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Original Article

First Order of Magnitude Calculations of Strain of Gravitational Waves from Man-Made sources, Pulsar, Supernova, and Binary Black Holes and Omni-directionality of Laser Interferometer Gravitational-Wave Observatory(LIGO) using Antenna Pattern Function and Simple Trigonometry

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Abstract - *Gravitational Waves (GW) are energy spreads in the form of disturbance in the spacetime fabric. GW is produced when the quadrupole moment of the system changes; the quadrupole moment is the geometric distribution of the particles in the system. Passage of gravitational waves from an object changes the shape and size of the body, and it is measured in terms of gravitational wave strain, which is the relative change in the size of the object. This paper presents two findings: (i) first-order calculations of strain produced by man-made source, pulsar, supernova and binary black hole collision. The strain calculations are done using the first-order approximation of the change in the quadrupole moment of the system. The order of magnitude of strain by man-made source is of the order of 10^{-43}, by a rotating pulsar is 10^{-26}, and by supernova and binary black holes are of the order 10^{-21}. The second finding of the paper is (ii) simple proof of omni-directionality to LIGO and VIRGO detectors, except for $\pi/2$ and $3\pi/2$ angles with respect to the plane of the detector. LIGO and VIRGO are gravitational wave detectors that work on the principle of laser light interferometer.*

Keywords - *General Theory of Relativity, Gravitational Waves, Man-Made Source, Black Holes, Pulsar, LIGO, Omni Directionality.*

1. Introduction

Black holes, Expansion of the Universe, Einstein's Cross and Gravitational Radiations were some of the key predictions of Einstein's General Theory of Relativity(GTR) [1, 2, 3]. Black holes are objects with a few times the mass of the Sun. The gravity of a black hole is so strong that even light cannot escape from inside of a black hole, or to say, can never come out of the event horizon [4]. The expansion of the universe and the existence of Dark Matter and Dark Energy[5, 6] are also predicted by Einstein's GTR and hold a theoretical background for explaining the cosmic microwave background radiation and the Big Bang [7, 8]. Bending of light from the edge of massive objects such as the Sun [9] or from galaxies [10], also known as Einstein's Cross, are all known astronomical observations. Gravitational Waves were predicted by Einstein in the year 1916-17 [11], and the first astronomical evidence was found in the year 1964 with the discovery of the binary pulsar PSR B1913+16. The orbital decay of this binary pulsar was satisfying the calculations of GTR [12]. The orbital decay of binary pulsars and the emission of gravitational radiation are the multiple sources of gravitational waves.

Einstein's General Theory of Relativity holds true, and there are multiple strong theoretical as well as observational-supporting astrophysical phenomena observed in the past and this century. A few of the supports are the perihelion shift of the Mercury's orbit holds theoretical as well as observational support [13], the detection of Cosmic microwave radiation in the year 1963, which holds observational support [14], capturing the first-ever image of a supermassive black hole M87 using the event horizon telescope in the year 2017 [15]. The detection of gravitational waves from the binary pulsar is an indirect detection of gravitational waves and not a direct one; it took around 100 years after the prediction of gravitational waves to detect the first gravitational wave signal from the collision of two black holes by the twin LIGO detectors on 14th September 2015, an event titled as GW150914 [16, 17]. This event has registered a signal from the collision of two stellar-mass black holes of mass $36\ M_{\odot}$ and $29\ M_{\odot}$ that merged to form a black hole of mass $62\ M_{\odot}$ and releasing an energy of $3\ M_{\odot}$ as gravitational radiation, this was the first-ever direct detection of gravitational waves. Till now, 10's signals from the collision of black holes and neutron stars have been detected. The discovery of gravitational waves by LIGO detectors won the Nobel Prize in physics in the year 2017 to Weiss, Barish and Thorne for their contribution. With this discovery, a new field in astronomy has emerged, known as gravitational waves astronomy.

