SSRG International Journal of Applied Physics
ISSN: 2350-0301/ https://doi.org/10.14445/23500301/IJAP-V11I2P101

Volume 11 Issue 2, 1-7, May-Aug 2024

Original Article

Experimentally Determining the Relationship between Hydrodynamic Drag and Salinity

Agastyaa Vishvanath

Lancers International School, Haryana, India.

Corresponding Author : agastyaa.agastyaavishvanath@gmail.com

Received: 09 April 2024 Revised: 18 May 2024 Accepted: 01 June 2024 Published: 13 June 2024

Abstract - *This research paper aims to explore the relationship between the salinity of water and the Hydrodynamic Drag experienced by bodies moving through it. More specifically, it attempts to experimentally establish a generally applicable quantitative relation between the two and hypothesize the potential causes for it. This was done through experimentally determining the variation in fluid density and coefficient of Drag with salinity, as these are the only two variables that affect Drag force that has the potential to vary with salinity. The experiment involved measuring the mass and volume of saline solutions and then measuring the amount of time taken for a body with known characteristics to fall under gravity through a specified height of saline water. The data showed that Hydrodynamic Drag does appreciably increase with salinity in an approximately linear fashion. Moreover, the individual trends in density and coefficient of Drag further demonstrated that it was purely the increase in density with salinity that was responsible for the observed increase in Drag.*

Keywords *- Salinity, Hydrodynamic Drag force, Drag coefficient, Fluid density, Differential equation modelling.*

1. Introduction

As bodies move through a fluid such as water, they experience a resistive force known as hydrodynamic Drag [6]. This force occurs due to the rapid displacement of fluid molecules that occurs during the motion of the body, as well as the friction the body experiences due to the fluid molecules moving against it [10]. As a variety of different industries involve the motion of bodies through or against water, understanding the general behaviour and ways to reduce hydrodynamic Drag force, in the case of water in particular, is of incredible importance. The design and manufacture of ships [7], submarines [1], pipelines [4], hydraulic applications [8], and spillway dams [5], among many others, all depend heavily on the proper implementation of hydrodynamics and the reduction of hydrodynamic Drag force.

Many Characteristics of the water can, at times, alter the Drag force experienced by bodies moving through it. One such characteristic, which is the focus of this research paper, is the salinity of the water. Very little recent research has been done in an attempt to directly investigate the effect of salinity on the hydrodynamic Drag experienced by bodies moving through the water. Most of the research that has been done in the field either involves incredibly specific scenarios or fails to provide a quantitative trend between the two variables, as seen in [2] and [3]. Thus, the research detailed in this paper will aim to investigate the extent to which salinity affects hydrodynamic Drag force in water and to experimentally establish a general quantitative trend between the two that is as generally applicable as possible.

As seawater is incredibly saline in nature, the results of this research will likely be incredibly useful in the water transportation industry, as a variety of water-fairing vehicles, such as ships and submarines, routinely encounter large amounts of Drag force from saline water [7]. The presence or absence of a trend will help clarify whether salinity affects hydrodynamic Drag or not and, thus, if measurements and conclusions about Drag made in pure or fresh water are at all applicable to seawater. Moreover, the precise nature of the trend will aid in articulating the extent to which salinity affects Drag and will thus help better inform the design and deployment of these vehicles.

Hydrodynamic Drag force is primarily affected by the density of the water, the frontal surface area of the body, and the velocity of the body. It is also affected in a lesser capacity by many other variables, the most notable of which is the shape of the body. The effect of all of these other variables on the Drag force is summarized in a single unitless coefficient known as the coefficient of Drag (C_d) [6][11].

As salinity only has the potential to change the density of the water and also possibly the coefficient of Drag of the body, its effect on only these two variables will be experimentally investigated in this paper. Drag force is directly proportional to the product of density and the coefficient of Drag [6]; thus, the trend of this 'combined coefficient' (hereafter denoted with the symbol '$\boldsymbol{\gamma}$') with salinity will be observed and taken as representative of the trend of Drag force will salinity.

