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Original Article

Determination of Orbital Time Period of Revolution Dwarf Planets Around the Sun using Kepler's Third Law of Planetary Motion and Python - Pluto, Eris, Haumea, Makemake, Gonggong, Quaoar, Sedna, Orcus and Salacia

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Abstract *- Dwarf Planets do not classify as planets as most of them do not clear the neighborhood around their orbit, for example, Pluto. Some of the biggest dwarf planets are found farther than Neptune's distance, which is 30 AU. This makes it hard for an observational study of these dwarf planets the determination their orbital period around the sun. In this paper, we use Kepler's third law of motion to determine the theoretical orbital period of some of the biggest dwarf planets. Firstly, using the orbital time and semi-major axis of the eight planets in the solar system, namely, Mercury, Venus, Earth, Mars, Jupiter, Saturn and Neptune, the mass of the sun using average and graphical methods is calculated, the results were* $2.0355 * 10^{30}$ *Kg and* $2.2289 * 10^{30}$*Kg, respectively. Using Kepler's third law,* $T = \sqrt{(4\pi^2/GM)R^3}$*, where R is the semi-major axis, G is the universal constant of gravity, M is the mass of the sun, and the orbital time period(T) of dwarf planets is determined. This paper suggests the orbital time period of the dwarf planets Pluto, Eris, Haumea, Makemake, Gonggong, Quaoar, Sedna, Orcus and Salacia are about 236, 910, 213, 310, 794, 261, 736, 320, 265 years.*

Keywords *- Dwarf planets, Planets, Kepler's Law, Orbital period, Semi-major axis.*

1. Introduction

The invention of the telescope in the 17th century by Galileo Galilei opened new dimensions for humans. In the year 1609, Galileo heard about what was then known as "Danish perspective glass" and built a basic telescope with a magnification of 3 times; later, Galileo improved telescopes with eight and thirty times magnification. Galileo, with his telescope, observed Jupiter's moon, Earth's moon Sun, etc. His observations supported the theory that the Sun is the center of the solar system and not the Earth. The Earth being the center of the solar system, known as Ptolemy's geocentric model, was followed for hundreds of years. The order of the solar system, according to Ptolemy's model, is Earth, Moon, Mercury, Venus, Sun, Mars, Jupiter and Saturn. Galileo assisted in the development of observational astronomy.

Newton is a very important scientist in the field of physics and astronomy. He was born in the mid 17th century in England. He was one of the first people to use mathematics to formulate the laws of physics. He contributed to the field of light, lens, motion, gravitation, thermodynamics, derivatives, integration etc. His contribution to the field of physics started the development of pure theoretical physics. Three important developments for this paper: (1) The laws of kinematics for a constant acceleration, related to the final velocity with an initial velocity, displacement traveled with time, and final velocity with displacement. (2) The development of laws of light allowed to study the motion of light inside lenses, the deviation of light inside the lenses and the properties of the lenses themselves; this allowed the creation of more sophisticated telescope to observe some of the faintest objects in the sky, such as nebulae. (3) The law of gravitation suggests that the force between two masses is proportional to the product of the two masses and inversely proportional to the square of the distance between them. This law is a theoretical tool to measure the gravitational force between two forces. The telescope developed by Galileo is an observational tool in the field of physics, whereas the equations of motion, law of gravitation etc, developed by Newton stands as the theoretical tools in the field of physics. Kepler's discovery of the planetary laws was the first and foremost astronomical observation which could be explained using Newton's laws of motion; this makes them a theoretical background in the field of astronomy.