Gravitational waves are produced when heavy mass objects, such as black holes and neutron stars, change their shape or when they create a change in the spacetime structure around them. These waves travel at the speed of light and carry energy with them; gravitational waves are the fluctuations of spacetime itself, and when they pass through an astronomical object such as a planet, they change the shape and size of the planet. Some of the strongest sources of gravitational waves are collisions of black holes or neutron stars, asymmetry of the surface of a neutron star and supernova. These are some of the strongest sources of gravitational waves [18]. This makes it extremely difficult to detect gravitational waves as the quantum fluctuations in any device at room temperature could be a million times more than that of this strain. Thus advancement in cryogenic physics was required to make this detection possible [19]. Different types of detection devices were invented; two of the most famous were Weber's Bar and Wave Interferometer. In 1966, Joseph Weber invented a device, which is now known as Weber's bar, and it uses transducers that would pick up the pressure difference created by the passing of gravitational waves from the bar and amplify it for data collection. Weber's bar never detected any detection as the noise in the instrument was way more than the gravitational wave signal strength. The other device is known as a Wave Interferometer; it uses a laser beam and splits it into two equal-intensity beams that travel at 90 degrees with respect to each other.

The two beams reflect back after traveling the same distance from a mirror and are made to interfere with a phase difference of 180 degrees. Thus, the resultant interference is a destructive interference and, specifically, a null interference. When gravitational waves pass through these two arms of the interferometer, they change the length of these arms simultaneously in such a way that the two split beams of laser phase difference are no longer 180 degrees and this creates a non-zero intensity interference on the screen. This is the fundamental principle of LIGO detectors as well. There are two twin LIGO detectors in the USA, one at Handford, Washington and the other in Livingston, Louisiana. Some other important detectors are VIRGO in Italy, KAGRA in Japan, and GEO600 in Germany. In the future one LIGO detector will be built in India in the Hingoli district, and one detector named LISA will be sent into space for gravitational waves detection from supermassive black holes.

The detection of gravitational waves depends on (i) a correct measurement of the range of the gravitational wave strain and (ii) its passage from the gravitational wave detector. In this paper, we will study the first-order calculations of strains of gravitational waves from different possible sources, such as man-made sources, pulsars, supernovas and binary black holes. We will also be studying the direction dependency of detection for the LIGO detectors; in other words, we will try to understand whether the LIGO can detect gravitational waves coming from any direction in space or whether it has some limitations. For comparison, a light-collecting optical telescope can only collect light that enters the aperture of the telescope and it cannot see the entire sky but a small part of the sky at a time.

2. Methodology

This section discusses the methodology used to conduct the research.

2.1. Aim of the Study

The study aims to assess the order of magnitudes of gravitational wave strain from man-made sources, pulsar, supernova, and binary black holes, as well as the omni-directionality of LIGO/VIRGO detectors.

2.2. Research Design

This paper utilizes the first-order formula of gravitational wave strain to calculate theoretical limits from various sources. Also, the study of the properties of the antenna pattern function is done to find out the omni-directionality of LIGO/VIRGO detectors.

2.3. Tools Used

General Theory of Relativity, Basic Trigonometry and Algebra.

4. Result and Discussion

According to Alber Einstein's General Theory of Relativity, Gravitational Waves (GW) are produced when a body or system of masses undergoes acceleration or when there's a quadrupole moment of the system changes[20]. Quadrupole moment is a physical parameter that measures the orientation of the masses of the system. A drastic change in quadrupole moment generates gravitational wave strain, which is typically denoted by the letter 'h'. In this section, we will calculate the gravitational wave strain for man-made sources, pulsar, supernova, and binary star systems. The first-order strain can be calculated by calculating the change in the quadrupole moment of the system:

$$h = 2MG{v_{N.S}}^2/rc^4 \quad ...(1)$$

Where M is the mass of the source, r is the distance between the source and the detector, and $v_{N.S}$ stands for the velocity of the non-spherical part of the system. Using this formula, we will calculate the gravitational wave strain 'h' for different possible man-made and astrophysical sources.

3.1. Man-Made Sources

Man-Made sources are any sources that are present on earth or the result of an experiment that is done on earth. It includes gravitational waves emitted due to the acceleration of a car, ferris wheel, etc. We will perform two thought experiments: (i) A car performing circular motion, (ii) A dumbbell-shaped heavy object mimicking two objects rotating around their center of mass.

