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Original Article

Classical Calculation of the Radii of Proton and Electron

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***Abstract** - In his model, Bohr had the electron orbit around the center of gravity of the hydrogen atom. He manipulated classical physics by demanding freedom from radiation for this accelerated motion. In this article, his model is used, and the radiation of electromagnetic energy is explicitly permitted. The energy remains in the atom so that there is no loss to the outside. For the surfaces of the proton and electron, which are assumed to be spherical, a direct proportionality to the atomic volume and the radiation power and an inverse proportionality to the binding energy are derived. The binding energy of the hydrogen atom known from the literature is used to calculate the particle radii. The radiation powers of the proton and electron are determined using their kinetic data. The volume of the hydrogen atom is calculated using values from the literature. With these data, a proton radius in the order of $6*10^{-16}$ m to $19*10^{-16}$ m can be derived. For the electron, this results in a radius range of approximately $3*10^{-19}$ m to $10*10^{-19}$ m. With the numerical values of the two authors, the proton radius can be narrowed down to a range of $8.42*10^{-16}$ m and $8.83*10^{-16}$ m. The corresponding electron radii can be calculated as $4.59*10^{-19}$ m and $4.81*10^{-19}$ m.*

***Keywords** - Bohr's atomic model, Atomic volume, Radiation power, Energy flow, Energy density, Surface radiation balance.*

1. Introduction

Since the middle of the 19th century, it has been known that there are positively and negatively charged particles, that the electric charge of these particles is the same in magnitude and differs only in sign, and that there is a repulsive or attractive electrostatic force [1], [2]. In 1903, J. J. Thomson formulated an atomic model on this basis, in which the positively charged mass evenly fills the atomic volume, and the negatively charged electrons move between them [3].

The first successful measurements on this topic were carried out at the beginning of the 20th century in the working group of E. Rutherford and culminated in 1911 in the hypothesis of a small and massive atomic nucleus that contained almost the entire atomic mass [4]. Rutherford dealt with radioactive radiation and derived his conclusion from scattering experiments of alpha rays on metal atoms. He calculated a differential scattering cross-section. $\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford}$ for the elastic scattering of alpha particles, which was confirmed experimentally by Geiger and Marsden [5].

The mathematical instrument of the scattering cross-section was extended in 1929 by N. F. Mott to describe the scattering of a point-like particle on large nuclei [6] and was supplemented in 1950 by M. N. Rosenbluth with electric and magnetic form factors that take into account the spatial expansion of the target [7]. The size of these factors depends on the transmitted quadruple pulse Q.

The form factors can be extracted from the measurement results using the so-called Rosenbluth line. In the limiting case of a very low momentum exchange ($Q^2 \rightarrow 0$), the electric and magnetic proton radius can be calculated because the proton structure is not resolved. In this context, the electric radius of the proton is defined as [8]

$$\langle r_p^2 \rangle = -6 \frac{dG_E(Q^2)}{dQ^2}\bigg|_{Q^2=0} \tag{1}$$

With $G_E(Q^2)$ for the electrical form factor. This method of differential scattering cross-section can also be used to narrow down the electron radius via fermion pair formation [9].

The electric radius of a proton has been determined since the 1950s, initially in scattering experiments using the elastic interaction of the proton with electrons. The results were calculated using the method described above [10]. Because an extrapolation to $Q^2 = 0$ is required for the calculation, the result is influenced not only by the experimental setup and how the measurement is carried out but also by the selection of the measured values.

Another approach to determining the proton radius is laser spectroscopy on electronic hydrogen atoms. This utilizes the fact that in the s-states of the hydrogen atom, the probability of the electron being in the nuclear region is greater than zero, resulting in a slightly lower binding energy ΔE_B [11]:

$$\Delta E_B = \frac{2 m_{red.}^3 (Z\alpha)^4}{3 n^3} \boldsymbol{r}_p^2 \tag{2}$$

With the reduced mass $m_{red.} = \frac{m_{electron} * m_{proton}}{m_{electron} + m_{proton}}$, the nuclear charge number Z, the fine structure constant α, the principal quantum number n and the proton radius $\mathbf{r}_p$.

