

Original Article

Critical Investigation of the Bed Load Evaluation Models for the Mountainous Stream

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Abstract - For the computation of the bed load transport rate at a particular section of stream, the selection of any approach depends upon the ambient hydraulic condition as well as the nature of the sediment and the bed of the stream. The present study evaluates bed load transport rate in the upper reaches of a mountainous stream, which consists of large non-uniformity in the bed load. The best-suited instrument for bed load collection should have features to take care of the non-uniformity of the bed load and the bed material. In the present study, the USGS Sampler (a modified version of the Helley-Smith bed load Sampler) has been used. The sampling time is an important criterion as far as the direct measurement of the bed load from the natural stream is concerned. The natural channels are non-prismatic in nature, so the velocity distribution will vary in both directions of the stream. The non-uniformity of the velocity interacts in a complex fashion with the non-uniform bed material, which later on is converted into the bed load of the stream. In the present study, the comparison of the measured bed load transport rate and the computed bed load transport rate has been carried out using several approaches for the gravelly bedded stream. In the present study, thirteen equations have been selected for the bed load prediction at the selected sites. The best-suited approach for the bed load prediction is A. Recking's approach, which scores 0.663 and 0.843 on the scale, subject to a maximum score of 1.

Keywords - Non-Uniformity, Sampling, Boundary shear stress, Bed load, Bed material.

1. Introduction

A key component of fluvial processes, sediment transport is closely connected to the morphological changes. The bed load and suspended load are the two primary types of sediment transport. When there is a bed load, the sediment grains may roll, slide, or move in a series of modest jumps near the bed surface called as the saltation [1]. Bed load transport in gravel-bed rivers has been predicted using a range of formulae from simple regressions to complex multiparameter formulations. Earlier, the absence of bed load transport data for gravel bed rivers has hindered testing these formulae over many field sites [1, 4]. Prediction of the bed load transport rate is very important in various domains of Engineering, like Environmental Impact Assessment (EIA), Reservoir planning and sedimentation, Risk management, and Reservoir life of dams are of concern. Evaluation of bed load transport rate in the natural stream by the manual collection of the bed load in the stream is very desirable. It requires very rigorous manual work and a large sampling time for bed load collection from the bed of the stream [2, 10, 11]. The recent techniques (Machine learning) may be used for the computation of bed load transport rate through the stream. Recently, Basumatary et al. (2026) have used the machine learning approaches

(Random Forest, Extra Trees, Extreme Gradient Boosting and CatBoost) for the computation of Bed load. But a total of 624 datasets were used, out of which 70% (436 datasets) were used for training purposes of model, and 30% (188 datasets) were used for the testing of models [3]. In most cases, a hydraulic engineer uses the field measurements to calibrate a sediment curve. It requires collecting the bed load samples from the bottom of the natural stream in order to validate a particular sediment formula developed for that particular reach [4].

The amount of sediment passing a cross-section of a stream can be determined by various approaches or methodologies, but the approach used is a function of the hydraulic condition and the gradation of bed material or bed load [5]. Large streams show less fluctuation of sediment transport in the lateral direction than smaller streams [7]. When the flow conditions exceed the conditions similar to those of the incipient motion of the bed, then the bed of the stream becomes mobile, and its mobility is a complex function of the hydraulic conditions and the gradation of the bed material [8, 23]. If the motion of sediment particles is such that it is transported by rolling, sliding or sometimes by jumping, then the sediment transported in this manner is called as bed



load [10, 11]. In a gravelly bedded channel, sediment transport rate can be calculated by direct measurement using an appropriate sampler. In this approach to bed load transport rate, a number of bed load samples are taken from the bed of the stream [12]. The sampling of the bed load is accomplished by a special kind of sampler that takes care of the complex interaction of bed load and the prevailing hydraulic condition. Apart from the complex interaction of bed load and the prevailing hydraulic condition of the stream, the bed configuration also plays a vital role as far as the bed load measurement by the sampler is concerned [13]. In the present analysis, the Helley-Smith bed load sampler, having 1.54 efficiency has been used considering all the parameters [23]. The criterion of the bed load sampling is the size of the bed load to be sampled, the velocity of the stream and the sampling time. The sampler used in the current study (USGS Sampler) is able to sample the grain size varying from 5 mm to 16 mm and up to 3m/s stream velocity [14, 15]. The sampling duration should be chosen so as not to negatively affect the sampler's hydraulic efficiency [16, 17]. The hydraulic efficiency of the sampler can be defined as the ratio of the velocity of the stream passing through the sampler to the velocity of the stream passing through the same width in the absence of the sampler [18]. On the other hand, the ratio of the amount of bed load passing through the sampler to the amount passing through the same width in the absence of a sampler is known as the sampler efficiency.