3.1.1.Car Performing Circular Motion

Consider a car of mass 2000 kg and performing a circular motion with a speed of 50 m/s. In this case, $M = 2000\,kg$, $v_{N.S} = 50\,m/s$ and r, that is, the distance between the source and the detector must be at least one wavelength distance apart from the source. Let's say that the car is covering 20 circles per second, thus $f_{orbiting} = 50\,Hz$ using, $f_{gravitational-waves} = 2f_{orbiting}$, we get, $f_{gravitational-waves} = 100\,Hz$. As we also know that the speed of gravitational waves is equal to the speed of light, using this, we can calculate the wavelength of gravitational waves to be $\lambda_{gravitational-waves} = c/f_{gravitational-waves} =$

$$3 \times 10^8/100 = 3 \times 10^6\,m$$

Then,

$$r = \lambda_{gravitational-waves} = 3 \times 10^6 m$$
$$M = 2000\,kg$$
$$v_{N.S} = 50\,m/s$$

The gravitational wave strain is,

$$h = 2MG{v_{N.S}}^2/rc^4$$

$$= (2 \times 2000 \times 6.67 \times 10^{-11} \times 50^2)/(3 \times 10^6 \times (3 \times 10^8)^4)$$

$$h = 2.75 \times 10^{-45}$$

3.1.2. A Dumbbell-Shaped Heavy Object Mimicking Two Objects Rotating Around their Center of Mass

A dumbbell-shaped object with each blob of mass $10^4 kg$ rotating about its center of mass with a thin massless rod. The rod rotates at a frequency of 50 times per second, that is, $f_{orbiting} = 50\,Hz$ and at a velocity of $v_{N.S} = 50\,m/s$.

Then,

$$r = 3 \times 10^6 m$$
$$M = 10^4\,kg$$
$$v_{N.S} = 50\,m/s$$

The gravitational wave strain is,

$$h = 2MG{v_{N.S}}^2/rc^4$$
$$= (2 \times 10^4 \times 6.67 \times 10^{-11} \times 50^2)/$$
$$/(3 \times 10^6 \times (3 \times$$

$$h = 1.37 \times 10^{-43}$$

The strain produced in these man-made experiments is impossible to detect as quantum fluctuations are higher in magnitudes than this [21].

3.2. Gravitational Wave from Pulsar

A Pulsar is a neutron star rotating at a very fast rate. According to some models of neutron stars, neutron stars are not fully spherical in shape but can have irregularities, that is, bumps on their surface [22]. Let say that a neutron star has a mass, radius, and frequency of spinning, M, R and f, respectively. Considering the mass of the bump to be 10^{-5} times the mass of the pulsar M.

The time period of spinning a crab pulsar is 33.5 ms, its radius is around 10 km, and it is at a distance of 2200 pc [23].

Then,

$$T = 33.5\,ms$$
$$R = 10\,km$$
$$r = 2200\,pc$$
$$M = 2.78 \times 10^{30}\,kg$$
$$m = 10^{-5} M$$

The gravitational wave strain is,

$$h = 2mG{v_{N.S}}^2/rc^4$$
$$v_{N.S} = 2\pi Rf = 2 \times \pi \times R \times f$$
$$= 2 \times \pi \times 10^4 \times \frac{1}{33.5 \times 10^{-3}} = 1.87 \times 10^6 m/s$$

$$h = (2 \times 10^{-5} \times 2.78 \times 10^{30} \times 6.67 \times 10^{-11} \times (1.87 \times 10^{6})^{2})/(2200 \times 3 \times 10^{16} \times (3 \times 10^{8})^{4})$$

$$h = 2.45 \times 10^{-26}$$

This strain is not in the detectable range of LIGO/VIRGO. Using long-term detection techniques, one can still detect the gravitational waves from a pulsar with confidence.

3.3. Supernova

Massive stars die of some of the most violent events in the universe. Depending on the star's mass, it either becomes a black hole or neutron star, in a process known as a supernova. In a supernova, the star, which has most of its mass, collapses inward within a fraction of a seconds. If the collapse is unsymmetrical, then the center of mass will accelerate, and this will create gravitational waves. For a supernova, the gravitational wave strain is given by $h = \frac{1}{\pi f r}\sqrt{\frac{E}{T}}$. 'E' is the energy emitted in the form of gravitational waves and 'T' is the time required for supernova collapse.