2. Theoretical Background

The Drag force experienced by bodies moving through a particular fluid with a particular velocity has been experimentally shown to obey the following relation [6]:

$$F_d = \frac{\rho A C_d}{2} v^2 \quad (1)$$

*ρ = *Fluid Density A= Cross-sectional Area* C_d= *Coefficient of Drag v= velocity*

The experimental method used throughout this paper will involve bodies falling under gravity from rest through water of different salinities. As gravity exerts an approximately constant force on bodies near the earth's surface, a theoretical description of the motion of a body being propelled by a constant force through a fluid will prove incredibly useful.

Let us first acknowledge the fact that net force on the body at any time will be equal to the sum of the propulsive force (F_p) acting on the body and the Drag force (F_d) experienced by the body ($F_{net} = F_p - F_d$).

As the mass of the body in question is assumed to be constant throughout the course of its motion, we can further make use of Newton's second law of motion in order to frame the following relation between the body's velocity and the net force acting on it:

$$\frac{dv}{dt} = \frac{F_{net}}{m} \quad (2)$$

m= mass

Substituting for F_{net}, we can obtain the following differential equation:

$$\frac{dv}{dt} = \frac{F_p - F_d}{m}$$

$$\frac{dv}{dt} = \frac{F_p}{m} - \frac{\rho A C_d}{2m} v^2 \quad (3)$$

Solving (3) would yield a function v(t) describing the velocity of this body as a function of time. As the propulsive force acting on the body, the body's mass, its cross-sectional area, its coefficient of Drag, and the density of the fluid it is moving through remain constant throughout the course of its motion, and we can make further calculations more coherent by reparenting $\frac{F_p}{m}$ as the variable A, and representing $\frac{\rho A C_d}{2m}$ as the variable B. Doing so yields the following differential equation:

$$\frac{dv}{dt} = A - Bv^2 \quad (4)$$

This differential Equation (4) can be solved through the separation of variables (the full method has been outlined in the appendix), yielding the following general solution:

$$v(t) = \frac{\sqrt{A}Ce^{2\sqrt{AB}t} - \sqrt{A}}{\sqrt{B}Ce^{2\sqrt{AB}t} + \sqrt{B}} \quad (5)$$

*C can be any real number

We can now substitute the value of t as zero in order to obtain a relation between the constant (C) and the initial velocity of the body (u):

$$u = \frac{\sqrt{A}C - \sqrt{A}}{\sqrt{B}C + \sqrt{B}}$$

We can then algebraically rearrange this relation to make C the subject, and we can then substitute the value of C into our original function in order to introduce the initial velocity of the body into the function in order to obtain our final function:

$$C = \frac{\sqrt{A} + u\sqrt{B}}{\sqrt{A} - u\sqrt{B}}$$

$$v(t) = \frac{\sqrt{A}\left(\frac{\sqrt{A}+u\sqrt{B}}{\sqrt{A}-u\sqrt{B}}\right)e^{2\sqrt{AB}t} - \sqrt{A}}{\sqrt{B}\left(\frac{\sqrt{A}+u\sqrt{B}}{\sqrt{A}-u\sqrt{B}}\right)e^{2\sqrt{AB}t} + \sqrt{B}} \quad (6)$$

As the experimental method used in this appears to involve gravity acting on bodies at rest, we can assume the initial velocity to be zero to obtain the following function:

$$v(t) = \frac{\sqrt{A}e^{2\sqrt{AB}t} - \sqrt{A}}{\sqrt{B}e^{2\sqrt{AB}t} + \sqrt{B}} \quad (7)$$

A general definite integral of the function (7) can then be taken to yield the distance travelled by the body as a function of time, which would be a full theoretical description of the body motion:

$$s(t) = \int_0^t \frac{\sqrt{A}e^{2\sqrt{AB}x} - \sqrt{A}}{\sqrt{B}e^{2\sqrt{AB}x} + \sqrt{B}} dx$$

$$= \frac{\ln\left(e^{2\sqrt{AB}t} + 1\right)}{B} - t\sqrt{\frac{A}{B}} - \frac{\ln(2)}{B} \quad (8)$$

*Assuming A, B, and t are positive

This is an incredibly important and versatile function, which will be used heavily throughout the paper.