Replacing the electron with a heavier muon provided even more accurate results because the muon is closer to the nucleus and has a higher probability of being in the nucleus than the electron. The proton measurement with muonic hydrogen led to a significantly smaller radius than the other measurements, which was extensively discussed in the literature under the term "proton radius puzzle" and calculated with various correction options. In the meantime, laser spectroscopic measurement results with electronic hydrogen [12] and scattering experiments with electrons [13] exist, which indicate a comparably small proton radius as the measurements with muonic hydrogen.

Both the practical measurements and the theoretical calculation of the proton radius are time-consuming for all the variants mentioned above. In this article, an easily accessible method is presented as an alternative, with which the radii of the proton and electron can be quickly calculated using literature data. The considerations are based on a completely different approach to the topic of particle size than the previous ones.

2. Behavior and Effect of Electron and Proton in the Bohr Model

The starting point is the ground state of the hydrogen atom, as described in Bohr's atomic model. There, the electron and proton are in an accelerated circular motion around a fictitious common atomic center of gravity. Experience has shown that accelerated charges radiate energy. This phenomenon has not been observed in the hydrogen atom. It, therefore stands to reason that the radiated energy remains in the atomic volume V_{H1}. This means that the electron and proton emit electromagnetic energy in the hydrogen atom through their accelerated movements and can be used again to continue the accelerated movements through subsequent absorption. A direct energy exchange between the accelerated charges, as described by Wheeler and Feynman in another context [14], [15], is not necessarily required in this article.

In the first step, the electron is considered, and its definitions known from the literature for the Bohr radius, the orbital velocity, the orbital period that can be derived from it and the binding energy in the hydrogen atom are compiled. The corresponding values for the proton are then determined, and the respective radiation power is defined for both particles. Assuming a constant surface radiation balance, the radius values of the proton and electron can be extracted in the final step.

2.1. Behavior of the Electron in the Bohr Model

According to Bohr, the electron orbits the center of mass of the hydrogen atom on a circular path with the radius r_{H1} [16], [17], [18]

$$\boldsymbol{r}_{H1} = \frac{4\,\pi\,\varepsilon_0 h^2}{m_{red.}\, e_0^2} \tag{3}$$

This orbital radius depends on the electric field constant ε_0, Planck's quantum of action h, and the reduced mass $m_{red.}$ and the elementary charge e_0. The orbital velocity of the electron $\mathbf{v}_{H1}$ can be calculated as

$$\boldsymbol{v}_{H1} = \frac{h}{2\,\pi\,\boldsymbol{r}_{H1} m_e} = \alpha\, c \tag{4}$$

It corresponds to the product of the dimensionless fine structure constant α and the speed of light c. The duration t_{H1} of a path circulation results in

$$t_{H1} = \frac{2\pi r_{H1}}{v_{H1}} \tag{5}$$

and the binding energy of the hydrogen atom in the ground state is

$$E_{binding\ H1} = -\left|\frac{e_0^4 m_r}{32\pi^2 \varepsilon_0^2 \hbar^2}\right| \tag{6}$$

with the reduced Planck constant $\hbar$.

2.2. Behavior of the Proton in the Bohr Model

Equation (6) takes into account the co-motion of the proton in the hydrogen atom. If the electron were bound to an infinitely heavy atomic nucleus at rest, the reduced mass $m_{red.}$ in Equation (6) would have to be replaced by the mass of the electron m_e:

$$E_{binding\ heavy\ nucleus} = -\frac{e_0^4 m_e}{32\pi^2 \varepsilon_0^2 \hbar^2}. \tag{7}$$

The difference between the two amounts of energy in Equations (6) and (7) must be assigned to the kinetic energy of the proton.

