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Original Article

Inside the Event Horizon of a Black Hole – A Visible Movement through 1D Space Like Time

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Abstract *- In the present paper, we are mainly focusing on the inversion of the role of space and time inside the event horizon of any black hole. Inside the event horizon, all the world lines are only directed towards the singularity. As a result, every particle has to move towards the singularity – the only destination and destiny, without any deviation in any other direction. Outside the event horizon of the black hole, everywhere, we are aware that the arrow of time is in the forward direction from past to present and from present to future, and we cannot avoid this movement through time. So, there is a clear analogy between our unidirectional movement through 'time' from present to future outside the event horizon and the unidirectional movement of any particle through 'space' from the interior surface of the event horizon towards the singularity. Only the difference between them is that our motion through 1D 'time' outside the event horizon is invisible. In contrast, the motion of any particle through 1D 'space' inside the event horizon is quite visible. Because of this analogy, the inevitable movement of the particle inside the event horizon is 'time-like' movement – as if the particle is moving through time from any present to the future, which is the only destination and destiny and which is nothing but the singularity. Therefore, the role of space and time is interchanged there. We have established this inversion of the role of space and time logically as well as mathematically. As a prerequisite, we discussed briefly at the beginning, spacetime and spacetime curvature, straight path movements of planets and stars, spacetime interval and light cone, the ever constant speed of light, strongest spacetime curvature inside the event horizon.*

Keywords *- Black Hole, Event Horizon, Singularity, Time Dilation, Spacetime, Curvature Of Spacetime, Spacetime Interval, Light Cone, Time Like Space, Space Like Time.*

1. Introduction

Suppose suddenly I ask you all a very simple but curious question, 'Can you stop your movement from today to tomorrow? Or, more generally, can you stop the flow of time? I am sure that all of your answers will be 'No – it is impossible'. So, being on today, tomorrow is a must to come, quite inevitable, which can never be avoided at any cost. It is cent percent certain to come to all of us, more generally to all living beings. We have all moved from our past to present and are warming up (like the warm-up of a footballer or cricketer, etc.) to move to our future. This is what is called by all astrophysicist – "the unidirectional arrow of time" in their own terminology.

Now, let us permit me to slightly elongate the question further by asking you, 'Can you see your movement while you are passing (or moving) form today to tomorrow?' More generally, can anybody see the unidirectional forward flow of time? You may laugh at me and make the answer, 'No, it is quite impossible to see our movement from past to present and thence from present to future. We cannot see the flow of time. We can only see our movement while we are moving in space, but not in time. We cannot see that we are moving from past to present and then from present to future, as we can see, we are moving from one place to another.

Now, suppose I ask you 'Can you move forward, backward, left, right or at any angle?' I am sure that your answer is 'Yes, of course, if there is no obstruction ahead or aside'. Even if there is an obstruction, you can remove it and do the needful. Now, if I ask you, 'This time, can you see your movement?' I am sure that all of you will answer 'yes'. So, all of us can see our movement in space. Our movement in space is quite visible – unlike our movement in time, which is invisible. Now if I ask you, 'Can you stop your movement or alter your direction of movement, while you are moving away at any direction?' I am sure that once again your answer is 'yes'. So, you can stop or change your direction of movement while you are moving in space – unlike your movement through time (whose direction can in no way be changed, you cannot stop moving to the future or you cannot move back to the past!).

Summing those all up:

1. We cannot see our movement through time (we cannot see the flow of time).
2. We cannot stop our movement through time (we cannot stop the flow of time).

Whereas,

3. We can see our movement through space (we can see we are moving from here to there).

4. We can stop or redirect our movement in space (we can stop or change our direction while moving)

Can anyone suggest a miraculous region where, although the flow of time is inevitable (we cannot stop the flow of time), it is quite visible? Similarly, can you suggest a region where the movement in space is somewhat free with some constraints (we can move in any direction of our choice – forward, backward, left, right or at any angle) and we can see our movement?

I am sure that some of you has already guessed that region of miracle, from the title of my topic. Yes, it is inside the event horizon of a black hole.

Summing all those above up, inside the event horizon:

1. The movement of an object through 'time like' space is visible (we can see the flow of redefined 'time')- dissimilar to conventional time.
2. We cannot stop the movement of an object through 'time like' space (we cannot stop the flow of 'time') - similar to conventional time.

Whereas,

3. We can see the movement of the object through 'space like' time (we can see a moving object in redefined 'space') – similar to movement in conventional space.
4. We can stop or redirect the movement of the object through 'space like' time (we can stop or change the direction while moving, with some constraint that the movement should be inside the local light cones each time, otherwise it would exceed the speed of light, which is impossible) – similar to conventional space.