Jun-De Li et al. (2018) introduced the concept of two travel lengths for the rolling and lifting sand grains in the bed layers. The probability of the rolling and lifted sediment grains, as well as the anticipated journey length and time scale, which are highly dependent on the flow, have been used to create a new bed load sediment transport rate function [19]. Van Rijn introduced the terms particle velocity (u_b), saltation height (δ_b) and bed-load concentration (C_b). Further, the bed load transport rate is the product of these three parameters [20]. When compared to the unisize case, Wilcock et al. (2003) include a function that decreases the mobility of smaller sizes and boosts the mobility of coarser sizes. The reference shear stress variation is provided by the hiding function [21]. Laboratory and natural stream observations have demonstrated that the bed load transit rate at a section fluctuates noticeably over time [23].

The reason behind this phenomenon can be explained by the sheltering and hiding effect of the bed load. Figure 1 explains the sheltering and hiding effect of the bed load in the gravelly bedded stream. From Figure 1, it is evident that as the larger particle passes the cross-section of the stream, the bed load transport rate increases at that particular instant of time. It is due to a rise in the rate of transportation of hidden sediment particles behind the larger particle. Thus, it becomes very important to select the sampling time in such a manner that it considers all these parameters [24].

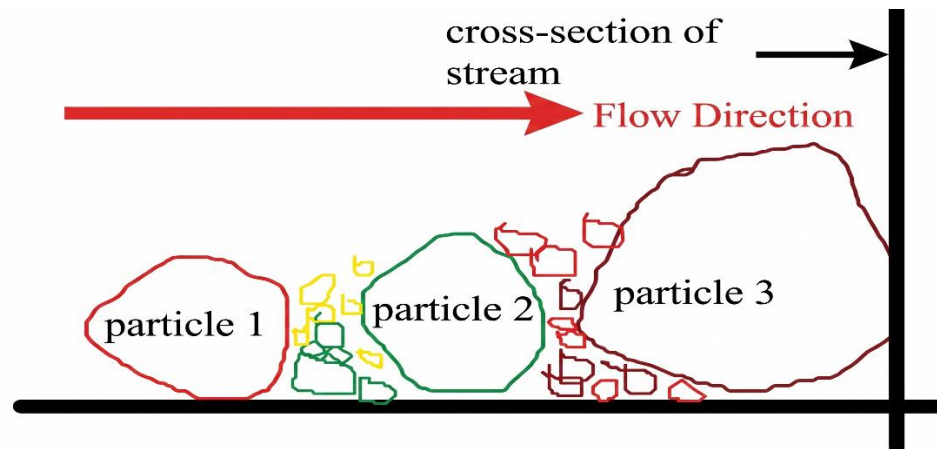


Fig. 1 Hiding and sheltering effect

The theories put forward during the earlier age of development of sediment transport are based on experimental results from a flume. During the 1970s, several researchers put forward their efforts to correlate the observed and the results obtained from bed load transport rate equations, but the results obtained from bed load transport rate equations vary from the observed results by several orders of magnitude [25]. This variation of observed bed load transport rate from the calculated bed load transport rate is due to the complex interaction of bed load and bed material with the ambient hydraulic conditions. However, it is difficult to find a prismatic channel in natural conditions. The velocity of flow

will vary in both longitudinal and lateral directions of the stream. The ambient hydraulic conditions also vary from point to point along the stream's longitudinal direction [17, 19]. The behaviour of the sediment transported through the channel will also be altered from point to point. The bed load transport rate in longitudinal and lateral directions will be altered [18]. The spatially large and temporally fine measurement of the sediment transportation is required for a representative measurement of the bed load transport rate. The sediment transport for a particular river reach depends upon various factors, out of which climate change and anthropogenic changes factor is important. Over the past 60 years, the Yellow

River's sediment load has dropped by almost 90%, despite once being the world's greatest carrier of fluvial material. The Huang He, often known as the Yellow River, used to be the world's most sediment-filled river, but since the 1950s, its yearly sediment load has steadily declined. Over the last 60 years, the annual sediment loads at the major gauging sites along the Yellow River have all shown significant declining trends ($p < 0.01$). Maintaining perennial vegetation cover is one of the best ways to stabilise soils and reduce erosion. The sediment flux into rivers has been greatly decreased by extensive vegetation regeneration programs, especially the grain-for-green initiative started under the Loess Plateau. A well-known technique for lowering the flow of sediment into river systems is terrace farming [26]. The sediment supply to the stream is highly variable due to rapid urbanisation and human activities. The construction of river corridors with concrete and stone masonry embankments, upstream sand and gravel mining, and stone quarrying are examples of anthropogenic activities that have further modified natural flow paths and changed the conveyance capacity of river channels by raising the river bed through sedimentation [28]. The objectives of the current research are to estimate the bed load transport rate at selected cross-sections of the stream using the Soong and Cheiw approach (1998) and to compare the measured bed load transport rate with the computed bed load transport rate using bed load transport models. Further, based on this comparison, the ranking of these equations/models to be carried out. This is the first effort to measure the bed load of any Indian stream. The present analysis has been done for the gravelly bedded, steep-sloped stream, where the entire theory related to sediment transport developed till the last decade of the 20th century, is unable to estimate the sediment transport. [27].

