The Mathematics of The Tympanic Membrane As An Acoustics Device

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Abstract

Hearing loss and hearing sensitivity to Frequencies have occupied the front burner in many recent researches. Frequencies are thought to affectthe ear; including the Vibration of the Tympanic membrane and the oscillation of the middle bones. The transduction of sound waves into electrical signals and many more. In this work, the Vibrational modes of the Tympanic membranewere derived. The Tympanic membrane was viewed as a circular Vibrating homogeneous membrane of Radius(a)under a Tension (T) and having a linear mass Density ρ*. The Vibrational Frequency of the membrane was found to be directly proportional to the square root of the Tension* T*, inversely proportional to the square root of the Density* ρ*, and inversely proportional to the Radius* (a)*. This suggests that the vibrational Frequencies of the membrane are high for a small Radius which suggestsa high sensitivity to high Frequency sound waves and Hearing loss at low Frequencies.*

Keywords: *Hearing loss, Vibrations, Radius, Tension, Frequency, Tympanic membrane and Density.*

I. INTRODUCTION

Hearing is one of the major senses and like vision it is important for distant warning and communication. It is a conscious appreciation of vibration perceived as sound. It can be used to alert and to communicate pleasure and fear. The human ear serves as a transducer, converting sound energy into mechanical energy and then mechanical energy into a nerve impulse which is transmitted to the brain. The ear's ability to do this allows one to perceive the pitch of sounds by detection of the wave's frequencies. The ears detect loudness of sound by detecting the wave's amplitude and the timbre of the sound at various frequencies which make up a complex sound wave [1]. Most hearing loss problems are traceable to the vibrations of the tympanic membrane. From literatures, some tympanic membranesareperforated while others have density outside the normal range giving rise to abnormal vibrational frequencies of the tympanic membrane. Some of the problems could either be from the outer ear, middle ear or inner Ear. It could also be the fault of either of all these.The pinna, which is the visible portion of the ear, is composed of cartilage and attaches to the skull on the temporal bone above the external auditory canal, which is commonly referred to as the ear canal. The auditory ossicles, which include the malleus, incus, and stapes, are the smallest bones in the body and are essential in conducting sound from the tympanic membrane to the inner ear [2]. The middle ear, also known as the tympanic cavity, is an air-filled compartment surrounded by the temporal bone. It is lined with a mucosal skin layer that extends over the tympanic membrane, creating the innermost layer of the tympanic membrane [3].The inner portion of the ear is responsible for transforming mechanical vibrations produced by a sound into neural impulses which can be interpreted by the brain. The structure of the inner ear that is most essential to this process is the cochlea. The cochlea's function is to convert sound waves to neural signals, but it is also responsible for breaking down Acoustical waveforms into more simplistic components [4]. The human cochlea is approximately 10mm wide and forms a shape which appears similar to the shell of a snail. The cochlea is a tube which is roughly 35mm long if outstretched [5]. The round window and the oval window are each covered by membranes and are located at the base of the cochlea. These membranes are the basilar membrane and the tectorial membrane respectively. Their functions are to vibrate and send pressure waves into the cochlea where approximately 30,000 hair follicles are located. These hair follicles convert vibrations to nervous impulses, which are gathered by approximately 19,000endings and these endings transfer the nerve impulses to the brain [6]. The human tympanic membrane(TM) also called eardrum is a soft tissue membrane which separates the outer ear from the Meddle Ear (ME). It is placed in the ear canal with a particular inclination, which allows it to have a bigger surface than the ear canal section itself [7]. The angle between the eardrum and the superior and posterior wall of the ear canal is 140°, while the angle between the eardrum and the inferior and anterior wall is 30° [8].It is slightly conically shaped with the apex pointing medially towards the ME. The sides of the cone formed by the eardrum are convex outwards. Its vertical axis ranges from 8.5 to 10.0 mm while the horizontal axis ranges from 8.0 to 9.0 mm and the total area is 85.0 mm^2 while the physiologically active area is 55.0 mm^2 [9]. It varies in elasticity and thickness, being thicker in the centre and in the periphery than in the intermediate zone, with an average thickness of 74 µm (minimal thickness: ~50 µm, maximal thickness: ~100 µm) [10].Macroscopic observation reveals the existence of three different parts: pars tensa, pars flaccida and annular ligament. The differences in the pars tensa and pars flaccida lie in the structure of their lamina propria. The pars tensa is the inferior part and is also the most extended part, its lamina propria consists of two sub epidermal connective tissue layers, between which there are two collagenous layers with radial (outer) and circular (inner) fibre orientation [11]. The pars flaccida is the superior part of the TM, its lamina propria is made up with loose connective tissue containing collagen and elastic fibres. The abundance of elastic fibres may account for its flaccid nature. The tympanic membrane is approximately 10mm in diameter and 80 micrometres in thickness on average in human adults,it is instrumental in converting sound pressure waves into mechanical vibrations [12].The normal range of human hearing is from 0 to 100 dB(A), before sound becomes uncomfortably loud.