Then,

$$r = 10^{4}\,pc$$
$$E = 10^{-7} M_{\odot} c^{2}$$
$$T = 1\,ms$$

The gravitational wave strain is,

$$h = \frac{1}{\pi f r}\sqrt{\frac{E}{T}}$$

$$h = \frac{1}{3.14 \times 10^{3} \times 10^{4} \times 3 \times 10^{16}}\sqrt{\frac{10^{-7} \times 2 \times 10^{30} \times (3 \times 10^{8})^{2}}{10^{-3}}}$$

$$h = 8 \times 10^{-21}$$

This strain is in the detectable range of LIGO and VIRGO.

3.4. Binary Star System

A thought experiment involves considering two equal-mass stars, each of mass M. These stars revolve around the center of their mass with a frequency of 'f' and are located at a distance 'r' from the earth, and R be the radius of the circular orbit of the binary star system.

Then,

$$M = 30 M_{\odot}$$
$$f = 100\,Hz$$
$$r = 400\,Mpc$$

The gravitational wave strain is,

For a circular binary star, the strain is given by $h = (2/r) M^{5/3} (2\pi f)^{2/3}$.

Thus,

$$h = (2/400 \times 10^{6} \times 3 \times 10^{16}) \times (30 \times 2 \times 10^{30})^{5/3} \times (2 \times \pi \times 100)^{2/3}$$

$$h = 10^{-20}$$

This strain is in the detectable range.

Detailed information on the source, strain and detection status is provided in Table 1.

Table 1. Gravitational wave source and strain magnitude

Sr No	Type	Source	Strain	Detection
1	Man-Made	Accelerating Car	2.75×10^{-45}	Technically Undetectable
2	Man-Made	Rotating Dumbbell	1.37×10^{-43}	Technically Undetectable
3	Astrophysical	Pulsar	2.45×10^{-26}	Not yet detected
4	Astrophysical	Supernova	8×10^{-21}	Not yet detected
5	Astrophysical	Binary Star System	10^{-20}	Detected

3.5. Antenna Pattern Function

General Theory of Relativity predicts two independent polarization states of gravitational waves, known as "plus polarization" and "cross polarization", typically denoted by h_+ and h_-.

$$h_+ = \frac{h_o}{2}(1 + cos^2\iota)cos\phi(t) \quad (2)$$

$$h_- = h_o cos\iota sin\phi(t) \quad (3)$$

Where, "h_o" depends on the mass of the system, luminous distance and the orbital frequency. "$\phi(t)$" is the signal's phase and "ι" is the angle made orbital momentum vector with the line of sight. In general, the produced strain is a combination of these two independent polarizations. The response of the gravitational waves on the LIGO/VIRGO detector can be better understood with what is known as antenna pattern functions; these are the responses of the gravitational wave detector when gravitational waves pass through them.

$$F_+ = \frac{1}{2}(1 + cos^2\theta)cos2\phi cos2\psi - cos\theta sin2\phi sin2\psi \quad (4)$$

$$F_x = \frac{1}{2}(1 + cos^2\theta)cos2\phi sin2\psi + cos\theta sin2\phi cos2\psi \quad (5)$$

The angular parameters θ , ϕ , and ψ are defined according to Figure 1.

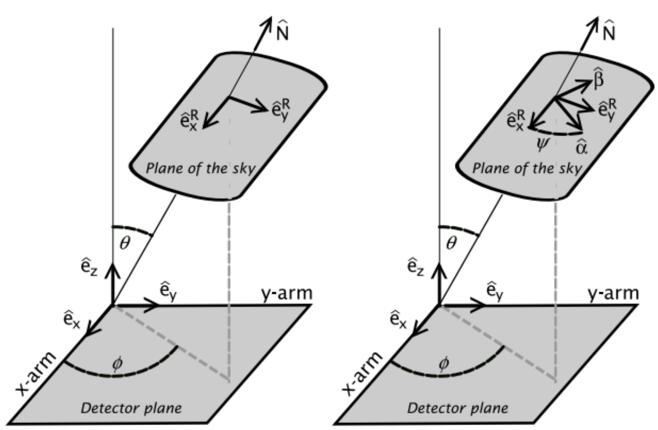

Fig. 1 Antenna pattern function angular parameter definition

For an inspiring circular rotating binary black hole system, the corresponding strain can be calculated as a combination of plus and cross polarization, and the antenna pattern functions.