$$\Delta E_{binding} = \left|\frac{e_0^4 m_e}{32\pi^2 \varepsilon_0^2 \hbar^2}\right| - \left|\frac{e_0^4 m_{red.}}{32\pi^2 \varepsilon_0^2 \hbar^2}\right| = \left|\frac{e_0^4}{32\pi^2 \varepsilon_0^2 \hbar^2}\right| * (m_e - m_{red.}) = \frac{m_P * v_{Hp}^2}{2}. \tag{8}$$

After transformation, this results in the orbital velocity of the proton $\mathbf{v}_{Hp}$

$$\boldsymbol{v}_{Hp} = \sqrt{\frac{e_0^4 * (m_e - m_{red.})}{16\pi^2 \varepsilon_0^2 \hbar^2 m_p}} \qquad (9)$$

Equation (11) results for the orbital radius of the proton via the requirement that the Coulomb approach

$$F_{Coulomb} = F_{circular\ motion\ proton} \qquad (10)$$

applies for the circular motion of the proton as well as for the circular motion of the electron:

$$\boldsymbol{r}_{Hp} = \frac{m_p \boldsymbol{v}_{Hp}^2 \boldsymbol{r}_{H1}}{m_e \boldsymbol{v}_{H1}^2} \qquad (11)$$

Further calculations show that the orbital period of the electron and the orbital period of the proton have the same value.

2.3. Radiation and Energy Exchange in the Hydrogen Atom

Electrons and protons carry the same amount of electrical charge. The only difference is the sign. The electric field between the two charges can be imagined in the form of field lines that are oriented from the positive to the negative charge. The number of these lines symbolizes the field density prevailing at a particular location. Due to the equal amount of charge, the number of lines on the surfaces of both particles is expected to be the same.

If one of the two particles is geometrically smaller than the other, there must, therefore, be a higher field density on the surface of the smaller particle than on the surface of the larger one. If the surfaces of both particles are enlarged concentrically until they are the same size, then the same field density prevails on both enlarged surfaces.

Coulomb's law and Gauss' theorem [19] enforce a constant and equally large flux ϕ_{field} of the electric field E through these surfaces for both particles:

$$\phi_{field} = \int \vec{E}\, dO_e = \int \vec{E}\, dO_p\ . \qquad (12)$$

These flows through the fictitious surfaces, therefore, also correspond to the flows from and to the surfaces.

Since the proton and electron in Bohr's atomic model orbit a common center of gravity as outlined above, they radiate electromagnetic energy with the radiation power P into the entire space as charges moving in a circle with the acceleration $\boldsymbol{a} = \frac{\boldsymbol{v}^2}{\boldsymbol{r}}$ [20]:

$$P = \frac{\mu_0 e_0^2 \boldsymbol{a}^2}{6\pi c} \qquad (13)$$

If analogous to the flow of the electric field, there is a flow of radiant power $\phi_{radiant\ power}$ from the respective particle surface in the form.

$$\phi_{radiant\ power} = \int P\, dO\ , \qquad (14)$$

a constant surface radiation balance is ensured by Equation (15)

$$P_e O_e = P_p O_p\ , \qquad (15)$$

which is equivalent to an equally large ratio of the radiant power of one particle per surface of the other particle:

$$\frac{P_e}{O_p} = \frac{P_p}{O_e}. \qquad (16)$$

3. Results and Discussion

Using the energy density D_{H1} as a quotient of binding energy and atomic volume V_{H1}

$$D_{H1} = \frac{E_{binding}}{V_{H1}}, \qquad (17)$$

an equilibrium condition can be formulated for the radiant power in the atomic volume

$$D_{H1} = \frac{P_e}{cO_p} = \frac{P_p}{cO_e}\ , \qquad (18)$$

which can be used to calculate the radii of the proton and electron.