3. Methodology

3.1. Research Aim

This paper aims to explore the relationship between salinity and Hydrodynamic Drag quantitatively. As previously stated, this will be done by observing how the combined coefficient γ (the product of fluid density and coefficient of Drag) quantitatively changes with salinity. This quantity is directly proportional to the Hydrodynamic Drag Force experienced by the body in the saline fluid, thus, its relationship with salinity can be taken as representative of that Drag. In fact, γ can be considered to be the Drag force experienced by the body per unit surface area per unit velocity squared; thus, it will be assigned the units $Nm^{-4}s^2$ throughout the remainder of the paper.

3.2. Data Collection

In order to determine this relationship, primary data was collected by conducting an experiment which yielded both the trend of density and the coefficient of Drag with Salinity. The results yielded by this method would allow for a quantitative and generally applicable relationship between salinity and hydrodynamic Drag force to be established.

3.3. Experimental

The experiment conducted involved letting a 7mm solid steel ball fall under gravity through a measuring cylinder filled with water at the salinities 0, 1, 2, 3, 4, and 5 M at a predetermined height (38.4 cm) and measuring the time taken for the said ball to fall to the bottom of the measuring cylinder. All relevant data, such as the ball's diameter, volume, frontal surface area, and mass, were recorded prior to the experiment.

The ball was fully submerged in the water prior to its release to prevent any energy loss due to splashing from causing errors in the final calculations. Moreover, the balls were dropped using a Vernier Calliper to ensure that the path taken by the balls was as consistently straight as possible, and any trials where this was not the case were immediately discarded. In order to effectively determine the amount of time taken for the ball to fall, the experimental trials were recorded with a high-speed video camera that was capable of capturing 240 frames per second.

The raw number of frames between the point of the ball's release and its contact with the bottom of the measuring cylinder was then manually counted for each trial. This method removed or, at the very minimum, greatly decreased the human error involved with the data collection and allowed for an accurate measurement to within 0.0041 s. In order to further ensure the accuracy and consistency of the data, five trials were conducted at each salinity, and the final data point was considered to be the arithmetic mean of all the concordant data points. In addition to this time data, the mass and volume of the solution were also recorded at each salinity in order to derive its density.

After having obtained this time data, the distance travelled function derived in the theoretical background section could then be used to determine the coefficient of Drag. This function is applicable here as the ball moves from rest through a fluid (the saline solution) while being propelled by a constant force, which in this case will be the difference between the constant gravitational force accelerating it downwards and the constant buoyant force accelerating it upwards.

As the ball traverses the height of the measuring cylinder in the measured time, an equation could be set up using the function, equating the height of the cylinder to the value of the function at the measured time. This would be an equation in terms of A and B but as $A = \frac{F_p}{m}$, both of which are known for a given trial; this can be reduced to an equation in terms of 'B'. This equation was then numerically solved to calculate the value of 'B' for each of the different salinities. As $B = \frac{\rho A C_d}{2}$, the coefficient of Drag was then algebraically determined as the values of B, density, and frontal surface area were all known.

This allowed for an experimental relation between salinity and coefficient of Drag to be formed, and as the relation between Density and Salinity was already known from the mass and volume measurements made during the experiment, the relation of the γ with salinity was also derived. To further interpret the data, linear regressions were also taken to observe the general quantitative trend and gauge the statistical correlation of the data.

The use of this method, in particular, was quite advantageous as it allowed the coefficient of Drag. This quantity can otherwise only be measured with fairly advanced and intricate experimental equipment, to be measured with acceptable accuracy through the use of an incredibly rudimentary experimental apparatus. This is achieved through the heavy use of mathematics and theoretical modelling in conjunction with the experimental data, and while this large amount of mathematical and theoretical abstraction introduced a fairly large possibility for error in the final results, attempts were made to minimize this error wherever possible through the aforementioned use of multiple trials, highly precise time measuring equipment, and the consideration of linear regressions as opposed to the raw results.