Kepler's laws of planetary motion are natural laws in nature and can be derived using Newtonian mechanics. There are three of Kepler's planetary laws of motion namely, first - the planets in the solar system travel around the sun in elliptical orbits, with the sun at one focal point. The second law - the planet's speed is not uniform in its orbit but is such that the line segment from the sun to the planet sweeps out equal areas in equal time intervals, and, finally, the third law - for all the planets, the square of time required for the planet to complete one orbit around the sun, divided by the cubed of its semi-major distance from the sun, is the same constant. Kepler's laws of motion are used to study the motions of planets and moons in our solar system, as well as the exoplanets and exomoons, which are planets and moons outside our solar system. The planets, moons, comets and asteroids in our solar system follow Kepler's laws of motion. One can study the trajectory of astronomical objects, predict astronomical phenomena, and determine the physical properties of the planet, such as its density, mass, surface gravity, etc, using Kepler's laws of motion. In our solar system, the Sun is the parent star, and there are eight planets, four terrestrial planets, which are Mercury, Venus, Earth, and Mars, and there are four gaseous planets, which are Jupiter, Saturn, Uranus and Neptune. The solar system also has astronomical objects such as asteroids, for example, Pallas, comets for example, Halley's comet, dwarf planets for example Pluto and a huge amount of dust particles.

Sun is the main star of the solar system; it is a main sequence star with a surface temperature of about 5500 degrees K, it is about 4.6 billion years old, and with a mass of about 2×10^{30}kg, with this, it has 99.8 percent of the total mass of the solar system. Thus, we can consider a simple model of the solar system in which the sun is steady at a point in space and planets, comets, etc., rotate the sun by following Kepler's law of planetary motion.

The distance between these planets is millions to billions of km. Each planet has a different core composition, surface composition, atmosphere, mass, and size due to its position with respect to the Sun; the closer the planet, the hotter the planet, and the farther away the planet, the colder the planet, with some exceptions, of course. These properties of the planets depend on the availability of matter around the 'planet-forming mass'. If there is limited availability of gasses and matter around the 'planet-forming mass', the object would not become a successful planet. It would then be termed as a dwarf planet.

The dwarf planets are smaller astronomical objects that are not planets, and they have not managed to gain enough mass. This could be due to many reasons, such as less availability of gaseous and molecules around the object while forming, or maybe be presence of a bigger astronomical object that eats up more gasses from the region and, thus, allows the object to consume much material from around it. For this reason, these objects are not termed as planets, but they are known as dwarf planets [1]. A technical definition that planets must satisfy is (i) size condition - In the local region of space, the object must be larger compared to the surrounding objects. (ii) mass condition - In the local region of space, the gravity of the astronomical object must be the highest, such that it dominates the local space gravitationally. (iii) neighborhood condition - It removes most of the objects around it from its orbit, such that not many objects are present near, nor the orbit must overlap with any other orbit. Most of the dwarf planets, for example, Pluto, miss the (iii) condition, and hence Pluto was later declared as a dwarf planet[2]. Some of the biggest size dwarf planets are found far from the orbit of Neptune and their distance is so large that doing an observational study to determine their orbital period around the sun is technically impossible. Thus, in this paper, we aim to theoretically determine the orbital period of a list of the biggest dwarf planets in the solar system using Kepler's laws of motion. The dwarf planets are Pluto, Eris, Haumea, Makemake, Gonggong, Quaoar, Sedna, Orcus and Salacia. The majority of dwarf planets are found after the orbit of Neptune, thus making them in a location that is billions of km away from the Earth. This creates two technical issues: (i) The size of these objects is relatively small. Thus, it is hard to detect them using a telescope. (ii) Due to their large orbital distance, dwarf planets have a very large orbital time period, for example, 250 years for Pluto. Thus a direct detection of these planets and estimation of their orbital time period is technically an insensitive process. In a simplistic model of Kepler's law, the mass of the dwarf planets can be neglected in comparison to the Sun's mass; it contains about 99.8 percent of the total mass of the solar system. In this paper, the Kepler laws are used as a theoretical tool to measure the orbital period of dwarf planets such as Pluto, Eris, Haumea, Makemake, Gonggong, Quaoar, Sedna, Orcus and Salacia.