Before we are going to mathematically establish the inversion of the role of space and time, i.e. inversion of the spacetime coordinate system inside the event horizon, let us have some brief preliminary discussion about spacetime and curvature of spacetime due to self-gravity and velocity of an object.

The paper is organized in this way. Section II describes 'spacetime and curvature of spacetime'. Section III describes the 'straight path movement of planets and satellites in a 4D spacetime frame. Section IV describes 'spacetime interval and light cone'. Section V describes why it is impossible to exceed the speed of light.

Section VI describes the strongest spacetime curvature and spacetime distortion inside the event horizon. Section VII describes the inversion of the role of space and time inside the event horizon. Section VIII describes the mathematical establishment of the inversion of space and time inside the event horizon. Section IX is the conclusion.

2. Spacetime and Curvature of Spacetime

Space and time are not different independent dimensions of the events of any object. However, they are a single unique dimension, called 'spacetime' of the event of the object while it is moving along a particular world line.

Spacetime is four-dimensional. In that, three spatial coordinates, 'x', 'y ', and 'z', while the last but not least one is the time coordinate 't'.

We can never absolutely measure either 'space' or 'time' of any object. Thus, we cannot absolutely measure the 3D position of an object without measuring the 'time' of the observer. Similarly, we cannot absolutely measure the 1D time of the incident of an object without measuring the 3D position, i.e. 'space' of the object.

The object, under observation, is having a particular 'space time coordinate frame' – similarly, the observer is also having his own 'spacetime coordinate frame'. So, both the 'space' and 'time' of the moving object are relativistic with respect to the 'spacetime coordinate frame' of the observer at the point of observation.

As an example, suppose there are two observers at two different places (i.e. with different coordinate frames). Also, suppose that two different incidents are occurring in space (they are either close or far away). Further suppose that one observer is measuring that the two incidents are occurring simultaneously, or say just one after another.

The other observer may observe that the two incidents are occurring at significantly different time instants. Thus, the time must be measured in a relative manner. Similarly, the different observers should measure the spatial position of the same object differently due to the difference in their 'spacetime frame'

So, each object in this universe has its own 'spacetime frame', which is different from other objects. It is dependent on the 'velocity' as well as the 'self-gravity' of that object. Thus, in general, the velocity and self-gravity of one observer relative to another heavily affect the measurements (of space and time) of the point of observation made by the observers. These measurements are bound to be different (because they are relativistic to the spacetime coordinate frame).

At low velocities, these differences in measurements are not at all prominent, and these reduce to situations described by classical physics in Newton's Laws of Motion. However, these differences in measurements become heavily prominent only at velocities of the order of light (3×10^{10} cm/sec.).

2.1. Gravity Creates a Curvature of Spacetime

Suppose a continuous beep sound after every 1 sec. is occurring on the surface of a black hole or a neutron star.

When you are measuring the time interval between two beeps from one remote place of an absolute space time coordinate system (it is beyond any influence of gravitational force and is at rest), you will find the time interval is not 1 sec. but much larger – as high as 1 year or even more. It may also happen that you are not hearing the second and subsequent beeps.

Due to the huge self-gravity of the black hole or neutron star, there is a huge curvature of space time. Due to that curvature, the time gap at the remote place of the absolute space time coordinate system is too much elongated.

As an example, if 'T' is the measured time at the remote place while 't' is the time at the huge gravitating body, then

$$T = t/\sqrt{1-(\frac{rs}{r})}$$

Where 'rs' is the event horizon radius of the black hole/neutron star and 'r' is the distance of the black hole/neutron star from the remote place.

As $rs \leq r$, $T > t$

Thus, the measured time gap at the remote place is much higher than the time at the place of occurrence.

2.2. Velocity Creates a Curvature of Spacetime

According to general relativity, it is not only mass, but energy, momentum, and pressure that can cause spacetime to curve or a curvature in spacetime. A high-velocity object has more energy/momentum, therefore causing more curvature.

Suppose an astronaut is moving away from earth and coming back with a spacecraft with an extremely high velocity, say of the order of light. (Of course, an object cannot move with the velocity of light, because in that case its mass would be infinite and thus it would cease any movement). Say the time measured by the astronaut for the to-and-fro journey is 5 Sec.

Observation 1: On coming back after his to and fro journey, the astronaut will find that the earth, along with all its belongings, has grown older by approximately) 50-60 years or even more. This is because the time of 5 Sec of the astronaut (corresponding to his huge velocity and his space time) is different from the time of 50-60 years of earth (corresponding to its comparably low velocity and its space time).

