2.5], and C4 ∈[0.7,0.9+0.15]. This can be simplified to: C1 ∈[27,34], C2 ∈[14,17], C3 ∈[53,59.5], and C4 ∈[0.7,1.05]. This set of parameter ranges was used to construct an experimental matrix comprising 27 experiments, as detailed in Table 8.

Table 8. Experimental matrix for case 3

Alt.	C1	C2	C3	C4	y
#1	34	15.5	59.5	0.875	0.90679
#2	30.5	14	56.25	0.7	0.91021
#3	34	15.5	56.25	1.05	0.89554
…	…	…	…	…	…
#26	27	15.5	56.25	0.7	0.90731
#27	27	14	56.25	0.875	0.84506

Following a procedure similar to Example 1, the score for each experiment was calculated and is summarized in the last column of Table 8, from which the regression equation was developed as shown in formula (11). Following the same process as in Example 1, the alternatives in Table 7 were ranked, and the results are summarized in Table 9.

$$y = 1.28057 + 0.00754C_1 + 0.01567C_2 - 0.01428C_3 - 0.55354C_4 + 8.46564 \cdot 10^{-5}C_3^2 + 0.21087C_4^2 \quad (11)$$

Table 9. Ranking of alternatives in example 3 using different methods

Alt.	Proposed Method	PSI	TOPSIS	SAW	MOORA	RAM	COPRAS
A1	3	3	3	3	3	3	3
A2	1	1	1	1	1	1	1
A3	2	2	2	2	2	2	2
A4	4	4	4	4	4	4	4
Spearman	-	1	1	1	1	1	1

In this example, we also observe that the rankings of all alternatives are perfectly consistent across all methods used, including the one proposed in this study.

Naturally, the Spearman's Rank Correlation Coefficient between the proposed method and each of the other methods is equal to 1. In summary, this example further affirms the accuracy and validity of the proposed method.

3.4. Example 4

The three numerical examples above have demonstrated that the proposed method is as effective as existing methods. In this example, we address a real-world problem: selecting the best solution for extracting Trametes versicolor mushrooms. Table 10 summarizes fifteen different extraction scenarios for Trametes versicolor [23].

Each scenario was conducted by varying the drying temperature, carrier content, and feed rate. Six criteria were used to evaluate each alternative: yield (%), powder moisture content (%), ability to reduce antioxidant capacity (%), ability to reduce TPC (%), ability to reduce TFC (%), and ability to reduce TTC (%), denoted as C1 to C6. Among these, only C1 is a B-type criterion (larger is better), while the others are C-type criteria (smaller is better). The data for this example are compiled in Table 10.

Table 10. Data for example 4

Alt.	C1	C2	C3	C4	C5	C6
	B	C	C	C	C	C
A1	28.87	3.72	9.69	4.09	2.61	1.1
A2	50.37	3.32	11.3	5.41	3.29	1.83
A3	29.78	3.98	9.92	4.87	2.83	1.25
A4	58.06	2.28	9.43	4.49	2.42	1.02
A5	56.72	2.56	13.49	8.51	5.49	2.74
A6	57.89	2.29	9.96	4.37	2.89	1.43
A7	49.06	3.89	10.46	5.42	3.06	2.46
A8	46.81	3.02	11.35	6.29	3.35	2.26
A9	39.45	3.79	9.98	4.9	2.98	1.58
A10	57.82	3.05	11.37	6.27	3.86	2.35
A11	59.26	2.47	9.53	4.76	2.35	1.24
A12	29.76	3.82	10.17	5.05	2.81	2.01
A13	54.58	2.95	14.32	8.32	5.87	3.43