As bed load collection from the bottom of the stream involves lots of tedious work and hydraulic uncertainty, most of the earlier developed theories are based on laboratory flume data. The models developed using the laboratory flume data underpredict or overpredict the actual sediment transport rate by several orders of magnitude (of the order of 1000). The collection of bed load physically through the apparatus is a tedious task. Hence, in the current study, attempts have been made to redress this gap. Further, various models/ approaches based on the laboratory flume experiments, basic laws of Physics, Numerical and Regression Analysis have been investigated. The Nash-Sutcliffe Model Efficiency have been evaluated for bed load passing through both the cross-sections.

2. Study Area and Methodology

2.1. Study Area

Vandsa is a town that comes under the district of Navsari, Gujarat. This city is located at a distance of 95 km from the city of Surat. This region comes under the famous Saputara hill range of the Western Ghats of India. The study area is situated at a distance of 143 KM from the sea coast. The selected reach has been situated above 122 m above M.S.L., having a latitude of $20^{\circ}43'23''$ N and a longitude $73^{\circ}20'10''$ E. The field work has been carried out at the Chottey Cauvery stream located near the village of Gangpur, Vandsa, for collecting the repetitive samples. The Chottey Cauvery stream originates from the Saputara hills of the Western Ghats. The stream bed consists of cobble, sand and gravel. The basin is largely infested with sparse shrubs. The stream has been selected for the collection of the samples because the depth of flow at this section varies from 0.50 - 1.20 m. Figure 2 represents the study area.



Fig. 2 Location of study are

3. Methodology

3.1. Field Data and Bed Load Measurement

Field data were collected at two cross-sections, and each of the cross-sections has been divided into five sub-sections. In this way, four intermediate points between the banks of the stream have been established. So, eight intermediate points were selected. In order to counteract the temporal variation, the sheltering and hiding effect of the sediment transport rate

at a particular section of the stream, repetitive sampling is preferred rather than a single sampling at the particular point of the cross-section of the stream. At each selected point, three samples have been taken at a particular cross-section. A total of 24 bed load samples were used to carry out the study. The selected reach has a stable flow and is free from turbulence. Flow depth, velocity, bed load transport rate and the bed slope were measured simultaneously at each of the cross-sections.

The width of each cross-section was divided into several sub-sections. The flow depths at the selected points of the cross-section were measured by the surveying tape. The flow velocity at the selected points was measured by the Pigmy type of current-meter having a cup diameter of 25mm at 0.60 times the flow depth from the free surface of the stream. To determine the bed slope of the reach, the flow has been assumed to be uniform. Thus, bed slope is represented by the water surface slope. In order to determine the slope of the reach, the seven points have been chosen in the longitudinal direction of flow, and the corresponding depths have been measured by the surveying tape in association with the depth measuring rod, as shown in Figure 3. The flow depth was measured using a depth-measuring rod, while the corresponding longitudinal distance between points is measured using the measurement tape.

The best-fit line has been drawn between the longitudinal direction and the corresponding depth. The slope of the reach is 0.005 (1 in 200) and the R^2 -value of the best-fit line is 0.961, as shown in Figure 3. The USGS Sampler (a modified version of the Helley-Smith bed load sampler) was used for bed load measurement. It consists of mild steel with a Zinc coating over it with an opening of 7.62 x 7.62 cm. Also have a polyester bag with a mesh size of 0.25 mm. The sampling time is the primary concern while collecting the bed load from the bottom of the stream. The sampling time would be such that the sampling bag should not fill more than 30% of its capacity. Field observations showed that a sampling duration of 10-12 minutes is adequate to collect samples without violating the sampling duration criteria. Thus, in the present study, the sampling duration of 10 minutes has been selected.