Clyde W. Tombaugh discovered Pluto in the year 1930; the discovery was a result of a 25 years of long search. With the technology of the mid 20th century, not much information could have been able to be extracted about the planet from the observational search. The three parameters that were known about Pluto were its distance from the Sun, eccentricity and angle of inclination 49.6 AU, 0.25, 17 degrees. At the end of the 20th Century, with an advancement in technology, Pluto's surface could be seen using a telescope. It was found that Pluto has volatile CH_4, ice N_2 and CO [3]. Eris is orbiting the Sun at a distance of 98 AU, with an unusually high inclination of 44 degrees. With the application of the William Herschel telescope, the obtained spectrum of the surface suggests that it has methane ice and N_2 on the surface of Eris [4]. Haumea is an interesting dwarf planet. It is the first object in the Kuiper belt to be known to have rings [5]. It is present at a distance of about 36 AU, and not much is known about its surface composition. It has two moons named Namaka, which is the inner moon and Hilaka, which is the outer moon of the dwarf planet. Makemake is present at a distance of about 45 AU from the sun, and it has a diameter of about 1400 km in size. On the surface, CH_4 and N_2 are found, which is similar to that of the surface of Pluto [6]. Gonggong is 1000 km in size, placed at a distance of about 88 AU from the sun and has an eccentricity of about 0.3 [7]; Quaoar is of size of about 1200 km in size and present at a distance of 42 AU, whereas the size of Sedna is about the same size of that of Quaoar. The surface composition of Gonggong, Quaoar, and Sedna was studied not until recently with the James Webb telescope the surface contents, (C_2H_4, H_2O, CO_2), (C_2H_4, H_2O, CO_2 , HCN), and (C_2H_6, C_2H_2 , C_2H_6), respectively [8]. Orcus has a diameter of about 960 km, and spectroscopy of the surface shows the presence of icy methane and water. Orcus is present at a distance of about 48 AU from the Sun. Salacia is as big as Ceres, but it is far away from Ceres; Salacia is presented at a distance of about 42 AU from the sun. It has a diameter of 900 km. It can be seen that all of the dwarf planets have a surface composition which can be studied using spectroscopy using the advanced technological instruments on telescopes such as Hubble and James Webb telescopes.

The scientific study of these dwarf planets is crucial, and it will reveal some core information about (i) planetary system formation, (ii) planet formation (iii) atmosphere formation dynamics, which includes global warming on planets. In this paper, a key parameter of dwarf planets, which is the orbital period, that is, the time required to complete one complete revolution about the sun, is estimated by using Kepler's third law as a theoretical tool.

2. Methodology

This section discusses the methodology used to conduct the research.

2.1. Aim

To determine the orbital time period of revolution around the sun of dwarf planets Pluto, Eris, Haumea, Makemake, Gonggong, Quaoar, Sedna, Orcus and Salacia using Kepler's third law of motion and Python.

2.2. Research Design

Firstly, using Kepler's third law the mass of the sun will be calculated using the information of orbital distance and time period of the planets in the solar system. Once the mass of the sun is determined, using the distances of the dwarf planet and the mass of the sun, the theoretical time period of the dwarf planets will be calculated.

2.3. Tools Used

For calculations and plotting, Python is used.

2.4. Data Collection Procedure

Scientific papers and authorized/legal websites are used to collect information on planet's distances and time periods, and dwarf planet's distances.

3. Results and Discussion

The dwarf planets are planets that miss at least one of the conditions: (i)is by far the largest object in its local population, (ii) is the gravitationally dominant object in the region along its orbit, (iii) has cleared the neighborhood around its orbit.