$$h(t) = F_+h_+ + F_xh_x \quad (6)$$

This can be simplified further as a cosine function using some defined variables.

$$A_+ = \frac{1}{2}F_+(1 + cos^2\iota) \text{ and } A_x = F_x cos\iota \quad (7)$$

Using, A_+ and A_x, we can simply h(t) further as:

$$h(t) = Ah_o cos(\phi(t) - \phi_o) \quad (8)$$

Where,

$$A = \sqrt{A^2{}_+ + A^2{}_x} \text{ and } tan\phi_o = A_x/A_+ \quad (9)$$

3.6. Omni Directionality of Gravitational Wave Detector - LIGO/VIRGO

In this section, we will show the omnidirectionality of the gravitational wave detector using the antenna pattern function (equations 4 and 5). This could be done using simple trigonometry and indirect methods of finding out conditions on the directional parameters for which the gravitational wave strain is null, even when an actual passage of the gravitational wave signal from the detector is within the detectable range.

From equation 6,

$$h(t) = F_+h_+ + F_xh_x \quad ...(6)$$

The equation (6) is the effective response that is recorded by the gravitational wave detectors. Let us consider an astrophysical even that produces strain which is in the detectable region of LIGO/VIRGO detectors, thus h_+ and h_- are non-zero, intrinsically. Let us try to obtain conditions on the directional parameters, which are θ, ϕ, and ψ, such that for which F_+ and F_- are null. Without the loss of generality, let us set $\psi = 0$, the detection patterns becomes(from equation 4 and 5):

$$F_+ = \frac{1}{2}(1 + cos^2\theta)cos2\phi \quad (10)$$

$$F_x = cos\theta sin2\phi \quad (11)$$

Both the above equations, 10 and 11, cannot be set to zero simultaneously by choosing any value of ϕ. Thus, we set equation 11 to null; by setting $\theta = \pi/2$, we get $F_x = 0$, we. Substituting $\theta = \pi/2$ in equation number 10, we get,

$$F_+ = \frac{1}{2}cos2\phi \quad (12)$$

Thus, to set $F_+ = 0$, the $cos2\phi$ term has to be zero, which is true for $\phi = \pm\pi/4$ and $\phi = \pm3\pi/4$

4. Comparison

LIGO twin detectors and VIRGO detectors are designed to detect gravitational waves in a certain frequency and strain amplitude region, and the reason is that for different frequencies of the gravitational waves, the external noise increases, thus a decrease in the signal-to-noise ratio. These detectors are created to detect gravitational waves from 'compact binary inspiral gravitational waves', 'burst of gravitational waves' and 'stochastic gravitational waves'. In the category of 'compact binary inspiral gravitational waves', one can have (i) binary neutron star collision, (ii) binary black hole collision, and (iii) black hole and neutron star collision.

LIGO twin detectors detected the first detection of a gravitational wave on September 14, 2015, and the event formally known as GW151226 is from the merger of two black holes of mass $36M_{\odot}$ and $29M_{\odot}$, with the final black hole mass being $62M_{\odot}$ and $3M_{\odot} \times c^2$ amount of energy released in the form of gravitational waves. The peak gravitational wave strain has an amplitude of 10^{-21} [24]. The estimated order of magnitude of the strain is $10^{-20} - 10^{-21}$, hence in the range of the detected peak strain. GW170717 is an event of observation of gravitational waves from a binary neutron star inspiral. The event was recorded on August 17, 2017. The masses of the neutron star were estimated to be in the range of 1.17 - 1.60 $M_{\odot}$, the total mass of the system was $2.74M_{\odot}$, the peak gravitational wave strain is about 8×10^{-20} [25], again in the range as estimated by this paper.

Detection of gravitational waves due to a rotating neutron star and a supernova has not yet been recorded by the LIGO detectors. This could be due to two main reasons: (i) the signals are not in the working frequency range of LIGO, i.e. higher than that of LIGO's working frequency and secondly, (ii) The signal is weaker than that of the detectable range of LIGO working strain region. Also, higher masses of black holes, i.e. supermassive black hole collisions, are not yet detected by LIGO, and this is due to the higher frequency of the gravitational waves produced by supermassive black holes. The LIGO team is planning to create a space-based gravitational wave observatory, which would be called LISA. The mission is in progress, and the launch is planned for the mid 2030's. In the paper of LIGO: The Laser Interferometer Gravitational-Wave Observatory, the blind spots are described as the spots on the sky from which the gravitational waves enter the detector from; the LIGO detectors would detect a null-detector. This paper describes the direction of the blind spots with respect to the arms of the detector, which are at 45 degrees and 135 degrees with respect to any of the arms [26].