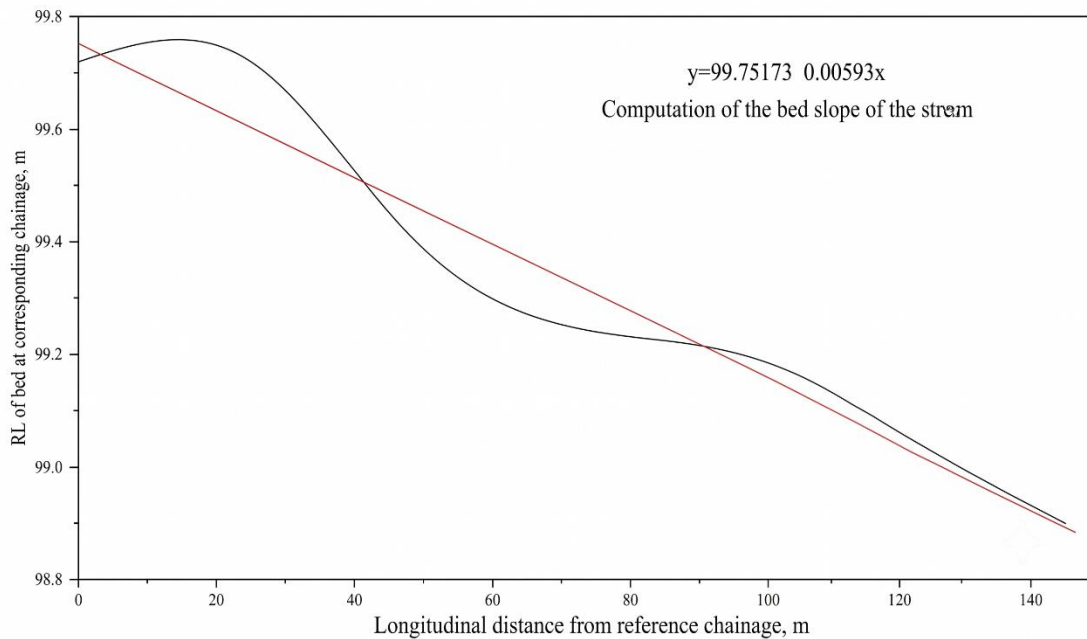


Fig 3. Bed slope computation

3.1.1. Measurement of the Bed Load using Heli-Smith Bed Load Sampler

Bed load mobility in the natural stream is due to the work done by the fluid. To accomplish this work done, the fluid receives an appreciable amount of stream power. This stream power is a function of the flow cross-section normal to the direction of flow. The flow cross-sectional area will be maximum during the monsoon period (the months during which more than 80% of annual rainfall takes place). The Western Ghats of India receive 80% of annual rainfall from the 2nd week of June to the 2nd week of September. During this period, the stream flux is appreciable. In order to get stream flux in the selected stream, the bed load samples have been collected from 14th September to 3rd October 2016. The

sampling of the bed load transport rate at the selected sub-sections of width can be carried out by the various methods like the Single Equal Width Increment bed load method (SEWI), Multiple Equal Width Increment bed load method (MEWI) and Unequal Width Incremental bed load method (UWI). The sampling point on the sub-section should be selected such that the sampler can be laid without any inclination relative to the bed. Due to the geological diversity of the stream bed, the Unequal Width Incremental bed load method (UWI) has been selected. In the UWI of the bed load sampling, the width of the particular cross-section is divided into several unequal sub-sections, and then repetitive sampling is done at those selected points. The width of the cross-section was divided into five unequal sub-sections. A

total of twelve samples has been collected at each of the cross-sections of the stream. The appropriate sampling time for the Helley-Smith bed load Sampler is (60-300) seconds. But the sampling time is purely a function of the extent to which the sampler bag has been filled. For that reason, the sampling time has been taken to be 10 minutes. The bed load transport rate for each of the cross-sections is calculated using the equation put forward by Soong and Cheiw (1998) in association with the trapezoidal rule for the integration of bed load transport rate through the sub-section:

$$Q_b = \frac{1}{2} [(L_1 * W_{tb1}) + (W_{tb1} + W_{tb2}) * L_2 + \dots + (W_{tbi} + W_{tbi+1}) L_i] \quad (1)$$

Where Q_b represents the bed load discharge (g/s), L represents the length between two points (m), and W_{tb} represents the dry weight per unit time and per unit width (g/s/m).

$$W_{tbs} = \frac{M_s}{W_s n_s t_s} \quad (2)$$

The summary of the fractional bed load transport rate and average bed load transport rate passing through cross-section 1 and cross-section 2 is mentioned in Tables 1 and 2.