4. Results

The first data that was obtained was that of the density of the fluid at different salinities. This was calculated by dividing the mass of the saline water by its experimentally measured volume. The initial 1200 ml of pure distilled water was known to have a mass of 1.1964 Kg, and a mass of 70.128 g of pure salt was added per unit increase in salinity.

Additionally, it was experimentally observed that the volume of the salt solution increased by approximately 25 ml per unit increase in salinity, thus, this was also taken into account during the calculations. Performing said calculations yields the data visible in Table 1.

Table 1. Fluid Density of water for different Salt concentrations in water

Salinity(M)	Fluid Density (Kg/m^3)
0	997
1	1033.9
2	1069.33
3	1103.36
4	1136.09
5	1167.58

This data can then be graphed, and a linear regression can be conducted to allow for further interpretation, as has been done in Figure 1.

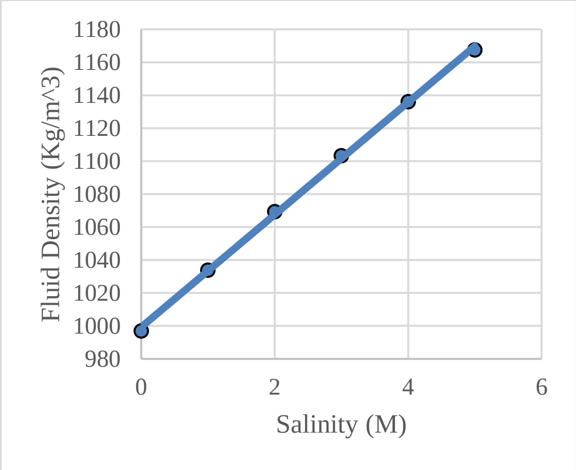

Fig. 1 Fluid Density of water for different Salt concentrations in water

It is clearly visible from this graph that the density of fluid follows a strong linear trend with respect to salinity, as would be expected. More specifically, it is found to have a Pearson moment correlation coefficient of 0.999, indicating that the data follows a very clear positive trend and that the predictions made by the linear regression are highly representative of the data. Moreover, the density is found to increase by approximately 34.1 Kg/m^3 per unit increase in salinity, with a 17.1% increase in density from a pure solution to the maximum salinity measured. Thus, it can be said that fluid density has a significant, positive, and linear relationship with salinity.

As for determining the coefficient of Drag, the first basic measurements of the ball and the solution were made in order to collect the necessary data for the mathematical calculations. These included the mass of the ball, the diameter of the ball, and the height of the lower meniscus of the solution in the measuring cylinder. These measurements were then used to derive further data points necessary for the calculations, such as the frontal surface area of the ball, the volume of the ball, and the gravitational force acting on the ball. These data points have been summarized in Table 2 below:

Table 2. Basic measurements taken of experimental apparatus

Mass of Ball (Kg)	1.51×10^{-3}
Diameter of Ball (m)	7.1×10^{-1}
Frontal Surface Area of Ball (m^2)	7.918×10^{-5}
Volume of Ball (m^3)	1.874×10^{-7}
Gravitational Force Experienced by Ball (N)	1.48×10^{-2}
Height of Solution in Measuring Cylinder (m)	3.84×10^{-1}

In addition to these readings, other data points, such as the density of the fluid and the buoyant force acting on the balls, were also required to undertake the mathematical calculations; however, as both of these properties vary with the salinity of the solution, they were individually calculated for every salinity used in the experiments. Their values have been summarized in Table 3 below:

Table 3. Buoyant force against salinity

Salinity (M)	Fluid Density (Kg/m^3)	Buoyant force Experienced by the balls (N)
0	997	1.83×10^{-3}
1	1033.9	1.90×10^{-3}
2	1069.33	1.97×10^{-3}
3	1103.36	2.03×10^{-3}
4	1136.09	2.10×10^{-3}
5	1167.58	2.15×10^{-3}