The TM has been the subject of many studies, focusing mainly on its structure, composition, and mechanical properties.The tympanic membrane is primarily conical in shape with the apex forming an umbo [13]. The TM is divided into two main sections; the pars flaccidaa membrane, which is the upper level of the tympanic membrane which can be identified by a small inverted triangle which is above the short process of the malleus, and the pars tensa is the lower portion of the tympanic membrane which is located around the umbo [14].It is very important if experiments are conducted in the long term as data can be recorded with pictures at time intervals, as was done to view the TM healing time in rats when a perforation was made in the membrane [15]. The resolution of images captured with microscopes can be very high, on the order of submicron. The contrast, however, suffers from backscattering of light or over illumination depending on the type of microscopy being utilized. Direct microscopic observation is incapable of quantitatively measuring internal structures of samples[16].

The tympanic membrane was viewed as a rectangular model, assuming the tympanic membrane is homogeneous and that one point on the membrane can move in only one direction. The tympanic membrane has an approximately constant volume density, somewhere between that of water (1.0gcm^{-3}) and that of undehydrated collagen (1.2 gcm^{-3}) [17]. A perforated eardrum is considered as a hole in the tympanic membrane as a common consequences of ear injury or infection. A perforated eardrum is often accompanied by decreased hearing and occasional discharge. Pain is usually persistent. The causes of perforated eardrum are usually from trauma (injury) or infection such as when the ear is struck squarely with an open hand or a skull fracture or after a sudden explosion[18].

This work aimed to obtain an expression for normal modes of operation of the tympanic membrane. The objectives are;to consider the tympanic membrane as a circular model with a radius, R; to generate an expression for the normal vibration of the tympanic membrane; to use the mathematical expressions to bring out important parameters of the tympanic membrane and to use the parameter above to guide audiologists, acousticians, surgeons, etc.

II. METHOD

In this work, the basic assumptions are; the mass of the membrane per unit area (density) is constant; that is, the tympanic membrane is homogenous. The tympanic membrane is perfectly elastic and does not offer any resistance to bending; the effect of gravitational force on the membrane is negligible. The Motion of the membrane is small (transverse vibration); hence, the membrane is fixed at its circular boundary and therefore, the transverse displacement is zero at the boundary i.e. $u(a, \theta, t); r = a$.

2D WAVE EQUATION

Starting the derivation with the Newton second law of motion:

$$F = ma = m\frac{\partial v}{\partial t} = m\frac{\partial^2 u}{\partial t^2} \qquad (1)$$

Density(ρ) of the Membrane is define as mass per unit area express in Kg/m^2.

$$\rho = \frac{m}{A} \quad (2)$$

Tension of the Membrane is the force F per unit length.

$$T = Fdl$$

$$T = m\frac{\partial^2 u}{\partial t^2}dl \quad (3)$$

$$F = \rho\frac{\partial^2 u}{\partial t^2}dxdz \qquad (4)$$

The net force on the element $dxdz$ due to the pair of tensions Tdz is:-

$$Tdz\left[\left(\frac{\partial u}{\partial x}\right)_{x+dx} - \left(\frac{\partial u}{\partial x}\right)_{x}\right] = T\frac{\partial^2 u}{\partial x^2}dxdz \quad (5)$$

And that due to the pair of tension given as

$$Tdx = T\frac{\partial^2 u}{\partial z^2}dxdz \quad (6)$$

Equating the sum of equation (5) and (6) to the product of the element's mass $\rho dxdz$ by its acceleration $\frac{\partial^2 u}{\partial t^2}$ gives

$$T\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right)dxdz = \rho dxdz\frac{\partial^2 u}{\partial t^2} \quad (7)$$

$$\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) = \frac{\rho}{T}\frac{\partial^2 u}{\partial t^2} \quad (8)$$

$$\frac{1}{c^2} = \frac{\rho}{T} \quad (9)$$

$$\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \quad (10)$$

This is the **$2D$ wave equation** of a stretch membrane is in rectangular coordinate.
In terms of its **$3D$ rectangular polar coordinate**, it is given as:

$$\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \quad (11)$$

Equations (10) and (11) are time dependent that is in transient state which is called **Poison's equations** and it can be reduced to Laplace's equation where the time derivative $\frac{\partial^2 u}{\partial t^2} = 0$, that is, in a steady state.