Thus, the estimates of gravitational wave strain generated in this paper match with the real observational data of gravitational wave collisions of black hole mergers and neutron star mergers. The detection of gravitational waves from supernovae and from a rotating neutron star has not yet been detected, but a hot topic in the field of gravitational waves. With LISA in future, we would expect collision of supermassive black holes and supernovae, and pulsar's gravitational wave data to be detected. In regards to the detection of gravitational waves, to increase the sensitivity of the detector and reduce the chances of a null detection, LIGO is planning to construct a third observatory in India in the Hingoli district of Maharashtra.

5. Conclusion

Detectable gravitational waves are produced by some of the most violent astrophysical events, such as binary black holes or neutron star collapse, supernovas, etc. The produced gravitational wave strain by these events is detectable by LIGO and VIRGO detectors. It is impossible to produce these waves in the detectable region by any experiment on Earth, so-called "Man-Made" sources. Gravitational wave strain by binary black holes and neutron star collisions are detected by LIGO/VIRGO detectors. Even though the strain due to pulsar and supernova is in the detectable region, its detection is much anticipated.

Detectors such as LIGO and VIRGO use a technology known as laser interferometers in which laser light is split into two beams and made to bounce off mirrors after traveling a certain and equal distance and made to interfere. The interference is a null interference when no gravitational waves are passing the detector, and when a partial interference is observed, it is a detection. Gravitational Wave detectors are almost omnidirectional, which means that they can detect gravitational waves coming from all directions except for two specific directions, for which the directional parameters are

$$(\theta = 0, \psi = 0, \phi = \pm\pi/4) \text{ and}$$

$$(\theta = 0, \psi = 0, \phi = \pm 3\pi/4).$$

which means it cannot detect gravitational waves coming at an angle of 45 degrees and 135 degrees with respect to any of the arms.

References

[1] Von A. Einstein, *Approximate Integration of the Field Equations of Gravitation*, Seat of Prussian Academic Sciences, pp. 688-696, 1916. [Google Scholar] [Publisher Link]

[2] Albert Einstein, *Gravitational Waves*, Seat of Prussian Academic Sciences, pp. 154-167, 1918. [Google Scholar] [Publisher Link]

[3] Daniela Bettoni et al., "A New Einstein Cross Gravitational Lens of a Lyman-Break Galaxy," *The Astrophysical Journal Letters*, vol. 873, no. 2, pp. 1-5, 2019. [CrossRef] [Google Scholar] [Publisher Link]

[4] Edwin F. Taylor, John Archibald Wheeler, and Edmund Bertschinger, *Exploring Black Holes*, San Francisco: Addison Wesley Longman. [Google Scholar] [Publisher Link]

[5] G.F.R. Ellis, R. Maartens, and S.D. Nel, "The Expansion of the Universe," *Monthly Notices of the Royal Astronomical Society*, vol. 184, no. 3, pp. 439-465, 1978. [CrossRef] [Google Scholar] [Publisher Link]

[6] D. Comelli, M. Pietroni, and A. Riotto, "Dark Energy and Dark Matter," *Physics Letters B*, vol. 571, no. 3-4, pp. 115-120, 2003. [CrossRef] [Google Scholar] [Publisher Link]

[7] Eric Gawiser, and Joseph Silk, "The Cosmic Microwave Background Radiation," *Physics Reports*, vol. 333-334, pp. 245-267, 2000. [CrossRef] [Google Scholar] [Publisher Link]

[8] Richard H. Cyburt et al., "Big Bang Nucleosynthesis: Present Status," *Reviews of Modern Physics*, vol. 88, no. 1, pp. 1-22, 2016. [CrossRef] [Google Scholar] [Publisher Link]