The list of the biggest dwarf planets in the solar system is mentioned in the below table:

Table 1. Orbital distances of dwarf planets from the sun in AU

Dwarf Planet Name	Semi-Major Axis(in AU)
Pluto	39.236 [9]
Eris	96.375[10]
Haumea	36.694[10]
Makemake	47.055[10]
Gonggong	88[11]
Quaoar	42[8]
Sedna	83.646[10]
Orcus	48.07[10]
Salacia	42.359[12]

We will calculate the time period of the dwarf planet using Kepler's third law of motion, which states:

$$R^3/T^2 = GM/4\pi^2 \tag{1}$$

Where,

T - Orbital time period of revolution the (dwarf)planet

R - Semi-major distance of the (dwarf)planet

G - Universal constant of gravity

M - Mass of sun

Table 1 provides the semi-major axis distance of all the dwarf planets, to use Equation (1) to calculate the orbital time period of revolution (T), we need to calculate the mass of the sun. To calculate the mass of the sun, we will use the eight planets' orbital distance and time period. The mass of the sun can be calculated in two different methods: Method 1 - Average Method: To calculate the mass of the sun using Equation (1) for all planets separately and then taking the average of all the masses. Method 2 - Graphical Method: To plot R^3 vs T^2 and calculate the slope of the line. The slope of the line can be solved to get the mass of the sun. In this paper we will be using both the methods - average and graphical method to determine the mass of the sun.

Table 2. Planets distance in AU from Sun and orbital period of revolution in the year

Planet Name	Semi-Major Axis(AU)	Time Period(Year)
Mercury	0.3870[13]	0.2408[15]
Venus	0.72822[14]	0.6153[16]
Earth	1	1
Mars	1.5236[15]	1.8809[17]
Jupiter	5.202[16,17]	11.86[18]
Saturn	9.9572[19]	29.46[19]
Uranius	20[20]	84[20]
Neptune	30[21]	165[21]

The semi-major axis distance and orbital period of revolution around the sun for the eight planets in the solar system are provided in Table 2

We will calculate the mass of the sun using Kepler's third law of planetary motion, Equation (1). To do this, we will use the orbital period and semi-major axis of the planets, as provided in Table 2. Two different methods can be used to do this: (i) Average method and (ii) Graphical method

Method 1: Average method

In this method, the ratio of R^3/T^2 is calculated for each of the planets. Let us call this ratio Kepler-Ratio(KR); the KR is equal to $GM/4\pi^2$Thus, the mass of the sun can be written in terms of KR as $M = (4\pi^2 * KR)/G$ . . . (2).

The Python code below is used to calculate the Mass of the sun for each of the planets.

Code 1: Calculation of Mass of Sun Using Kepler's Third Law.

```
# Determination of Mass of sun
import numpy as np
import matplotlib.pyplot as plt

# Names of Planet
#planets = [‘mercury’, ‘venus’, ‘earth’, ‘mars’, ‘jupiter’,
‘saturn’, ‘uranus’, ‘neptune’]
# distances in AU
# 1AU = 1.49*10^11 m - distance between Earth and the Sun
distances = [0.3870, 0.72822, 1, 1.5236, 5.202, 9.9572, 20,
30]
# orbital period is the time required to complete one
complete revolution around
# The sun
# The orbital period is given in years
orbital_period = [0.2408,0.6153, 1, 1.8809, 11.86, 29.46, 84,
165]
# we aim to calculate the kepler constant
constant = np.zeros(len(distances))
for i in range(0, len(distances)):
  constant[i] =
(distances[i]*1.49*10**11)**3/(orbital_period[i]*365.256*2
4*60*60)**2

# To get the value of the mass of sun
sun_mass = np.zeros(len(distances))
for i in range(0, len(distances)):
  sun_mass[i] = (constant[i]*4*np.pi**2)/(6.67*10**(-11))

sun_mass_average = np.average(sun_mass)
print(sun_mass_average)
```

Output: The average mass of the sun is $2.0355 * 10^{30}$ *Kg.*

Method 2: Graphical Method

In this method, a R^3 vs T^2 is plotted, and the slope of the line is calculated. The slope of the line gives the Kepler-Ratio. This will later be solved to get the mass of the sun. The below Python code is used to calculate the mass of the sun.