A14	31.41	3.52	10.02	5.02	3.11	0.94
A15	42.13	3.67	13.21	8.21	5.19	2.99

Once again, in this case, we assume that a new alternative may be added to the ranking list beyond the 15 existing options in Table 10. To create more diversity in the examples, the expansion of the criteria value range is performed differently here. Specifically, in Table 11, the criteria value ranges are C1 ∈[28.87,59.26], C2 ∈[2.28,3.98], C3 ∈[9.43,14.32], C4 ∈[4.09,8.51], C5 ∈[2.35,5.87], C6 ∈[0.94,3.43]. Now, the range is expanded by selecting values both greater and smaller than the limiting values of the criteria in Table 11. Specifically, alternatives with the following criteria values will be accepted for inclusion in the ranking list: C1 ∈[25,65], C2 ∈[2,4.5], C3 ∈[8,18], C4 ∈[3.5,10], C5 ∈[2,7], C6 ∈[0.8,5]. This new set of criteria was used to construct an experimental matrix of 54 experiments, as detailed in Table 11. Following a procedure similar to Example 1, the score for each experiment was calculated and is summarized in the last column of Table 11, from which the regression equation was developed as shown in formula (12).

Table 11. Experimental matrix for example 4

No.	C1	C2	C3	C4	C5	C6	y
#1	65	4.5	13	3.5	4.5	2.9	0.53953
#2	65	2	13	10	4.5	2.9	0.48605
#3	25	3.25	18	6.75	4.5	0.8	0.66708
#4	45	2	18	6.75	2	2.9	0.58341
…	…	…	…	…	…	…	…
#53	45	3.25	8	10	4.5	0.8	0.71920
#54	45	2	8	6.75	7	2.9	0.47768

$$y = 1.51727 + 0.00160C_1 - 0.05488C_2 - 0.01372C_3 - 0.04579C_4 - 0.09088C_5 - 0.21977C_6 + 0.00562C_2^2 + 0.00035C_3^2 + 0.00226C_4^2 + 0.00673C_5^2 + 0.02526C_6^2 \quad (12)$$

Following a similar procedure to Example 1, the alternatives in Table 10 were ranked, and the results are summarized in Table 12.

In this case, all methods used were consistent in identifying A4 as the optimal alternative (rank 1), A11 as rank 2, and A6 as rank 3. Therefore, in terms of finding the optimal alternative among the available options, the proposed method is evaluated as being equivalent to the other existing methods. Although the Spearman's coefficients between the proposed method and the MOORA and COPRAS methods are relatively low, at 0.4786 and 0.3964, respectively, the top-ranked alternatives (A4), rank 2 (A11), and rank 3 (A6) are consistently identified by all three of these methods, as well as the remaining alternatives. Furthermore, the Spearman's coefficients between the proposed method and the PSI, TOPSIS, SAW, and RAM methods are all very high, at 0.9750, 0.9821, 0.9714, and 0.9500, respectively. This indicates that the proposed method's performance is comparable to these methods. In summary, this example also proves the effectiveness of the proposed method.

Table 12. Ranking of alternatives in example 4 using different methods

Alt.	Proposed Method	PSI	TOPSIS	SAW	MOORA	RAM	COPRAS
A1	4	4	4	4	11	4	12
A2	8	8	7	8	5	6	5
A3	6	7	8	6	13	8	13
A4	1	1	1	1	1	1	1
A5	13	13	13	13	8	13	7
A6	3	3	3	3	3	3	3
A7	10	9	11	9	6	10	6
A8	11	12	10	11	7	11	8
A9	7	6	6	7	9	7	9
A10	12	10	12	10	4	9	4
A11	2	2	2	2	2	2	2
A12	9	11	9	12	14	12	14
A13	15	14	14	14	12	14	10

A14	5	5	5	5	10	5	11
A15	14	15	15	15	15	15	15
Spearman	-	0.9750	0.9821	0.9714	0.4786	0.9500	0.3964

The four examples above have all demonstrated the effectiveness of the proposed method in each scenario. However, in all four cases, the evaluation has only focused on the method's performance in ranking the existing alternatives. The next section will be dedicated to evaluating the effectiveness of the proposed method when a new alternative is added to the ranking list, especially when the criteria of the new alternative may fall outside the range of the corresponding criteria for the existing alternatives. This is a form of sensitivity analysis and will be presented in the next part of this paper.