Table 1. Fractional bed load transport rate at cross-section 1

Chainage from right bank	Set 1	Set 2	Set 3
	Sub-sectional bed load transport rate (W_{tb}) g/m/sec.	Sub-sectional bed load transport rate (W_{tb}) g/m/sec.	Sub-sectional bed load transport rate (W_{tb}) g/m/sec.
0	0	0	0
2	13.114	11.798	20
4	32.160	17.938	11.951
7	15.131	12.609	10.131
9	20.532	12.9167	14.188
11	0	0	0
Average bed load transport rate: (12.524 tons/day or 13.17760 g/m/s)			

Table 2. Fractional bed load transport at cross-section 2

Chainage from the right bank	Set 1	Set 2	Set 3
	Fractional bed load transport rate (W_{tb}) g/m/sec. (First Sampling)	Fractional bed load transport rate (W_{tb}) g/m/sec. (Second Sampling)	Fractional bed load transport rate (W_{tb}) g/m/sec. (Third Sampling)
0	0	0	0
2	19.634	22.653	7.280
4	10.657	9.188	8.530
7	16.798	16.118	7.916
8.50	13.33	20.855	13.330
9.50	0	0	0
Average bed load transport rate: (9.238 tons/day or 11.254 g/m/s)			

3.2. Bed Load Transport Rate

To compute bed load transport rate using existing models, a total of thirteen equations, namely (Du Boys, Meyer-Peter and Muller, Shield's equation, Yalin equation, Kalinske equation, Sato-Kikkawa and Ashida equation, Van Rijn equation, Roorkee approach, Englund and Fredsoe's equation, Samaga et al., A. Recking's approach, Bagnold's equation, Wu et al. equation) have been selected. These equations/models can be classified broadly in four approaches (Critical Shear Stress approach, Stream Power Approach, Approach based on Numerical and Regression analysis and Approach based on the theory of probability).

3.2.1. Critical Shear Stress Approach

Based on the critical shear stress approach, a total of nine bed load equations has been selected. These are the Du-Bois equation, Meyer-Peter equation, Shield equation, Yalin equation, Kalinske's equation, Sato-Kikkawa and Ashida equation, Van Rijn equation, Roorkee approach, and Englund and Fredsoe's equation. All the bed load equations based on the critical shear stress approach are based on the simple law of physics. When the bed shear stress exceeds the critical shear stress on the bed, then the bed load/material will become mobile.

Table 3. Fractional Bed-Load Transport Rates at Cross-Section 2

Model	Equations
Du Boys equation	$q_b = A(\tau_0 - \tau_{0c})\tau_0$, Where $A = \frac{\Delta h \Delta V_{ys}}{2\tau_{0c}^2}$
Meyer-Peter and Muller equation	$\phi = 8(\tau_*' - 0.047)^{1.5}$
Shield's equation	$\frac{q_b (\rho_s/\rho_f - 1)}{q\gamma_f S} = 10 \left[\frac{\tau_0 - \tau_{0c}}{(\gamma_s - \gamma_f)d} \right]$
Yalin equation	$\frac{q_b}{u_* \gamma_s \Delta d} = C S_\tau \left[1 - \frac{2.303 \log(1 + a S_\tau)}{a S_\tau} \right]$

Kalinske's equation	$\frac{q_b}{d\gamma_s u_*} = 2.57 \frac{U_g}{u}$
Sato-Kikkawa and Ashida equation	$\frac{q_B}{\tau_0 u_* f \left(\frac{\tau_{0c}}{\tau_0} \right)} = \left(\frac{1}{40n} \right)^{3.5} \text{ for } n \leq 0.025$ $\frac{q_B}{\tau_0 u_* f \left(\frac{\tau_{0c}}{\tau_0} \right)} = 1 \text{ for } n > 0.025$
Van-Rijin equation	$\frac{q_B}{(\Delta\gamma_s/\rho_f)^{1/2} d^{1.5}} = 0.053 \frac{T^{2.1}}{D^{0.3}}$
Roorkee approach	$\phi = 4.6 * 10^7 \tau_*'^8 \text{ for } \tau_*' \leq 0.065$ $\phi = \frac{8.5 \tau_*'^{1.8}}{[1 + 5.95 * 10^{-6} \tau_*'^{-4.7}]^{1.43}} \text{ for } \tau_*' > 0.065$
Englund and Fredsoe's equation	$q_b = 9.4 \frac{\Pi}{6} p d \gamma_s u_* \left[1 - 0.70 \sqrt{\tau_{*c}/\tau_*} \right]$

3.2.2. Stream Power Approach (Bagnold's Theory)

Bagnold's theory is based on the basic law of physics. It shows that a flowing fluid will contain some amount of impelling power; that power is called stream power. This stream power will help in supplying the energy for the bed load movement, as the rate of change in energy per unit time represents the power. Based on the equilibrium condition of the stream power per unit bed area and the bed load work rate,

Bagnold has put forward the following model:

$$q_b = \frac{\rho_s}{\rho_s - \rho} \cdot \tau_0 \cdot \frac{e_b}{\tan \alpha} \quad (3)$$

Basically, Bagnold has put forward the approach for the total load transport rate, but if we consider only the expression above out of the total load transport equation, then it will represent the bed load transport rate.

$$q_t = \left\{ \frac{\tau_0 \rho_s V}{(\rho_s - \rho)} \cdot \frac{e_b}{\tan \alpha} \right\} + \left\{ \frac{\tau_0 \rho_s V}{(\rho_s - \rho)} \cdot e_s \cdot (1 - e_b) \frac{U_s}{\omega} \right\}$$

Term due to the bed load transport rate.