Code 2: Graphical Method

```
# Determination of Mass of sun
import numpy as np
import matplotlib. pyplot as plt
from scipy.optimize import curve_fit

# Names of Planet
#planets = [‘mercury’, ‘venus’, ‘earth’, ‘mars’, ‘jupiter’,
‘saturn’, ‘uranius’, ‘neptune’]
# distances in AU
# 1AU = 1.49*10^11 m - distance between Earth and the Sun
distances = [0.3870, 0.72822, 1, 1.5236, 5.202, 9.9572, 20,
30]
# orbital period is the time required to complete one
complete revolution around
# The sun
# The orbital period is given in years
orbital_period = [0.2408,0.6153, 1, 1.8809, 11.86, 29.46, 84,
165]
# we aim to calculate the kepler constant
distances_meters_cube = np.zeros(len(distances))
orbital_period_seconds_squard = np.zeros(len(distances))
for i in range(0, len(distances)):
  distances_meters_cube[i] = (distances[i]*1.49*10**11)**3
```

```
  orbital_period_seconds_squard[i] =
(orbital_period[i]*365.256*24*60*60)**2

plt.scatter(distances_meters_cube,
orbital_period_seconds_squard)
plt.plot(distances_meters_cube,
orbital_period_seconds_squard)
plt.yscale('log')
plt.xscale('log')
plt.xlabel("Orbital Period Squared")
plt.ylabel("Orbital Distance from Sun Cubed")
plt.title("Orbital distance cubed vs orbital period squared")
plt.show()

slope = (distances_meters_cube[6] -
distances_meters_cube[1])/(orbital_period_seconds_squard[6
]- orbital_period_seconds_squard[1])

mass = (slope*4*np.pi**2)/(6.67*10**-11)
print(mass)
```

Output: The R^3 vs T^2 graph is shown in Figure 1.

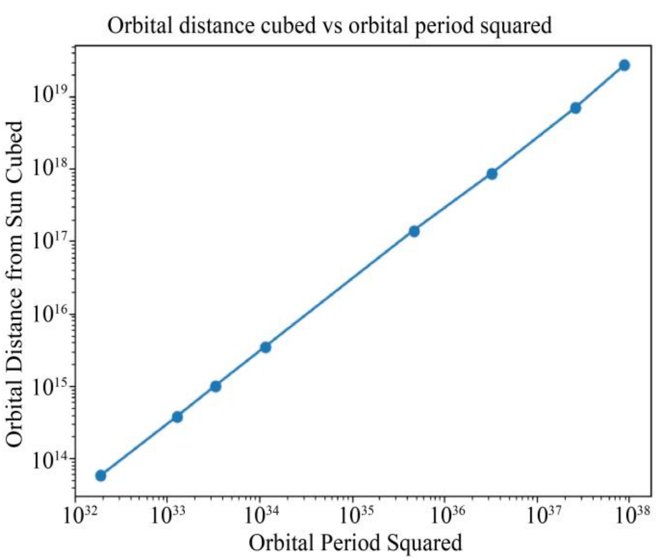

Fig. 1 R^3 vs T^2 plot

To calculate the slope, the second and seventh data points are chosen. The mass of the Sun from this method is 2.2289 * 10^{30} Kg. For the calculation of the orbital period of the dwarf planet, the mass of the sun will be taken as

$M = (2.0355 * 10^{30} + 2.2289 * 10^{30})/2 = 2.1322 * 10^{30}$ Kg . . . (3),

that is the average of the two methods

The code 3 is used to determine the time period of the dwarf planet. For this, the distances of the dwarf planet are taken from Table 1, and the mass of the sun from Equation 3.