To determine the proton radius, the part of the Equation in (18)

$$D_{H1} = \frac{P_e}{cO_p} \qquad (19)$$

is first converted to O_p and $\mathbf{r}_{Hp}$ and then converted to Equation (20) by inserting Equation (17):

$$|r_p| = \sqrt{\frac{P_e * V_{H1}}{c * 4\pi * E_{binding}}}. \qquad (20)$$

The part of the Equation in (18) that relates to the electron radius is converted in the same way:

$$|r_e| = \sqrt{\frac{P_p * V_{H1}}{c * 4\pi * E_{binding}}}. \qquad (21)$$

The binding energy of the hydrogen atom is known from measurements. The radiation powers of the particles can be calculated using the model data. The radius of the hydrogen atom, on the other hand, is subject to a wide range [21]-[24]. Using the radius data from reference [22], a spherical volume in a range of $0.65*10^{-31}$ m³ to $6.2*10^{-31}$ m³ can be calculated for the hydrogen atom. Equations (20) and (21) can then be used to determine the radii of the proton and electron to a bandwidth of $6.1*10^{-16}$ to $18.8*10^{-16}$ m for the proton and $3.3*10^{-19}$ to $10.3*10^{-19}$ m for the electron. With two radius values [23], [24] favored in the relevant literature for the hydrogen atom, the proton radius can be narrowed down to the range from $8.42*10^{-16}$ m to $8.83*10^{-16}$ m. The corresponding values for the electron radius are $4.59*10^{-19}$ m and $4.81*10^{-19}$ m. All values are listed again in Table 1 for comparison.

Before 1950, proton radii of the order of $1*10^{-15}$ m to $2*10^{-15}$ m were discussed in the literature [25]. In the first half of the 1950s, Hofstadter et al. measured the proton and attributed a radius of $7*10^{-16}$ m to $8*10^{-16}$ m to it [10]. In the decades that followed, the data stabilized at radii between $8*10^{-16}$ m and $9*10^{-16}$ m, most recently with a tendency towards $8.4*10^{-16}$ m [26]-[34]. The radius range calculated in this article is close to the measurement results reported in previous years. In particular, the current trend of approx. $8.4*10^{-16}$ m [33] can be well reproduced with the data given by Cordero et al. [24].

The particle data group has reviewed the measured values published worldwide at regular intervals and calculated current mean values from them. For several years, two mean values were published in parallel due to the uncertainties known as the proton radius puzzle. Figure 1 provides an overview of the development over the last two decades, together with a classification of two of the values calculated here.

Table 1. Comparison of the radius values of proton and electron for different volumes of the hydrogen atom

Author	Volume V (m³)	Proton radius (m)	Electron radius (m)
Jaeck [22]	$0.65*10^{-31}$ to $6.2*10^{-31}$	$6.1*10^{-16}$ to $18.8*10^{-16}$	$3.3*10^{-19}$ to $10.3*10^{-19}$
Pyykkö [23]	$1.37*10^{-31}$	$8.83*10^{-16}$	$4.81*10^{-19}$
Cordero [24]	$1.25*10^{-31}$	$8.42*10^{-16}$	$4.59*10^{-19}$