4. Sensitivity Analysis

In this section, a sensitivity analysis will be performed for each of the four cases previously examined. The process of adding new alternatives to the ranking list in each example is also intentionally varied to ensure the most objective evaluation of the proposed method. The details for each case are presented below.

For Example 1, we add alternative A5 to the ranking list, as shown in Table 13. Alternative A5 has a C1 value of 11, which is the lowest C1 value among all alternatives, and a C2 value of 5.3, which is also the lowest C2 value. Its C3 value is chosen as 24 ∈[20,26.5], and its C4 value is 0.9 ∈[0.08,0.12]. It is important to note that the criteria values for A5 must satisfy the conditions set in section 3.1, specifically C1 ∈[11,14], C2 ∈[5.3,6.5], C3 ∈[18.5,26.5], and C4 ∈[0.07,0.12]. Using formula (8), we calculate the scores for alternatives A1 to A5 and rank them using the proposed method. The alternatives were also ranked using the PSI, TOPSIS, SAW, MOORA, RAM, and COPRAS methods.

Table 13. Data for Example 1 after adding A5

Alt.	C1	C2	C3	C4
A1	12	6.5	25	0.08
A2	13	6	25	0.12
A3	14	6	20	0.95
A4	12.5	5.8	26.5	0.10
A5	11	5.3	24	0.9

All results are summarized in Table 14. The Spearman's Rank Correlation Coefficient was calculated using formula (9) and is also included in the last row of Table 14.

Table 14. Ranking of alternatives for example 1 after adding A5

Alt.	Proposed Method	PSI	TOPSIS	SAW	MOORA	RAM	COPRAS
A1	5	5	5	5	5	4	5
A2	3	3	4	3	3	2	3
A3	1	1	1	1	1	1	1
A4	4	4	3	4	4	3	4
A5	2	2	2	2	2	5	2
Spearman	-	1	0.9	1	1	0.4	1

We observe that all methods consistently rank A3 as 1st, A5 as 2nd, and A1 as 5th. Furthermore, the Spearman's coefficients between the proposed method and the PSI, TOPSIS, SAW, MOORA, and COPRAS methods are all very high. This demonstrates a high degree of consistency in the rankings produced by the proposed method compared to those from existing methods. It is crucial to note that even with the addition of A5, ranking the alternatives using the proposed method only requires the single use of the regression Equation (8), whereas other methods would necessitate a complete re-calculation from scratch. In summary, the proposed method's effectiveness is confirmed in this case. For Example 2, we add alternative A6 to the ranking list, as shown in Table 15. Alternative A6 has a C1 value of 60, which is the highest C1 value among all alternatives, a C2 value of 12.5, which is the lowest C2 value, and a C3 value of 6.2, which is also the lowest C3 value. Again, the criteria values for A6 must meet the conditions mentioned in section 3.1, which are C1 ∈[51,60], C2 ∈[12.5,15.2], and C3 ∈[6.2,7.5].

Table 15. Data for example 2 after adding A6

Alt.	C1	C2	C3
A1	57	12.5	6.8
A2	55	14	6.2
A3	51	13.8	6.5
A4	57	14.2	7
A5	54	14	7
A6	60	12.5	6.2

Using formula (10), we calculate the scores for alternatives A1 to A6 and rank them using the proposed method. The alternatives were also ranked using the PSI, TOPSIS, SAW, MOORA, RAM, and COPRAS methods.

All results are summarized in Table 16. he Spearman's coefficient was calculated using formula (9) and is also included in the last row of Table 16.