To transform the rectangular polar coordinate to spherical coordinate, the spherical transformation operators was used which is given below.

TRANSFORMATION OF LAPLACE'S EQUATION

The Cartesian form of Laplace's equation for the stretch membrane in 3D was derived from the Newton second law of motion and was obtain as:

$$c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 0$$

$\frac{\partial^2 u}{\partial t} = 0$ and u is Satisfies.

The Laplace's equation can be transformed to spherical polar form by using the Transformation

$$x = r\cos\theta \quad (12a)$$
$$y = r\sin\theta \quad (12b)$$
$$z = z \quad (12c)$$

So that, $r^2 = x^2 + y^2$ and $\tan\theta = \frac{y}{x}$,

And $\frac{\partial r}{\partial x} = \frac{x}{r} = \cos\theta$, $\frac{\partial r}{\partial y} = \frac{y}{r} = \sin\theta$

$$\frac{\partial \theta}{\partial x} = -\frac{\sin\theta}{r},\ \frac{\partial \theta}{\partial y} = \frac{\cos\theta}{r} \quad (13)$$

By the way of transforming for the x-component we have:

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial u}{\partial \theta}\frac{\partial \theta}{\partial x} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial x} \quad (14)$$

Substituting the values of the transforms into equation 14, it becomes:

$$\frac{\partial u}{\partial x} = \left(\frac{\partial u}{\partial r}\cos\theta + \frac{\partial u}{\partial \theta}\left(-\frac{\sin\theta}{r}\right)\right) \quad (15)$$

Factor u, we have:

$$\frac{\partial u}{\partial x} = u\left[\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\right] \quad (16)$$

This equation become:

$$\frac{\partial}{\partial x} = \cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial \theta} \quad (17)$$

Equation 18 is called the operator equation in the x-direction

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) \quad (18)$$

$$\frac{\partial^2 u}{\partial x^2} = \left(\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\right)\left(\frac{\partial u}{\partial r}\cos\theta + \frac{\partial u}{\partial \theta}\left(-\frac{\sin\theta}{r}\right)\right) \quad (19)$$

With similar expression for $\frac{\partial^2 u}{\partial y^2}$ and $\frac{\partial^2 u}{\partial z^2}$, the Cylindrical coordinate is obtained as

$$\boldsymbol{\nabla^2 u = c^2\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)} \quad (20)$$

The equation above is the wave equation for a circular membrane. Where $\nabla^2 u = 0$, the 3D circular equation reduces to Laplace's equation.

THE TYMPANIC AS A CIRCULAR MEMBRANE OF RADIUS *R*

In circular coordinate *(r, θ, t)*

$$\boldsymbol{\nabla^2 y = \frac{\partial^2 y}{\partial r^2} + \frac{1}{r}\frac{\partial y}{\partial r} + \frac{1}{r^2}\frac{\partial^2 y}{\partial \theta^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}} \quad (21)$$

From equation (21), the membrane is assume to be thin and independent of the z coordinate

$$y(a, \theta, t) = 0,\ \ r = a$$

In other to solve (21); assuming a harmonic solution which is a function of (r, θ, t).

$$\boldsymbol{y(a,\theta,t) = 0}, \qquad \boldsymbol{r = a}$$

$$\boldsymbol{y(r,\theta,t) = R(r)\Theta(\theta)e^{i\omega t}} \qquad (22)$$

This is a product solution which can be solved using separation of variables. Differentiating equation (22) twice with respect to time gives: -

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2 R(r)\Theta(\theta)e^{i\omega t}$$

Taking

$$y = \varphi e^{i\omega t} \qquad (23)$$

$$\varphi(r,\theta) = R(r)\Theta(\theta) \qquad (24)$$

Equation (23) becomes

$$y = \varphi(r,\theta)e^{i\omega t} \qquad (25)$$

Putting (24) into (25),

$$y = R(r)\Theta(\theta)e^{i\omega t} \qquad (26)$$

Putting equation (26) into (21)

$$\Theta e^{i\omega t}\frac{d^2R}{dr^2} + \frac{\Theta e^{i\omega t}}{r}\frac{dR}{dr} + \frac{Re^{i\omega t}}{r^2}\frac{d^2\Theta}{d\theta^2} = R\Theta e^{i\omega t} \qquad (27)$$