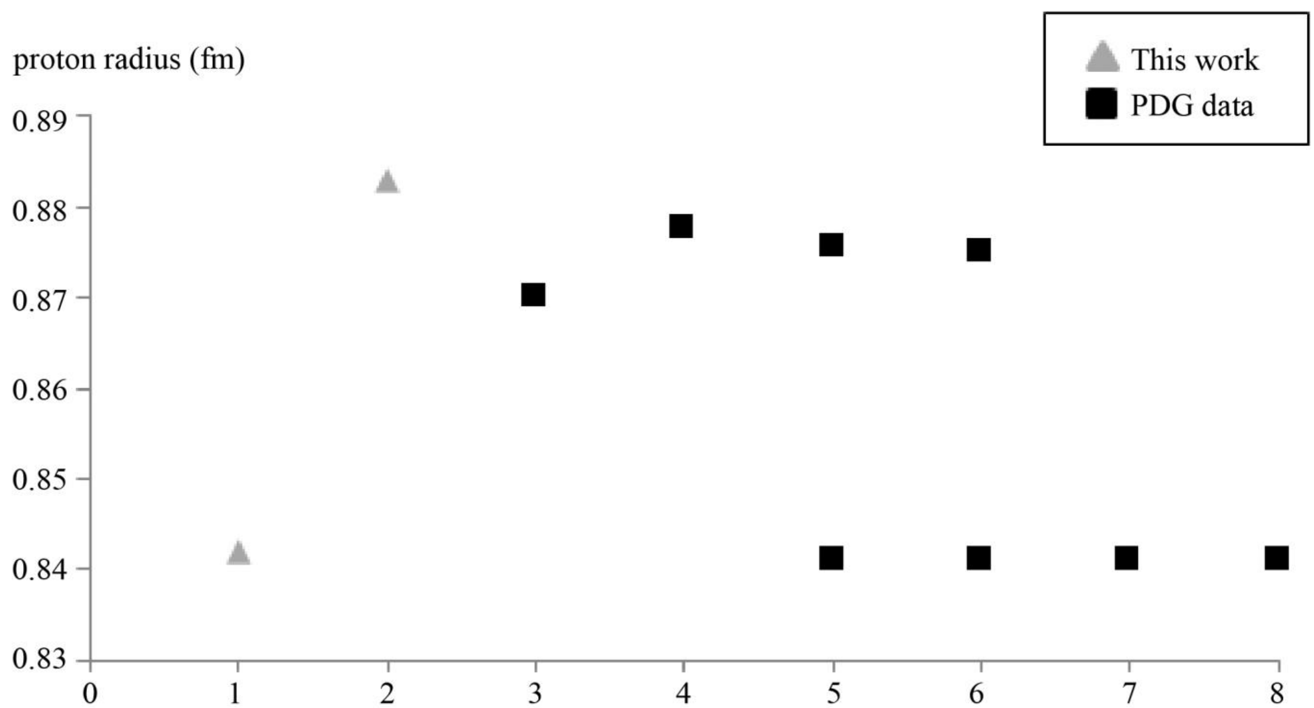

Fig. 1 Calculated proton radii and average values of the Particle Data Group (PDG) from different years in femtometers (fm). (1) Calculated with data from [24], (2) calculated with data from [23], (3) PDG 2002 [35], (4) PDG 2012 [36], (5) PDG 2014 [37], (6) PDG 2016 [38], (7) PDG 2020 [39], (8) PDG 2022 [33]