Observation 2: If an observer from earth is watching the movements of the spacecraft, he would have the feeling that the spacecraft is moving very slowly (in comparison to its actual velocity and space-time). On the contrary, if the astronaut is watching the movement of any object on earth, he would have the feeling that it is moving very fast (in comparison to its actual velocity and space time on earth). The spacecraft is having a space time of its own, depending on its velocity, while the observer on earth is having a different space time which depends on his or earths velocity.

The underlying reason behind this is that the 'space time" coordinate system of each object is different. The space time of one object will appear according to the reference space time coordinate system (i.e. of the observer).

As an example, if 't' is the measured time at the object moving at velocity 'v' (very close to the velocity of light 'c') while 'T' is the corresponding measured time at the object which is relatively more or less stationary, then,

$$T = t/\sqrt[2]{(1-(v/c)^2}\ )$$

Where 'v' is the velocity of light.
As $v \leq c$, $T > t$

Thus, the measured time at the object, which is almost stationary, is much larger than the measured time at the object, which is moving with a velocity 'v' very close to 'c'.

3. Straight Path of Movement of Planets and Satellites

In 4D spacetime coordinate system, the planets and satellites are orbiting the sun and the respective planets respectively in a straight path (geodesic or world line) along the curved spacetime created by the sun and the respective planets respectively.

The straight path does not imply a straight linear path. Rather, it is the shortest path between the different events in the world line as selected by the initial condition of the object and the spacetime curvature created by the sun and planets, respectively. No other alternative path is there for the motion.

The projection of this motion or movement in 3D space is essentially elliptical. As the time dimension is invisible to us in a 3D space coordinate system, we only see the projection of this movement in 3D space and the movement appears as elliptical, which is not the reality. When the time dimension is included to make the real 4D spacetime coordinate system, the actual motion is a helical path where the planets and satellites are moving forward in time along the time axis.

So, the planets and satellites are moving in a straight path along a geodesic or world line of the curved spacetime, but its projection appears as an elliptical path in 3D space, while it is actually a helical path moving forward in the time axis in 4D spacetime as well as elliptical in projected 3D space.

4. Space Time Interval – Light Conve

Before we mathematically justify the inversion of the role of space and time and thereby the spacetime coordinate system inside the event horizon of any black hole, let us have some fundamental ideas of space time interval of the two events of an object moving through spacetime, and the preliminary concept of the light cone.

Two successive events A and B of an object are such that information can be carried by a light signal from event A to event B. In other words, A and B are light signal connected. Thus, if B has to be the future event of A, then the light signal should be able to flow from A to B carrying information.

4.1. Four-Dimensional Space-Time Interval

We are presenting below the four-dimensional space-time interval of the two events of a moving object through spacetime.

This interval tells us whether the object moves from one event (say the present event) A to another event (i) B, (ii) C or (iii) D (future event) at a speed.

1. Lesser than that of light (matter particle)
2. Equal to that of Light (light/photon particle)
3. Greater than that of light (wave-particle; practically impossible)

The space time interval between two events P(x1,t1) and Q(x2,t2) is given by:

$$S^2 = (x2\text{-}x1)^2 - c^2(t2\text{-}t1)^2$$

[After dimension normalization of time to distance; 'c' is the velocity of light]

Here we have simplified the situation by considering that the space is represented by only one dimension 'x' and the time by another dimension 't'.

In general, if we consider space is 3 dimensional, then the space time interval between two events P(x1,y1,z1,t1) and Q(x2,y2,z2,t2) would be:

$$S^2 = (x2\text{-}x1)^2 + (y2\text{-}y1)^2 + (z2\text{-}z1)^2 - c^2(t2\text{-}t1)^2$$

[There is a similar terminology entitled Space Time Distance. The Space Time Distance between the above two events, P and Q are:

$$S^2 = (x2\text{-}x1)^2 + (y2\text{-}y1)^2 + (z2\text{-}z1)^2 + c^2(t2\text{-}t1)^2]$$

Space Time Interval S is such that:

4.1.1. Space Like Interval indicates $S^2 > 0$

Space Dominates Time; Object (wave particles, theoretical only; practically impossible – because causality relationship is broken down in that case) moves from event P to event Q at a speed greater than that of light

4.1.2. Time Like Interval indicates $S^2 < 0$

Time Dominates Space; Object (matter particle) moves from event P to event Q at a speed lesser than that of light

4.1.3. Null Interval indicates $S^2 = 0$

None of Space and Time Dominates the Other; Object (photon particle) moves from event P to event Q at a speed equal to that of light.