Term due to the suspended load transport rate.

Here; e_b (Bed load Transport efficiency) = $-0.028 \log(3.28 V) + 0.135$ for $D_a > 0.7$ mm

$\tan \alpha$ = Angle of internal friction, τ_0 = Bed Shear Stress, V = Mean Velocity, ω = fall velocity

The data set for the calculation of bed load transport rate has been mentioned in Tables 4 and 5.

Table 4. Data set for the calculation of bed load transport rate at cross-Section 1

Slope	1 in 200	Hydraulic radius	0.302 m	Width	11.00 m
Depth	14 cm -36 cm	Froude no.	1.2802	$d_a = 7.94$ mm	
Sediment size	7.94 mm (Arth. Mean size)	D_{90}	25 mm		
Relative density of sediment	2.65 (Assumed)	Velocity of flow V	2.20 m/sec.	$\tan \alpha = 0.375$	

Table 5. Data set for the calculation of bed load transport rate at cross-Section 2

Slope	1 in 200	Hydraulic radius	0.3080	Width	9.50m
Depth	18.6 cm -58 cm	Froude no.	1.0912	$d_a = 8.462$ mm	
Sediment size	8.4624 mm (Arth. Mean size)	D_{90}	28 mm		
Relative density of sediment	2.65 (Assumed)	Velocity of flow	1.9072 m/sec.	$\tan \alpha = 0.375$	

3.2.3. Approach based on Numerical and Regression Analysis Recking's Approach (Reach Averaged Approach)

Recking's approach considers the three parameters of sediment transport (Hydraulic, Shear stress & grain size

distribution of bed load). Here, shear stress and bed load transport rate are made dimensionless with the diameter D_{84} of the bed load, as it controls the mobility of gravel- and cobble-bedded streams. The approach was put forward during

the initial regime of development of the sediment transport rate. It is based on the data sets obtained from the laboratory flume as well as the data set obtained from actual field sampling. The models developed from the laboratory flume data don't give satisfactory results. The reason behind this discrepancy from the real transport is due to the sediment transport data set collected from the flume, which is under steady, uniform flow conditions and free from excessive turbulence. Apart from this hydraulic condition, the sheltering and hiding effect has no pronounced impact during the

sediment data collection. The problem related to the gravelly bedded stream can't be analysed using a conventional approach. Due to this, the reach-averaged approach has been selected for the reach of the stream. In the present study, Recking's approach has been selected, which is applicable for the steep-sloped gravelly bedded reach, and it considers the reach-averaged approach for the calculation of bed load transport rate. Recking's approach focuses on the evaluation of Einstein's dimensionless bed load transport rate, which uses more than 3000 field data.

Table 6. Model based on numerical and regression analysis

Serial No	Model	Equation
1.	A. Reckling's Model	$\phi = 14\tau_{84}^{*2.5} / [1 + (\tau_m^* / \tau_{84}^*)^4]$, $\tau_{84}^* = \frac{SR}{(s-1)D_{84}}$, $\tau_m^* = (5S + .016)(D_{84}/D_{50})^{4.4\sqrt{S}-1.5}$ $q_{sv} = \sqrt{g(s-1)D_{84}^3} \phi$, $Q_s = W\rho_s q_{sv}$ kg/sec
2.	Samaga et al.	uses a plot between $(\xi_B \tau_0' / \Delta\gamma_s d_i)$ & ϕ_B

Based on measured hydraulic parameters and sediment data at the two cross-sections of the selected reach, the fractional bed load transport rate at the selected sub-sections has been computed. Various attempts have been made to show the variation of the measured bed load transport rate and the computed bed load transport rate. Most of the earlier bed load transport rates have been calculated for the larger streams in which the temporal bed load transport rate is very less; but for the smaller streams, the temporal variation will be high. A total of 24 samples have been analysed. The error analysis or discrepancy ratio of the bed load measured and calculated has been evaluated for each of the selected model/ approach. Table 5 represents both the models based on Numerical and regression analysis.

Samaga et al Approach

The approach put forward by Samaga et al. is a stochastic one because this approach is based on the modification of Einstein's bed load function; it is applicable for particle size greater than 0.065 mm. Initially, the model put forward by Samaga et al. was developed for particle sizes ranging from 0.07 mm to 40 mm. In this approach, the hiding-cum-sheltering coefficient (ξ_B) has been introduced. The value of the dimensionless bed load transport rate is calculated from the plot of $(\xi_B \tau_0' / \Delta\gamma_s d_i)$ vs. ϕ_B . In this method, the fractional bed load transport rate of different particles has been calculated individually.