Divide through by $e^{i\omega t}$

$$\Theta\frac{d^2R}{dr^2} + \frac{\Theta}{r}\frac{dR}{dr} + \frac{R}{r^2}\frac{d^2\Theta}{d\theta^2} = \frac{-\omega^2}{c^2}R\Theta = 0 \qquad (28)$$

Recall,

$$\boldsymbol{k = \frac{\omega}{c};\ k^2 = \frac{\omega^2}{c^2}}$$

Equation (28) becomes

$$\Theta\frac{d^2R}{dr^2} + \frac{\Theta}{r}\frac{dR}{dr} + \frac{R}{r^2}\frac{d^2\Theta}{d\theta^2} + k^2R\Theta = 0 \qquad (29)$$

Multiply (29) by $\frac{r^2}{R\Theta}$;

$$\frac{r^2}{R}\frac{d^2R}{dr^2} + \frac{r}{R}\frac{dR}{dr} + \frac{1}{\Theta}\frac{d^2\Theta}{d\theta^2} + k^2r^2 = 0 \qquad (30)$$

$$\frac{r^2}{R}\left(\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr}\right) + k^2r^2 = -\frac{1}{\Theta}\frac{d^2\Theta}{d\theta^2} \qquad (31)$$

It can be seen clearly, LHS of (31) is a function of $\boldsymbol{r}$ only while the RHS is a function of θ. This is only possible if both functions are equal to some constant, say m^2 i.e.

$$-\frac{1}{\theta}\frac{d^2\Theta}{d\theta^2} = m^2$$

And

$$\frac{d^2\Theta}{d\theta^2} = -m^2\Theta$$

And

$$\Theta(\theta) = cos(m\theta + \varphi)$$

φ, is the phase angle.

Re-writing $Y(r,\theta,t)$ as $Y(r,\theta + 2\pi,t)$, and this becomes a periodicity; restricting m to integersvalues, i.e. $m = 0,1,2,3 \ldots$

Rearranging equation (30)

$$\frac{r^2}{R}\left(\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr}\right) + k^2r^2 = m^2 \qquad (32)$$

Multiply equation (32) by $\frac{R}{r^2}$

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + k^2R^2 = \frac{m^2R}{r^2}$$

And

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + \left(k^2 - \frac{m^2}{r^2}\right)R = 0 \qquad (33)$$

This is a Bassel equation where the solutions are the transcendental function of the first kind $\boldsymbol{J_m(kr)}$ and second kind $\boldsymbol{Y_m(kr)}$ of order m written:

$$R(r) = AC_1J_m(kr) + C_2J_m(kr) \qquad (34)$$

The solution$\boldsymbol{J_m(kr)}$ which has a finite limit as $\boldsymbol{kr}$ approaches zero is called Bassel function of the first kind and order m. The solution $\boldsymbol{Y_m(kr)}$ which has no limit (i.e. unbounded) as $\boldsymbol{kr}$ approaches zero in the Bassel function of the second kind and order m or Neuman function. The Bassel function of the first kind of order m is defined as:

$$\boldsymbol{J_m = (r)\sum_{x=0}^{\infty} = \frac{(-1)^x\left(\frac{r}{2}\right)^{m+2x}}{x!\sqrt{(m+x+1)}}} \qquad (35)$$

Bassel functions are oscillating functions whose amplitudes diminish as $\boldsymbol{kr}$increases, and $\mathbf{Y_m(kr)}$ become unbounded in the limit $\boldsymbol{kr \to 0}$.

Since the circular membrane extends across the origin ($r = 0$), the solution $\boldsymbol{y}$ must be finite at$\boldsymbol{r = 0}$. This means

$$\boldsymbol{B = 0}$$

And equation (34) becomes

$$\boldsymbol{R(r) = AC_1J_m(kr)} \qquad (36)$$

$$\boldsymbol{J_m(ka) = 0} \qquad (37)$$

Since $R(a) = 0$

If the values of m cause the function J_m to be equal to zero is designated by $(\boldsymbol{j_{mn}})$, then we have

$$J_m(j_{mn}) = 0 \qquad (38)$$

so that k again assume discrete values given by:

$$k_{mn} = \frac{j_{mn}}{a} \quad (39)$$

Values of j_{mn} for some of the zeros of Bessel function are normally given.