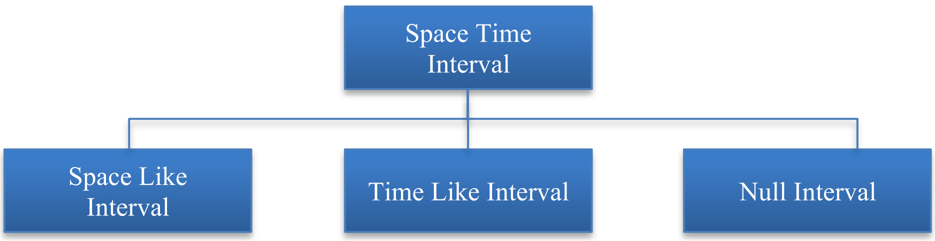

Fig. 1 Different Space Time Intervals

4.2. Light Cone

World lines indicate all possible paths for an object while it is moving from one event to another. However, what world line has to be followed by the moving object is settled by initial conditions (like velocity, acceleration, etc.) of the object.

The light cone indicates what are the allowed world lines when an object moves from event A to event B. An object cannot move from event A to event B along a world line if the world line is outside the light cone. In that case, the causality relationship between the events A and B would be broken down. Over and above the massive particle should attain a mass of infinity when the events of the particle tend to come out of the light cone – with that mass, no motion is practicable.

The light cone is the pictorially determining factor to decide whether an object is a matter particle (moves from one event to another at a speed lesser than light), light /photon particle (moves from one event to another at the speed of light) or wave particle (moves from one event to another at a speed greater than light).

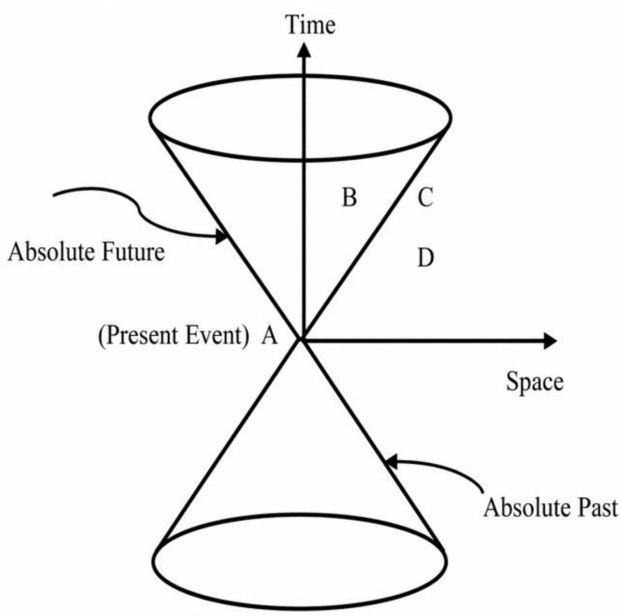

Fig. 2 A Light Cone indicating the location of world lines of a matter particle (B), photon particle (C) and wave particle (D)

If the object is moving with a velocity less than that of light (matter particle), then its world line is within the light cone. I

If the object moves with a velocity equal to that of light (light /photon particle), then the world line of the moving object is on the surface of the light cone.
If the object moves with a velocity greater than that of light (wave-particle, practically impossible), then the world line of the moving object is outside the light cone.

5. It is Impossible to Exceed the Speed of Light

No matter whether it is a massive (having some mass, e.g. electron, proton, neutron, etc.) or massless particle (without any mass, e.g. light particle, photon, graviton, gluon, etc.), it is impossible to exceed the speed of light.

5.1. Massive Particles

Massive particles cannot exceed the speed of light. This is because of the fact that in that case, the mass of the massive particle will approach infinity according to Special Relativity. No motion is practicable with that huge mass of the particle.

According to Special Relativity, if the rest mass is "m0" and the mass in motion is "m", they are related by

$$m = \gamma mo$$

Where $\gamma = 1/\sqrt{(1-(v/c)2}$

Here, v is the velocity of the object. c is the velocity of light.

So, when v = c, m = ∞. With that mass, any motion ceases to continue.

5.2. Both Massive and Massless Particles

Both massive and massless particles cannot exceed the speed of light. For both massive and massless particles, the causality relationship (cause of the event must precede the effect of the event) is broken down or distorted, in the case the particle moves with a speed greater than that of light, i.e. c. Below, we are going to establish these interesting phenomena

5.2.1. Object Moves at a Speed Less Than or Equal to That of Light

Suppose there are two events, A and B, of an object moving at a speed less than or equal to that of light, where A occurs prior to B, and event A is the cause of event B.