3.2.4. Approach based on the Theory of Probability Wu et al. Approach (Probabilistic Approach)

The theory put forward by Wu et al. considers the sheltering and hiding effect, which is based on the theory of probability. As far as the sediment transport in the gravelly bedded stream is concerned, the hiding and sheltering effect plays a vital role in sediment transportation. As the non-uniformity of bed interacts in a very complex manner with the

stream hydraulics, and this complex behaviour challenges the basic laws of physics; so, the approach based on the theory of probability gives a better result in the gravelly bedded stream.

The model put forward by Wu et.al. is as follows;

$$\phi_{bi} = 0.0053 \left[\left(\frac{n'}{n} \right)^{3/2} \frac{\tau_b}{\tau_{ci}} - 1 \right]^{2.2} \quad (4)$$

Here, the critical shear stress for the grain size equal to d_i is τ_{ci} . The equation involved for the calculation of τ_{ci} is as follows;

$$\frac{\tau_{ci}}{(\gamma_s - \gamma)d_i} = \phi_c \left(\frac{p_{ci}}{p_{hi}} \right)^m \quad (5)$$

Here, ϕ_c can be interpreted as the non-dimensional critical shear stress for the corresponding uniform sediment or the mean size of the bed materials. The wide range of ϕ_c is 0.03-0.06. For comparison in this study, the value of ϕ_c can be taken as 0.03. If $p_{hi,j}$ represents the probability that a particle of size d_j is hidden by d_i ;

The hiding and exposure probability is as follows,

$$p_{hi,j} = p_{bj} \frac{d_j}{d_i + d_j} \quad \& \quad p_{ei,j} = p_{bj} \frac{d_i}{d_i + d_j}$$

4. Statistical Analysis

To compare the performance of the various equations and methods, a classification system has been created. The ratio of the measured sediment transport rate to the projected sediment transport rate is used to categorise the formulas. The ratio is defined as follows for the categorisation of the transport rate forecasts:

$$\text{Ratio } (j) = \frac{q_{bp}}{q_{bm}}$$

The score is calculated as:

$$\text{Score} = \frac{\sum_{j=1}^n \text{factor } (j)}{n}$$

$$\text{Where factor } (j) = \begin{cases} \text{Ratio } (j), & \text{if ratio } (j) \leq 1 \\ \frac{1}{\text{Ratio } (j)} & \text{if ratio } (j) > 1 \end{cases} \quad (6)$$

In which n is the total number of tests, and j is the specific test. The maximum score of a particular approach/equation is subject to 1. The scores of the selected bed load equations have been assigned based on the absolute bed load transport rate and the measured bed load transport rate. The metric was chosen because it can be used to aggregate the performance measures in a dimensionless way and because it is insensitive to the units and magnitude of the measured bed-load transport rates. It also permits direct comparison between equations developed under different experimental and hydraulic conditions. The metric was chosen because it can be used to aggregate the performance measures in a dimensionless way and because it is insensitive to the units and magnitude of the measured bed-load transport rates. It also permits direct comparison between equations developed under different experimental and hydraulic conditions. Also, the scoring approach lowers the possibility of selecting an equation based on a single statistical characteristic, such as correlation or absolute error.

Further, after assigning the rank to each model/equation, A. Recking's model is best suited for the current study with a rank of 1. In order to assess the suitability of A. Recking's approach for the current study, the Nash- Sutcliffe model efficiency coefficient has been used. The mathematical form of the Nash-Sutcliffe model efficiency coefficient is as under,

$$\text{NSE} = \left| 1 - \frac{\sum_{x=0}^X (x_o^t - x_m^t)^2}{\sum_{x=0}^X (x_o^t - \bar{x}_o)^2} \right| \quad (7)$$

The Nash- Sutcliffe efficiency can be measured as one minus the ratio of the error variance of the modelled data set (Bed load predicted by A. Recking's approach) and the variance of the observed data set. In the case of a perfect model, the value of NSE will be equal to 1, while an NSE value of zero represents model has the same predictive skill as the observed value. The NSE value at cross-section 1 is 0.6586, while at cross-section 2 it is 0.8294. Hence, the performance evaluation of NSE at cross-section 2 reveals a very good ($0.75 < \text{NSE} < 1.0$) agreement between the selected model (A. Recking's Model) and the observed value. Cross-section 1 doesn't yield a higher NSE value. The error in

agreement between the predicted and measured value is primarily due to the complex interaction between the bed load/material load and ambient hydraulic conditions, and the hiding and sheltering effect of larger grain particles.