[9] Kayll Lake, "More on the Bending of Light!," *arXiv*, pp. 1-11, 2007. [CrossRef] [Google Scholar] [Publisher Link]

[10] K.U. Ratnatunga et al., "New "Einstein Cross" Gravitational Lens Candidates in HST WFPC2 Survey Images," *Symposium-International Astronomical Union*, Cambridge University Press, vol. 173, pp. 323-328, 1996. [CrossRef] [Google Scholar] [Publisher Link]

[11] Stuart L. Shapiro, Richard F. Stark, and Saul A. Teukolsky, "The Search for Gravitational Waves: Predicted By Einstein's Theory of General Relativity and Now on the Threshold of Being Detectable, Gravitational Radiation has Launched a New Field of Astronomy," *American Scientist*, vol. 73, no. 3, pp. 248-257, 1985. [Google Scholar] [Publisher Link]

[12] J.M. Weisberg, D.J. Nice, and J.H. Taylor, "Timing Measurements of the Relativistic Binary Pulsar PSR B1913+ 16," *The Astrophysical Journal*, vol. 722, no. 2, pp. 1-5, 2010. [CrossRef] [Google Scholar] [Publisher Link]

[13] Asher Yahalom, "The Weak Field Approximation of General Relativity, Retardation, and the Problem of Precession of the Perihelion for Mercury," *arXiv*, pp. 1-23, 2022. [CrossRef] [Google Scholar] [Publisher Link]

[14] Lyman Page, and David Wilkinson, "Cosmic Microwave Background Radiation," *Reviews of Modern Physics*, vol. 71, no. 2, 1999. [CrossRef] [Google Scholar] [Publisher Link]

[15] Event Horizon Telescope Collaboration, "First M87 Event Horizon Telescope Results. IV. Imaging the Central Supermassive Black Hole," *arXiv*, pp. 1-52, 2019. [CrossRef] [Google Scholar] [Publisher Link]

[16] B.P. Abbott et al., "GW150914: The Advanced LIGO Detectors in the Era of First Discoveries," *Physical Review Letters*, vol. 116, no. 13, pp. 1-12, 2016. [CrossRef] [Google Scholar] [Publisher Link]

[17] Andrzej Królak, and Mandar Patil, "The First Detection of Gravitational Waves," *Universe*, vol. 3, no. 3, pp. 1-12, 2017. [CrossRef] [Google Scholar] [Publisher Link]

[18] Kip S. Thorne, "Gravitational Waves," *arXiv*, pp. 1-25, 1995. [CrossRef] [Google Scholar] [Publisher Link]

[19] Jo van den Brand, "Gravitational Waves: Physics at the Extreme," *European Review*, vol. 26, no. 1, pp. 90-99, 2018. [CrossRef] [Google Scholar] [Publisher Link]

[20] Luc Blanchet, "Quadrupole-Quadrupole Gravitational Waves," *Classical and Quantum Gravity*, vol. 15, no. 1, pp. 89-111, 1998. [CrossRef] [Google Scholar] [Publisher Link]

[21] Craig J. Hogan, "Measurement of Quantum Fluctuations in Geometry," *Physical Review D*, vol. 77, no. 10, 2008. [CrossRef] [Google Scholar] [Publisher Link]

[22] Nathan K. Johnson-McDaniel, "Gravitational Wave Constraints on the Shape of Neutron Stars," *Physical Review D*, vol. 88, no. 4, 2013. [CrossRef] [Google Scholar] [Publisher Link]

[23] C. Germanà et al., "Aqueye Optical Observations of the Crab Nebula Pulsar," *Astronomy & Astrophysics*, vol. 548, pp. 1-7, 2012. [CrossRef] [Google Scholar] [Publisher Link]

[24] B.P. Abbott et al., "GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence," *Physical Review Letters*, vol. 116, no. 24, 2016. [CrossRef] [Google Scholar] [Publisher Link]

[25] B.P. Abbott et al., "GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral," *Physical Review Letters*, vol. 119, no. 16, 2017. [CrossRef] [Google Scholar] [Publisher Link]

[26] B.P. Abbott et al., "Observation of Gravitational Waves from a Binary Black Hole Merger," *Physical Review Letters*, vol. 116, no. 6, pp. 1-14, 2016. [CrossRef] [Google Scholar] [Publisher Link]