A occurs before the occurrence of B
B occurs after the occurrence of A

They should be connected by causality or cause and effect relationship. This is because of the fact that a light signal can flow from A to B, thus making A and B light signals connected. Information can flow from event A to event B through a light signal. Thus,

A is the past event of B
B is the future event of A

Thus, there is both chronological and logical dependency between events A and B.

From the Light Cone angle of view:
The light cone of event B (future event) is inside the light cone of event A (past event). In the two light cones (one inside the other), one world line is there which connects A to B. B is the future event of A. As the object is moving at a speed less than or equal to that of light, A and B are both in 'time like' or 'light like' interval - light can easily flow from A to B carrying information. Thus, B is the future event of event A.

Whatever the spacetime coordinate system frame, we will always be able to recognize that A is occurring prior to B, i.e. A is the cause, and B is the effect. That is, event B is the future event of A. This maintains the causality relationship.

5.2.2. Object Moves at a Speed Greater than that of Light

Suppose the object is moving at a speed greater than 'c'. Moreover, as before, there are two events, A and B, of the object such that A occurs before B and B occurs after A. There is no causal relationship between A and B. This is because of the fact that no light signal can flow from A to B carrying information, thus making A and B light signals disconnected. The causal relationship is broken down.

Here also as before
A occurs before the occurrence of B
B occurs after the occurrence of A

But

A is not the past event of B
B is not the future event of A

A and B are chronologically dependent but logically independent.

From the Light Cone Angle of View
Unlike before (where the object moves at a speed less than or equal to that of light), the light cone of B can in no way be inside the light cone of A. In fact, neither of the two light cones is inside the other light cone. Light cones of A and B are two completely different light cones. Thus, there cannot be any world line which will be able to connect the future event B to the past event A. As the object is moving at a speed greater than 'c', A and B both happen to be in a 'space like' interval. Light can in no way flow from A to B carrying information. Thus, B is not the future event of A.

In that case, in some spacetime coordinate system, B would appear as occurring before A. This implies that 'future' is occurring before the 'past'. This breaks down the causality relationship between A and B. Each event with a separate light cone is a separate event of a separate object without any causal relationship.

5.2.3. Some Informal Analogy

1. Suppose we are switching on a lamp.

Event A = Pressing the switch
Event B = The lamp glows

A occurs before B
B occurs after A

There is a wired connection between the two events, A and B. Current flows from the switch to the lamp, and the lamp glows. The switch and lamp are electrical signal connected. An electrical signal carries information. This only makes:

A is the past event of B
B is the future event of A.

Because of the electrical connection (signal flows), the causality relationship is maintained. This implies that event A is the cause of event B. Both chronological and logical dependencies are maintained. Whatever the space time coordinate system, event A must be realized or recognized to occur before event B.

2. Astronomical phenomena in two different galaxies.

Suppose two separate supernova bursts are occurring one after another in sequence in two different galaxies. Suppose the event supernova burst in galaxy G1 is symbolised as A, while the event supernova burst in the other galaxy G2 is symbolised as B.

A is occurring before B
B is occurring after A

But

A is not the past event of B
B is not the future event of A

This is because of the fact that the two objects (i.e. the two supernovae in two galaxies, G1 and G2) are completely different. This makes the light cones of the two events of the two objects completely different. The light cone of event B is not inside the light cone of event A.

We may or may not assume that light single can flow from one event to another to identify the supernova bursts - this is quite immaterial to decide whether one event is the past event of the other, because of the fact that the two objects are quite different.

Like before (object moving at a speed greater than ‘c’), in that case also in some spacetime coordinate system, B would appear as occurring before A. This implies that ‘future’ is occurring before the ‘past’. This breaks down the causality relationship between A and B. There is a chronological dependency but no logical dependency between A and B. Each event with a separate light cone is a separate event of two separate objects, without any causal relationship.

So, to maintain the causality relationship of the two events A and B, (i) the two events A and B should be of the same object, as well as (ii) the object should move at a speed $v \leq c$, where c is the velocity of light.