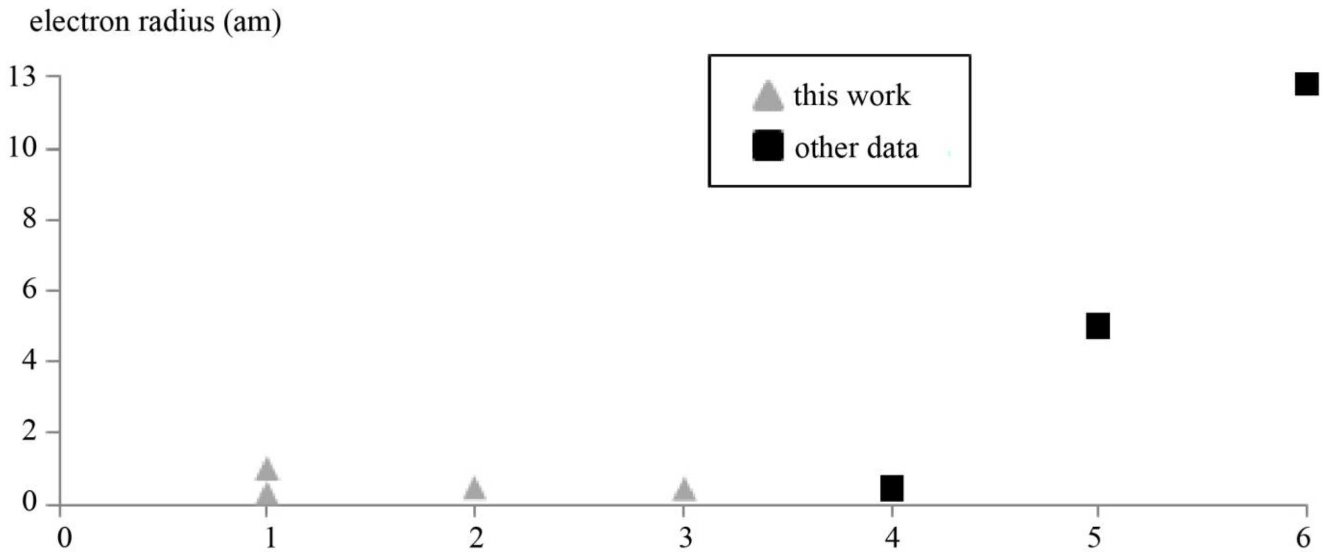

Fig. 2 Calculated electron radii from this and other works in attometers (am).
(1) Calculated range with data from [22], (2) Calculated with data from [23], (3) Calculated with data from [24], (4) Theoretical value from [45], (5) Experimental upper limit from [44], (6) Theoretical value from [42]

While the measured and calculated proton radii have been relatively close to each other for decades, the estimates for the size of the significantly lighter electron vary by several orders of magnitude in the specialist literature. In some theoretical approaches, the electron is defined as a mathematical point with an extension of 0 meters. Other approaches give it a finite radius up to a size of 10^{-12} m [40]-[43]. The experimental data are generally limited to the specification of upper limits for the electron radius. As an example, two articles are cited here in which the electron radius was experimentally located below the upper limit of $2.8*10^{-19}$ m [9] and the upper limit of $1*10^{-22}$ m [41]. The data situation is open, and the particle data group has not yet published any figures on the electron radius.

As shown in Figure 2, the electron radii calculated in this work with the data of Cordero [24] and Pyykkö [23] correlate with the value of $4.42*10^{-19}$ m calculated by Rosen [45] and fulfill the experimentally derived requirement of an upper limit of $5*10^{-18}$ m [44]. They do not agree with the significantly larger values of Storti [42] or Jimenez [40], whose calculations yield electron radii of about 11.8 attometers and 2.8 femtometers (the latter not shown in the figure).

4. Conclusion

The approach presented for calculating the radii of the proton and electron is based on a classic link between the Bohr atomic model and the radiation of electromagnetic energy by moving charge carriers. It introduces motion data for the proton that could not be found in this form in the literature. Using the example of the proton radius, which is extensively quantified in the literature, it is shown that the numerical solutions from this approach correspond to the reality of measurements. In addition, two further aspects will be addressed.

From a purely formal point of view, equation (18) corresponds to Maxwell's definition of radiation pressure [46]. Suppose the radiation power of the proton and electron is interpreted as radiation pressure. In that case, an estimate of the potential effect shows that this additional force is lower by orders of magnitude compared to the Coulomb force and is, therefore, negligible to a first approximation. The resulting small change in momentum is comparable to the trembling motion described by Breit [47] and Schrödinger [48].

The second aspect concerns the application of the presented model to muonic hydrogens, such as the laser spectroscopic studies on proton size described by Antognini et al. [32], in which a muon replaced the electron of the hydrogen atom. Since the volume of muonic hydrogen has not yet been determined, the proton radius cannot be extracted using the model presented. At best, atomic instability can be deduced from the model because one consequence of the electron exchange for a muon in the atomic shell is an imbalance. In the hydrogen atom, the lever arm products $m_p*\mathbf{r}_{Hp}$ and $m_{red}*\mathbf{r}_{H1}$ are identical. In muonic hydrogen, there is a small discrepancy in the ppb range. The discrepancy is significantly greater for the orbital periods around the common center of gravity. In the electronic hydrogen atom, the Bohr model predicts the same orbital periods for the players. In muonic hydrogen, the muon is ahead of the proton by a time factor of 1.0548. These indications suggest that muonic hydrogen - apart from the instability of the muon - has a different behavior than electronic hydrogen.

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