Table 16. Ranking of alternatives for example 2 after adding A6

Alt.	Proposed Method	PSI	TOPSIS	SAW	MOORA	RAM	COPRAS
A1	4	4	4	4	4	4	4
A2	3	3	2	3	3	3	3
A3	1	1	1	1	1	1	1
A4	6	6	6	6	6	6	6
A5	5	5	5	5	5	5	5
A6	2	2	3	2	2	2	2
Spearman	-	1	0.9429	1	1	1	1

In this case, we also find that the rankings for alternatives A1 to A6 are perfectly consistent when ranked by the proposed method compared to the PSI, SAW, MOORA, RAM, and COPRAS methods. There is only a very small difference in the ranks when compared to the TOPSIS method, but the Spearman's coefficient between these two methods is still very high at 0.9429. This indicates that the two methods have similar performance. A key takeaway is that even with the addition of A6, ranking the alternatives with the proposed method only requires the single use of the regression Equation (10), while other methods require re-calculation from the beginning. In short, the proposed method's effectiveness is also confirmed in this case.

For Example 3, we add alternative A5 to the ranking list. The criteria values for A5 must satisfy the conditions stated in section 3.3: C1 ∈[27,34], C2 ∈[14,17], C3 ∈[53,59.5], and C4 ∈[0.7,1.05]. The data for Example 3, after adding A5, is summarized in Table 17.

Table 17. Data for example 3 after adding A5

Alt.	C1	C2	C3	C4
A1	29	17	55	0.8
A2	32	16	53	0.75
A3	30	16	56	0.7
A4	34	15	57	0.9
A5	27	14	53	0.7

Using formula (11), we calculate the scores for alternatives A1 to A5 and rank them using the proposed method. The alternatives were also ranked using the PSI, TOPSIS, SAW, MOORA, RAM, and COPRAS methods. All results are summarized in Table 18. The Spearman's coefficient was calculated using formula (9) and is included in the last row of Table 18.

Table 18. Ranking of alternatives for example 3 after adding A5

Alt.	Proposed Method	PSI	TOPSIS	SAW	MOORA	RAM	COPRAS
A1	3	3	3	3	3	3	3
A2	1	1	1	1	1	1	1
A3	2	2	2	2	2	2	2
A4	4	5	5	5	5	4	5
A5	5	4	4	4	4	5	4
Spearman	-	0.9	0.9	0.9	0.9	1	0.9

Once again, we observe that the rankings of alternatives A1 to A5 show a high degree of consistency when ranked by the proposed method compared to the other methods. The Spearman's coefficients between the proposed method and the others are all very high, with the lowest being 0.9, further reinforcing the effectiveness of the proposed method in this case. It is worth reiterating that with the addition of A5, the proposed method only requires the single use of the regression Equation (11) for ranking, whereas other methods require re-calculation from scratch. For Example 4, we add alternative A16 to the ranking list. Its criteria values must satisfy the conditions set in section 3.4: C1 ∈[25,65], C2 ∈[2,4.5], C3 ∈[8,18], C4 ∈[3.5,10], C5 ∈[2,7], and C6 ∈[0.8,5]. Table 19 summarizes the data for Example 4 after adding alternative A16.

Table 19. Data for example 4 after adding A16

Alt.	C1	C2	C3	C4	C5	C6
A1	28.87	3.72	9.69	4.09	2.61	1.1
A2	50.37	3.32	11.3	5.41	3.29	1.83
A3	29.78	3.98	9.92	4.87	2.83	1.25
A4	58.06	2.28	9.43	4.49	2.42	1.02
A5	56.72	2.56	13.49	8.51	5.49	2.74
A6	57.89	2.29	9.96	4.37	2.89	1.43
A7	49.06	3.89	10.46	5.42	3.06	2.46

A8	46.81	3.02	11.35	6.29	3.35	2.26
A9	39.45	3.79	9.98	4.9	2.98	1.58
A10	57.82	3.05	11.37	6.27	3.86	2.35
A11	59.26	2.47	9.53	4.76	2.35	1.24
A12	29.76	3.82	10.17	5.05	2.81	2.01
A13	54.58	2.95	14.32	8.32	5.87	3.43
A14	31.41	3.52	10.02	5.02	3.11	0.94
A15	42.13	3.67	13.21	8.21	5.19	2.99
A16	40	3	12	6	4	2

Using formula (12), we calculate the scores for alternatives A1 to A16 and rank them using the proposed method. The alternatives were also ranked using the PSI, TOPSIS, SAW, MOORA, RAM, and COPRAS methods. All results are summarized in Table 20. The Spearman's coefficient was calculated using formula (9) and is included in the last row of Table 20.