6. Space Time Curvature inside the Event Horizon

6.1. General Relativity Redefines Gravity as Spacetime Curvature

There is extreme curvature of spacetime on and inside the event horizon surface towards the singularity. We have to redefine gravity there as not merely a force that pulls inside. General relativity supports inversion of space and time inside the event horizon. According to General Relativity, gravity is no longer a force but is a curvature of spacetime caused by mass and energy. Inside the event horizon, the redefined reality is towards one and only one inevitable ‘future’ direction – i.e. singularity of the black hole. So, for the rushing object towards the singularity, that singularity is not merely a spatial position but a time in the ultimate future. Spacetime curvature is so intense or severe that in that region, all possible paths only target inwards, irrespective of all efforts of an object to move away or resist. Every world line or trajectory, i.e. every redefined ‘future’ converges towards the singularity. This is creating the astonishing ‘inversion’ of space and time because of the severe dramatic change of ‘spacetime’ geometry. This is because of the fact that, according to General Relativity, the interpretation of the role of ‘spacetime’ coordinate systems is solely dependent on spacetime geometry.

6.2. Passage Towards Singularity Is the Only Future of an Object

An object inside the event horizon surface of a black hole has one and only one unavoidable future destination – the singularity. This is only because of the fact that the roles of space and time are surprisingly interchanged or ‘inverted’. What happens outside the event horizon? In that region, you are offered directions (left, right, forward, backward, any angle, etc.) to select your choice of movement in space, while you are forced or bound to move through time to the future or forward – you have not even a little option to select or deselect movement through time to the future. Inside the event horizon, the freedom to select directions in space completely disappears. The passage towards the singularity is the only destiny or ‘future’ and is quite unavoidable. Moreover, this is exactly the same as your movement through time outside the event horizon.

6.3. Strong Curvature of Spacetime Inside the Event Horizon

Inside a black hole, the phrase “infinite curvature of spacetime” refers to how the geometry described by General Relativity becomes progressively more extreme as one moves inward, ultimately reaching a limit where the theory itself ceases to give finite answers. Curvature, in this context, is not just a visual bending but a precise measure of how spacetime tells matter and light to move. As the radial coordinate decreases inside the event horizon, the curvature grows without bound, indicating that the gravitational field is no longer merely strong but mathematically singular.

This increase in curvature is most clearly understood through tidal effects. If two nearby particles fall toward the singularity, their separation does not remain stable. Instead, spacetime stretches them apart in one direction while compressing them in others. These tidal forces intensify dramatically as the singularity is approached. In the limit, the relative acceleration between nearby worldlines becomes unbounded, meaning that even infinitesimally close particles experience arbitrarily large distortions. This is the physical manifestation of "infinite curvature": not that a single trajectory becomes erratic, but that the spacetime fabric forces neighboring trajectories to diverge or converge without limit.

Another way to interpret infinite curvature is through the breakdown of geometric concepts themselves. In ordinary curved spacetime, one can still define distances, angles, and durations consistently. However, near the singularity, the metric that defines these quantities becomes ill-behaved. Spatial (transverse) separations can shrink toward zero, while directional (radial) distinctions lose clarity. The notion of a smooth, continuous geometry begins to fail, and the usual tools used to measure spacetime intervals no longer yield meaningful, finite results.

It is also important to note that this phenomenon is not directly tied to the "zig-zag" motion of particles or any irregularity in individual worldlines. A freely falling particle still follows a smooth geodesic path. The infinity arises not from the path itself, but from the way spacetime curvature acts on nearby paths. The geometry becomes so extreme that even the smallest separation between neighbouring worldlines is subjected to infinite distortion in finite proper time.

Finally, the appearance of infinite curvature signals a fundamental limitation of our current physical theory. In General Relativity, such divergences indicate that the classical description of spacetime is no longer adequate. The singularity is therefore not just a region of strong gravity, but a boundary beyond which the known laws of physics cannot reliably describe reality. It is widely believed that a more complete theory—likely involving quantum gravity—would replace this picture and provide a finite, well-defined description of spacetime at these extreme scales.

6.4. More About Strong Curvature of Spacetime Inside the Event Horizon

In General Relativity, strong curvature means

1) Worldlines experience strong relative acceleration.
2) All geometric measurements are uncontrolled or unstable

Both these two are detailed below:

1. Worldlines experience strong relative acceleration.

This can cause them to (1) converge (come closer), (2) diverge (move apart), or (3) do both in different directions. Thus, strong curvature means the separation between nearby worldlines changes rapidly, causing strong convergence or divergence depending on direction.

Example 'black hole tidal field': Near a black hole, along the radial direction, nearby particles get stretched apart. So worldlines effectively diverge in radial separation along the radial direction. However, along the transverse directions, particles get compressed together. So worldlines converge sideways (along the transverse direction). Thus, strong space time curvature = strong space time distortion = strong deformation of the separation of the world line, not simply either convergence or divergence in all directions, i.e. radial as well as transverse direction.