The experiment was then conducted, and the balls were dropped from the specified height through the solution at different salinities while being recorded with a 240fps camera. The raw number of frames taken for the ball to fall the full distance was then counted and averaged over the five trials conducted per salinity. This was then simply divided by 240 fs^{-1} in order to obtain the final time taken for the ball to travel the specified distance. Both the raw frame data and the final time data have been depicted below in Table 4 and Table 5, respectively:

Table 4. Raw frame data of fall time of ball against salt concentration of water

Salinity (M)	Frames counted
0	99.25
1	99.6
2	103.33
3	104
4	105
5	105.375

Table 5. Fall Time of ball against salt concentration of water

Salinity (M)	Time Taken (s)
0	0.414
1	0.415
2	0.430
3	0.433
4	0.438
5	0.439

This, in conjunction with the previously outlined measurements and data points was used to numerically solve large equations to calculate the coefficient of Drag of the ball in each of these cases. This final data is depicted in Table 6 below:

Table 6. Coefficient of drag of ball against salt concentration of water

Salinity (M)	Coefficient of Drag
0	0.223
1	0.215
2	0.235
3	0.231
4	0.230
5	0.225

This data can then be graphed, and a linear regression can be conducted to allow for further interpretation, as has been done in Figure 2:

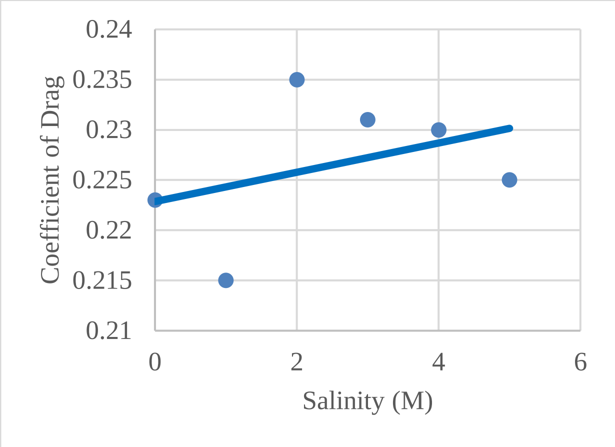

Fig. 2 Coefficient of drag of ball against salt concentration of water

As can clearly be seen in the graph, the data displays no clear trend but tends to oscillate around a mean value of around 0.227. The Pearson moment correlation coefficient for the data is only 0.384, indicating an incredibly weak correlation. While this correlation is found to be slightly positive, there is no statistically significant deviation from the horizontal and is likely simply due to the experimental error involved in the experiment. Thus, it can be said with confidence that the coefficient of Drag does not vary significantly with salinity to a significant extent.

After having obtained both the coefficient of Drag and density of the data, the two were finally multiplied together to provide the combined coefficient 'γ', which has been depicted in Table 7 below:

Table 7. Combined coefficient against salt concentration of water

Salinity (M)	Combined Coefficient (Kg/m^3)
0	222.331
1	222.289
2	251.293
3	254.876
4	261.301
5	262.706

As with the previous cases, this data can be graphed, and a linear regression can be conducted to allow for further insight and interpretation, as has been done in Figure 3:

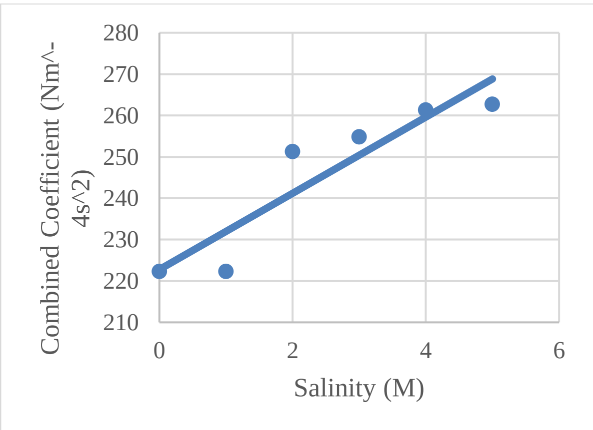

Fig. 3 Combined coefficient against salt concentration of water

As is clearly visible on the graph, the combined coefficient has a strong, positive, and seemingly linear trend with respect to salinity. It has a Pearson moment correlation coefficient value of approximately 0.923, indicating a strong positive correlation with salinity. Moreover, based on the linear regression, the combined coefficient increases in value by approximately 9.24 $Nm^{-4}s^2$ per unit increase in salinity and experiences an 18.16% increase from a pure solution to the maximum salinity measured. The data points do tend to deviate from the trend slightly; however this is likely due to the error involved in the experiment. Thus, it can be said that the combined coefficient and, by extension, the Drag force experienced by the body displays a significant positive trend with salinity.

5. Discussion

The results clearly show that, as expected, the fluid density of water displays a strong linear trend with respect to salinity. They further show that the coefficient of Drag of the body displays little to no trend with the salinity and instead remains around the same value, oscillating around 0.227. This oscillation is likely simply due to the high amount of error associated with the experiment, as the calculated values all stay within 5.3% of the mean value. There may perhaps be an incredibly minute trend or relation between salinity and coefficient of Drag that this experiment is not able to ascertain due to the high amounts of error associated with it. However, at the very minimum, it can be said that there is no substantial trend between the coefficient of Drag and salinity and that this trend plays very little part in affecting the Drag force experienced by the body.

The resulting strong positive trend of the combined coefficient with salinity aids in conclusively showing that Drag force is indeed affected by salinity and that the two have a clear and positive relationship. This conclusion can be made because, as previously mentioned, the combined coefficient is directly proportional to the Drag force experienced by the body. This relationship between the two also allows us to make conclusions about the extent to which the Drag force will increase based on the trend of the combined coefficient. Based on the data, the Drag force experienced by bodies moving through saline water will increase by 4.2% from its original value per unit increase in salinity. It will, as such, increase by a total of 25.62% at the saturation point of water at room temperature.

This is a substantial increase and aids in demonstrating that the effect of salinity on hydrodynamic Drag cannot be ignored and that most, if not all, of the measurements related to hydrodynamic Drag made in freshwater, are largely inapplicable to saline water (with the obvious exception of coefficient of Drag).

In addition to showing that Drag force increases with salinity, the fact that the density and coefficient of Drag were intentionally calculated separately and then multiplied together to produce the combined coefficient aids in providing further insight into the reason why salinity causes an increase in the coefficient of Drag. The density of a fluid can be considered as a combined representation of both how many particles there are per unit volume and how heavy these particles are. Thus, the Drag force experienced by bodies increases with density due to the fact that there are physically more and/or heavier particles that must be displaced for a body to complete its motion. All other minute effects and phenomena that occur within the fluid that could contribute to the Drag force are summarized by the coefficient of Drag. Thus, the fact that the resultant trend observed in the combined coefficient was almost completely due to the positive trend in density demonstrates that the Hydrodynamic Drag experienced by bodies increase with salinity almost purely due to the fact that increases in salinity cause a physically greater number of total particles which a body must displace in order to complete its motion. Moreover, all other minute ways in which salinity could potentially affect hydrodynamic Drag, whether positive or negative, are largely negligible due to the lack of a consistent trend in the coefficient of Drag.

6. Conclusion

This research clearly demonstrates that there is a measurable, quantitative, and highly significant relationship between hydrodynamic Drag and salinity and further shows that this trend is positive and approximately linear. More specifically, Drag force seems to increase by approximately 4.2 % per unit increase in salinity. In addition to the nature of the trend, the cause for this relationship can also be ascertained through the analysis of the individual trends of density and coefficient of Drag, which demonstrate that the increase in fluid density that accompanies salinity is the sole cause for the observed increase in Drag Force.