In this case, all methods consistently identify A4 as the optimal alternative (rank 1), A11 as rank 2, and A6 as rank 3. Thus, in terms of finding the optimal alternative, even with the addition of A16, the proposed method is evaluated as being equivalent to the other existing methods. Although the Spearman's coefficients between the proposed method and the MOORA and COPRAS methods are relatively low at 0.4676 and 0.4029, the top three ranked alternatives (A4, A11, A6) are consistently identified by all three of these methods and the rest. Moreover, the Spearman's coefficients between the proposed method and the PSI, TOPSIS, SAW, and RAM methods are very high, at 0.9676, 0.9794, 0.9676, and 0.9471, respectively. This shows that the proposed method's performance is comparable to these methods. It is again worth noting that with the addition of A16, ranking the alternatives using the proposed method only requires the single use of the regression Equation (12), while other methods would require re-calculation from scratch. In conclusion, the proposed method's effectiveness is confirmed in this case as well.

Table 20. Ranking of alternatives for example 4 after adding A16

Alt.	Proposed Method	PSI	TOPSIS	SAW	MOORA	RAM	COPRAS
A1	4	4	5	4	12	4	12
A2	8	10	7	8	5	6	5
A3	6	5	8	6	14	8	14
A4	1	1	1	1	1	1	1
A5	14	14	14	14	8	14	7
A6	3	3	3	3	3	3	3
A7	10	9	11	9	6	10	6
A8	11	11	10	11	7	11	8
A9	7	6	6	7	9	7	9
A10	13	12	12	10	4	9	4
A11	2	2	2	2	2	2	2
A12	9	7	9	12	15	12	15
A13	16	16	15	15	13	15	10
A14	5	8	4	5	10	5	11
A15	15	15	16	16	16	16	16
A16	12	13	13	13	11	13	13
Spearman	-	0.9676	0.9794	0.9676	0.4676	0.9471	0.4029

The four examples performed in section 3 all confirmed that the proposed method is as effective as other methods in ranking existing alternatives, meaning the rankings produced by the proposed method are highly similar to those from other methods. Notably, in section 4, we added new alternatives to the ranking list for each example, and this was done using different scenarios. In this section, the rankings produced by the proposed method also showed high similarity to those from other methods. The key advantage, however, is that ranking with the proposed method is done swiftly by simply calculating scores via a regression equation, whereas with any other method, the ranking process must be restarted from the beginning whenever an alternative is added to the list. This characteristic is considered a major distinction and a significant leap forward for the proposed method compared to existing ones.

5. Conclusion

Selecting the optimal alternative from a list of options based on multiple criteria is a complex task. This complexity increases significantly when a need arises to add new alternatives to the ranking list during the process. This study proposed a straightforward method that not only quickly ranks existing alternatives but also efficiently re-ranks the list when new options are added.

The key to the proposed method's novelty lies in the skillful combination of the Design of Experiments (DOE) methodology with the Preference Selection Index (PSI) multi-objective optimization method, a technique that does not require criteria weighting. In the future, combining DOE with other multi-objective optimization methods that do not require criteria weighting, such as the CURLI or R methods, could be explored to provide additional useful tools for rapidly ranking alternatives in various fields.

Author Contribution Statement

NHS and DXT conceived and designed the research. NHS and VTNU conducted experiments. BVB and NCB contributed new reagents or analytical tools. NHS and HNT analyzed data. NHS wrote the manuscript. All authors read and approved the manuscript.

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