Now there is a fundamental question. Why the two nearby worldlines (and thereby two particles traversing along those two worldlines) diverge along the radial direction, while they converge along the transverse direction? Here is the answer.

Why radial separation increases (divergence)

Imagine that there are two particles (along the two nearby worldlines), out of which one slightly closer to the black hole's singularity, while the other one is slightly farther away from the singularity. Thereby, the closer one must have to experience or feels stronger gravitational pull (i.e. tidal effect) compared to the further one. So, the closer one accelerates towards the singularity more strongly than the other one, and thereby it "pulls ahead" of the other. The consequence is that the two particles are stretched away along the radial direction. Their radial separation increases. This is also called radial stretching or divergence.

Why transverse separation decreases (convergence)

Consider two particles that are moving side by side (along the two nearby worldlines) at the same radial distance. Both are pulled towards the centre, but they are along two slightly different inward directions because of slightly different worldlines. Now, along slightly different inward directions, both trajectories tilt inward towards the same singularity, and thereby the two concerned particles move toward the same singularity at the centre. As a result, their sideways (transverse) separation continuously decreases, and near the singularity, they fall on each other and crush each other. Thus, the transverse separation decreases. This is also called transverse compression or convergence.

2. All geometric measurements are uncontrolled or unstable

Uncontrolled or unstable means, a small change in position, time slicing, coordinates or simultaneity choice can produce a very large change in geometric measurements such as distance, spatial position, and elapsed coordinate time. Thus, instability means geometric measurements become extremely sensitive to tiny changes in position or coordinates.

Symbolically:
small change in input → huge change in output

Summing them all up:

a) Outside the event horizon (moderate spacetime curvature), distance is coordinate-dependent, while it is well-behaved.
b) Just onto the surface or inside the event horizon of the black hole (strong spacetime curvature), distance is coordinate-dependent and more sensitive.
c) Near the singularity of the black hole (the strongest curvature of spacetime), distance is coordinate-dependent, highly unstable and approaches breakdown.
d) Onto the singularity (stronger than the strongest curvature of spacetime) distance is undefined.

Along with all the above, if ‘*ds’* is symbolized as an infinitesimal local spacetime interval, it is invariant and stable all throughout, i.e. everywhere, with the exception of the singularity, where it is not defined. However, the global interval, if symbolized as ‘∫ds’, although for a fixed path is well-defined, is highly unstable near the singularity due to huge path sensitivity. Finally, it becomes non-existent and undefined at the singularity itself.

7. Inversion of the Role of Space and Time

The roles of space and time are inverted or interchanged inside the event horizon. The radial direction towards the singularity is quite unavoidable and not a matter of choice, which is very similar to the movement through time outside the event horizon. The transverse direction towards the singularity is partially a matter of choice (so long as the particle is within the local light cone). Under that perspective, ‘space’ behaves more like ‘time’ and is called ‘time-like’ space. Moreover, ‘time’ behaves more like ‘space’ and is called ‘space like’ time. This is as explained in the subsections below.

7.1. Alteration or Inversion of the Spacetime Coordinate System

Due to this inversion of space and time, in the ‘spacetime’ coordinate system, what was previously termed as ‘space’ coordinate is now changed to ‘time’ coordinate, while what was previously termed as ‘time’ coordinate is now changed to ‘space’ coordinate. Now there is a fundamental question. Does the altered new time in the time coordinate have exactly the same feature as conventional time? Similarly, does the altered new space in the space coordinate have exactly the same feature of conventional space? The answer is negative. In the new inverted spacetime coordinate system, the new time axis should be better termed as a ‘spacelike’ time axis. Moreover, the new space axis should better be termed as a ‘time-like’ space axis. This is justified as below:

7.2. Justification for ‘Space like Time’ and ‘Time like Space’

When the object falling inside the event horizon of the black hole is rushing towards the singularity, there is an inversion of the ‘spacetime’ coordinate system, as told earlier. The new coordinate system is a ‘spacelike’ time & ‘time like’ space coordinate system.

In the new coordinate frame of ‘space like time’ and ‘time like space’, the particle is moving through ‘time’ (space like time – one dimensional), which is inevitable; the direction of movement cannot be altered (similar to movement through time outside the event horizon). However, unlike movement through ‘time’ outside the event horizon, which is invisible, here the movement is visible – thus this new inverted ‘time’ has some feature of space also. This is ‘time like’ space.

At the same time, with the new coordinate frame, it is moving through ‘space’ (time like space – three-dimensional), which is not inevitable; the direction of movement can be controlled or changed within the local light cone (similar to movement through space outside the event horizon). However, unlike movement through space outside the event horizon, in this case, the movement is invisible – this new inverted ‘space’ has some feature of time also. This is ‘space like’ time.