Code 3: Orbital Period of Dwarf Planet

```
mass = ((2.22+2.03)/2)*10**30
distance = [39.236, 96.375, 36.694, 47.055, 88, 42, 83.676,
48.07, 42.359]
orbital_time = np.zeros(len(distance))

for i in range(0, len(distance)):
  orbital_time[i] = (((4*np.pi**2)/(6.67*10**(-11)*mass)*
(distance[i]*1.49*10**11)**3)**0.5)/(365.256*24*60*60)

print(orbital_time)
```

Output: The orbital time of the dwarf planets in years from the above codes is provided in Table 3.

Table 3. Orbital periods of dwarf planets in years

Dwarf Planet Name	Orbital Time Period in Years
Pluto	236.3920
Eris	910.0230
Haumea	213.7954
Makemake	310.4662
Gonggong	794.0169
Quaoar	261.8060
Sedna	736.2192
Orcus	320.5656
Salacia	265.1699

4. Comparison

The orbital time period of dwarf planets can be estimated by using photometry, which is a method of measuring the light intensity of the dwarf planet with respect to the background reference star. The limit of this study is to have a high resolution of the instrument to be able to resolve the dwarf planet in the recorded images. For instance, A failed study on Eris by Rosemary Pike was done using time series photometry, in which 266 images had been collected with reference to 33 background reference stars, the data was collected using the swift ultraviolet optical telescope for 29 days. Due to low resolution, the orbital time period could not be estimated [22]. Not much study is done on other dwarf planets except Pluto and Eris. The best estimations are given by the team of Mike Brown from Caltech, who discovered the dwarf planet in the year 2005 [23].

The estimated orbital time period, by Mike Brown, is 557 years. Harvard Observatory conducted a study on Pluto's orbital time period in the 20th century. In this study, images of Pluto were taken by Humason on December 27, 1919 and on December 28 and 29 th, 1919. The position of Pluto was measured against six background reference stars. Other observations were taken in 1927 and 1930. The orbital time period of Pluto is about 251 years [24].

Thus, this paper gives a good estimate of the orbital time periods of different dwarf planets which are hard to resolve due to them being too far away. Our estimate of the orbital time period of Eris is 910; whereas the study of Brown suggests it to be about 570 years.

This discrepancy can be due to the fact that the study performed by Brown was a short duration study, thus can lead to error. Nevertheless, the order of magnitude is the same, and the difference in the orbital time period, as measured in this paper, is about 42 percent more than Brown et al.

The estimated orbital time period of Pluto by Nicholson is about 251 years, whereas this paper estimates it to be about 236 years. Thus, the difference is only about 5 percent.

Another reason for the difference in the calculations of the orbital time period of dwarf planets of different studies with this paper's estimates could be due to a high eccentricity and inclination of the dwarf planet about the solar system plane and perturbation in dwarf planets orbit due to gaseous planets in the solar system.

5. Conclusion

Kepler's laws of motion are a powerful theoretical tool to calculate the orbital time period of planets that are very far away from the Sun, which makes it technically hard to measure their time period with observational study. The biggest dwarf planets are found after the orbit of Neptune - Pluto, Eris, Haumea, Makemake, Gonggong, Quaoar, Sedna, Orcus, and Salacia. In this paper, the third law of Kepler is used to calculate the orbital time period of the above-mentioned dwarf planets. To do so, firstly, the mass of the sun is calculated using the semi-major axis and orbital time period of eight of the planets, namely, Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune. Two different methods were used to calculate the mass of the sun, namely the average method and the graphical method, and both methods resulted in almost the same mass of the Sun(up to the order of magnitude). Using the mass of the sun, we calculated the theoretical orbital period of the dwarf planets, which was otherwise impossible due to a very long time duration. The time period of the dwarf planets is provided in Table 3.

Limitation

- In this paper, it was assumed that the general form of Kepler's law given by Equation (1) is valid for all dwarf planets, independent of their eccentricity and orbital orientation.
- The slope is calculated assuming a straight line can pass through all the data points in the graphical method; for better results the curve fit method could have been used.
- The error in the orbital time period could not be calculated due to the limited availability of data.

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