The normal modes of vibration are therefore given as;

$$y_{mn}(r,\theta,t) = A_{mn} J_m(k_{mn} r) \cos(m\theta + \emptyset)\, e^{i\omega_{mn} t} \quad (40)$$

Where

$$\boldsymbol{k_{mn} a = j_{mn}} \quad (41)$$

And the natural frequencies

$$\boldsymbol{f_{mn} = \frac{1}{2\pi}\frac{j_{mn}}{a}\sqrt{\frac{T}{\rho}}} \quad (42)$$

III. RESULTS AND DISCUSSION

It should be noted from the derived equation, that the frequency *(f)*of the tympanic membrane is directly proportional to the square root of the tension *(T)*. Reduction in the *Tension T*, of the tympanic membrane due to perforation or other anomalies will result in decrease in the vibrational frequency of the membrane resulting to the inability of the ear to be adequately sensitive to high frequency sound waves. This implies that the ear will be deaf to high frequency sound waves or there will be high frequency hearing loss. On the other hand, if the tension increases either due to a mass or tumour on the middle ear or due to other anomalies, then the vibrational frequency of the membrane will increase and the ear would be sensitive to high frequency sound waves. In other words, the ear will have low-frequency hearing loss.It may also be noted that the vibrational frequency of the membrane is inversely proportional to the *radius 'a'* of the membrane, clearly as the radius of the membrane increases, the vibrational frequency of the membrane reduces. Perhaps, this suggests why children who have smaller radii are sensitive to highfrequency sound waves than adults. Also the vibrational frequency of the tympanic membrane is inverse proportional to the square root of the density.Fig. 1; Fig. 2 and Fig. 3 shows; The outer ear; Anatomy of the Human Ear and Anatomy of the Human Tympanic Membrane respectively.

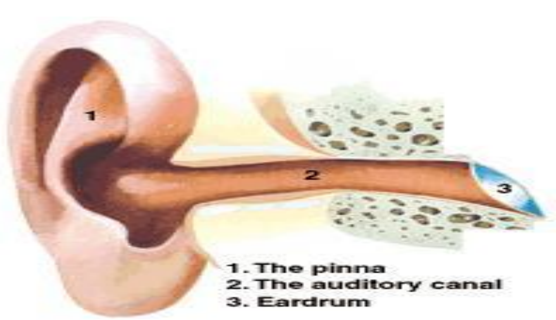

Figure 1: The outer ear [19]

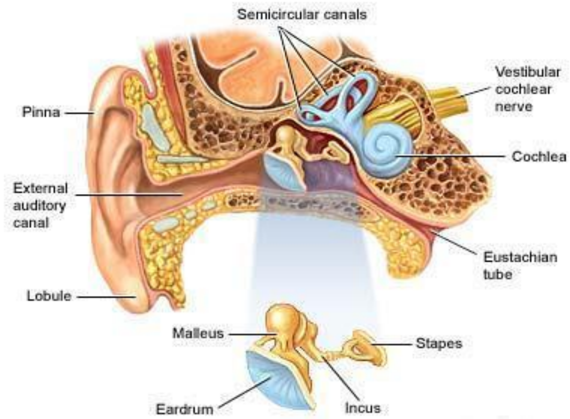

Figure 2: Anatomy of the Human Ear[20]

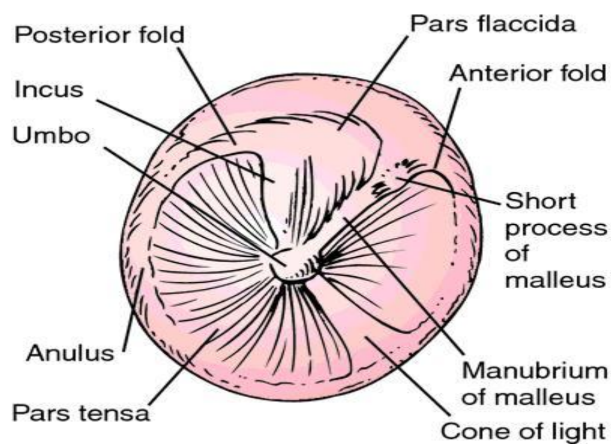

Figure 3: Anatomy of the Human Tympanic Membrane [21]

IV. CONCLUSION

$$\boldsymbol{f_{mn} = \frac{1}{2\pi}\frac{j_{mn}}{a}\sqrt{\frac{T}{\rho}}}$$

From the above equationit can be seen that the vibrational frequency*(f)* is inversely proportional to the *radius(a)* of the membrane. The vibrational *frequency* is inversely proportional to the square root of the *Density ρ*. The vibrational *frequency* is also directly proportional to the square root of the tension *T*. From the solutions of circular model, it can see that the *radius 'a'* of the membrane is inversely proportional to the *frequency (f)* of its oscillation. Moreover, if the radius of the membrane is increased, the frequencies of vibration decreases. Therefore, a child's tympanic membrane of small radius vibrates at higher frequencies when compared to an adult's tympanic membrane with a bigger radius.

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