As mentioned in the introduction, these results would, as such, be quite useful to the sea-faring vehicle industry, as they not only demonstrate that the hydrodynamic Drag force does change with salinity but also provide an indication of how much it increases and provide a trend that can be used to extrapolate data points based on their needs. All of this can greatly help inform the design, manufacture, and even operation of these vehicles.

7. Acknowledgement

I would like to thank my Research Supervisor, Dr. Mritunjay Sharma, for his unwavering support in this endeavour. I would also like to extend my heartfelt thanks to the Athena Research Program for guiding and facilitating the creation of this paper.

Appendix

Entry 1

The differential equation describing propelled motion through a fluid is of the following form:

$$\frac{dv}{dt} = A - Bv^2$$

This Differential Equation can be solved through the separation of variables followed by some Algebraic Simplification:

$$\int \frac{1}{A - Bv^2} dv = \int 1\, dt$$

$$\frac{ln\left|\frac{\sqrt{A} + v\sqrt{B}}{\sqrt{A} - v\sqrt{B}}\right|}{2\sqrt{AB}} = t + c$$

$$\frac{\sqrt{A} + v\sqrt{B}}{\sqrt{A} - v\sqrt{B}} = Ce^{2\sqrt{AB}t}$$

*The Modulus can be omitted so long as C is allowed to take both positive and negative values

$$v(t) = \frac{\sqrt{A}Ce^{2\sqrt{AB}t} - \sqrt{A}}{\sqrt{B}Ce^{2\sqrt{AB}t} + \sqrt{B}}$$

References

[1] Alex Lascelles, "*Submarine Drag Modelling and Hull Design*," Nuffield Research Project, 2012. [Publisher Link]

[2] A. Escobar et al., "Influence of Temperature and Salinity on Hydrodynamic Forces," *Journal of Ocean Engineering and Science*, vol. 1, no. 4, pp. 325-336, 2016. [CrossRef] [Google Scholar] [Publisher Link]

[3] Ahmed Kamel, and Subhash N. Shah, "Effects of Salinity and Temperature on Drag Reduction Characteristics of Polymers in Straight Circular Pipes," *Journal of Petroleum Science and Engineering*, vol. 67, no. 1-2, pp. 23-33, 2009. [CrossRef] [Google Scholar] [Publisher Link]

[4] Adrian Deaconu, Drag Reducing Agents for Pipelines/Conduits, Epcmholdings, 2021. [Online]. Available: https://epcmholdings.com/drag-reducing-agents-for-pipelines-conduits/.

[5] Imre M. Jánosi et al., "Turbulent Drag Reduction in Dam-Break Flows," *Experiments in Fluids*, vol. 37, no. 2, pp. 219-229, 2004. [CrossRef] [Google Scholar] [Publisher Link]

[6] F.R.S. Lord Rayleigh, "LIII. On the Resistance of Fluids," *The London, Edinburgh, And Dublin Philosophical Magazine And Journal Of Science*, vol. 2, no. 13, pp. 430-441, 1876. [CrossRef] [Google Scholar] [Publisher Link]

[7] M.D. Ibrahim et al., "The Study of Drag Reduction on Ships Inspired by Simplified Shark Skin Imitation," *Applied Bionics and Biomechanics*, vol. 2018, pp. 1-11, 2018. [CrossRef] [Google Scholar] [Publisher Link]

[8] T.R. Marrero, and G.S. Kuhlman, "Drag Reduction in Hydraulic Capsule Pipeline," *Handbook of Powder Technology*, vol. 10, pp. 513–520, 2001. [CrossRef] [Google Scholar] [Publisher Link]

[9] Wouter Remmerie, What Is a Drag Coefficient?, AirShaper, 2018. [Online]. Available: https://airshaper.com/videos/what-is-a-drag-coefficient/bEgoZ_dAg7o.
[10] Skin Friction Drag, Friction Drag, SKYbrary, 2021. [Online]. Available: https://skybrary.aero/articles/friction-drag.
[11] Hydrodynamics, Mecaflux, 2024. [Online]. Available: https://www.mecaflux.com/en/hydrodynamique.htm.