Thus, the object inside the event horizon is not exactly moving in an inverted ‘spacetime’ coordinate system, but more precisely in a ‘time like’ space and ‘space like’ time coordinate system.

8. Mathematical Establishment of the Inversion of the Role of Space and Time

Now we are at a position to mathematically establish the inversion of the role of space and time, i.e. inversion of the spacetime coordinate system.

The inversion of space and time inside the event horizon of the black hole, i.e. ‘space time’ coordinate system into {‘time like space’ ‘ space like time’} coordinate system, can be justified and established mathematically, from the Schwarzschild metric. This is nothing but a solution of General Relativity for any black hole.

Schwarzschild Spacetime Metric is as follows:

$$ds^2 = -\left(1-\frac{2GM}{rc^2}\right)c^2dt^2 + \left(1-\frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2d\Omega^2$$

Where:

- G= gravitational constant
- M= mass of the black hole
- c = velocity of light
- r= radial distance of the object from the singularity of the black hole (irrespective of whether it is inside or outside the event horizon of the concerned black hole)
- $d\Omega^2 = d\theta^2 + d\varphi^2 sin^2\theta + d\theta^2 sin^2\varphi$

[dependent on the radial position (r, θ, φ) of the object (either outside or onto or inside the event horizon surface of the black hole)]

And

- *ds* is the infinitesimal 'spacetime' interval for the object approaching the singularity of the black hole.
- *dt* is the infinitesimal time difference of the object while moving from one position to another.
- *dr* is the infinitesimal space difference (radial) of the object while moving from one position to another.

Now for the event horizon at a distance $'r_s'$ from the singularity of the black hole:

$$r_s = \frac{2GM}{c^2}$$

This is the well-known "Schwarzschild radius".

In the metric

c^2dt^2 has the coefficient $= -\left(1-\frac{2GM}{rc^2}\right) = -(1-\frac{rs}{r})$

dr^2 has the coefficient $= \left(1-\frac{2GM}{rc^2}\right)^{-1} = (1-\frac{rs}{r})^{-1}$

Now, when the object is outside the event horizon, i.e. $r > r_s$

c^2dt^2 coefficient $-\left(1-\frac{rs}{r}\right)$ is negative and the 'time' is 'time like'; time behaves as time in all respects.

dr^2 coefficient $(1-\frac{rs}{r})$ is positive and the space is 'space like'; space behaves as space in all respects.

Onto the event horizon where $r = r_s$

c^2dt^2 has the coefficient = 0

dr^2 has the coefficient = 0

Both time and space are neither 'time like' nor 'space like'; the inversion of time from 'time like' to 'space like' and space from 'space like' to 'time like' initiates there.

Now, when the object falls inside the event horizon, i.e. when $r < r_s$.

c^2dt^2 coefficient $-\left(1-\frac{rs}{r}\right)$ is positive and the 'time' is 'space like'; time behaves as space (partially, because the movement through that space is invisible, although that movement is not inevitable)

dr^2 coefficient $(1-\frac{rs}{r})$ is negative and the 'space' is 'time like'; space behaves as time (partially, because the movement through that time is visible, although that movement is inevitable)

This is the mathematical proof of the inversion of space and time inside the event horizon of a black hole.

9. Conclusion

The present work is an extensive study on the inversion of the role of space and time inside the event horizon of any black hole. The singularity of the black hole is a region of intense gravity where all the world lines are only directed and ultimately falls onto it. The only destination and destiny of any particle or object is the singularity, and there is no other deviation of direction. We have discussed in this paper how an analogy can be drawn between this unidirectional arrow of time outside and the unidirectional arrow of spatial movement of any particle towards the singularity inside the event horizon. We have thus confirmed that inside the event horizon, 'space' takes the role of 'time' and 'time' takes the role of space.

Moreover, we showed the inversion of space and time in this way. We came to the conclusion that inside the event horizon, the 'space' is one dimensional 'time' like and 'time' is three dimensional 'space' like. Although there is major similarity between 'time' outside and 'time like space' inside and 'space' outside and 'space like time' inside, the dissimilarity is also explored -the movement of a particle in 1D 'time like' space is visible unlike 'time' outside, and it is possible to stop or redirect the movement of a particle in 3D 'space like' time, unlike the movement through 'time' outside with the little constraint that the redirection should be inside the local light cone. Finally, we mathematically established the inversion of space and time with the help of the Solution of General Relativity for a black hole.

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