5. Result and Discussion

The bed load transport rate at section 1 and section 2 computed using Duboys equation ranges between (2.66-17.365) g/s/m, using Meyer-Peter and Muller ranges between (7.2807-20.537) g/m/s, using Shield's equation ranges between (6.734-21.857) g/m/s, using Yalin's equation ranges between (7.857-23.457) g/m/s, using Kalinske's equation ranges between (7.9167-20.537) g/m/s, using Sato-Kikkawa and Ashida equation ranges between (5.3478-22.657) g/m/s, using Van-Rijin's equation ranges between (0.02864-0.2169) g/m/s, using Englund and Fredsoe's equation ranges between (8.6132-28.212) g/m/s, using A. Recking's equation ranges between (3.2404-36.922) g/m/s, using Samaga et al. equation ranges between (0.000788619-4.7060) g/m/s, using Wu et al. equation ranges between (20.810-237.388) g/m/s, using Bagnold's approach the bed load transport rate ranges between (13.9227-15.57621) N/s/m and using Roorkee approach the bed load transport rate varies between (12.678-16.235) N/s/m. Bed load transport rate is highly variable in time and space due to turbulent flow patterns, local variations in bed shear stress, sediment availability, particle-size heterogeneity, and bed-shape development. Because sediment entrainment may occur only when the instantaneous hydraulic forces exceed the critical threshold for motion, bed load is transported intermittently in pulses rather than as a continuous process. As a consequence, large variations of bed load transport rate are recorded for a given stream cross-section. Repeated measurements are required to derive representative estimates of sediment transport. Variations in bed load transport rate are an intrinsic feature of sediment transport in alluvial rivers, reflecting the complex interactions of flow hydraulics, sediment characteristics and channel shape. Bed load transport, on the other hand, is highly discontinuous, unlike suspended sediment transport. It involves the rolling, sliding and saltation of sediment particles along the channel bed. Sediment entrainment occurs only when the instantaneous bed shear stress, produced by the turbulent flow, reaches the critical value for particle mobility. These oscillations are mainly controlled by turbulent flow patterns, regional and temporal bed shear stress variability, sediment supply, particle size heterogeneity and the migration of bedforms such as ripples and dunes. Moreover, the dynamic processes of scour, deposition, and channel adjustment constantly alter the hydraulic and geomorphic conditions, hence adding to the stochasticity of bed load transport. This variability is important for sediment transport modelling, river engineering and watershed management, as instantaneous observations may not be representative of the long-term average transport rate. Therefore, valid estimates of bed load transport can only be obtained by repeated field observations and by statistical

averaging. The recognition and consideration of these natural variations is crucial to improve the prediction capability of sediment transport models and the design and management of sustainable river systems. The statistical analysis and corresponding ranking of selected approaches/ models is summarised in Tables 6 and 7. From Table 6, it is evident that A. Recking's model is the best-suited approach for the computation of bed load transport rate at cross-section 1; because it scores 0.663 on a 1-scale. Apart from A. Recking's model, Du Boys' equation, Shield's equation, Sato-Kikkawa and Ashida approach can be rated to have a satisfactory score as it scores above 0.40. From Table 8, it is evident that A.

Recking's equation is best suited as an equation for the computation of bed load transport rate at cross-section 2, because it scores 0.843 on a 1-scale. Apart from A. Recking's equation, the Shield equation, Yalin equation, Sato-Kikkawa and Ashida approach, Du Boys model and Wu et al. model can be rated to have a satisfactory score as it scores above 0.40.

In the case of the Meyer-Peter & Muller equation, the value of the dimensionless grain shear stress (τ'_*) will be less than 0.047; consequently, the value of the dimensionless bed load transport rate (Φ) will be equal to zero.

Table 7. Ranking of selected bed load computation approaches/equations at cross-section 1

Approach	Equation	Score	Corresponding Ranking
Critical Shear Stress Approach	Du Boys	0.582	3
	Meyer-Peter and Muller Equation	0.318	8
	Shield Equation	0.495	5
	Yalin's Equation	0.642	2
	Kalinske's Equation	0.375	7
	Sato, Kikkawa and Ashida Equation	0.425	6
	Van Rijn's Equation	0.009	10
	Roorkee Approach	0.002	13
	Englund and Fredsoe's Equation	0.520	4
Stream Power Approach	Bagnold's Equation	0.008	11
Theory based on Numerical and Regression Analysis	A. Recking's Model	0.663	1
	Samaga et al. Model	0.003	12
Probabilistic Approach	Wu et al. Model	0.217	9

Table 8. Ranking of selected bed load computation approaches/equations at cross-section 2

Approach	Equation	Score	Corresponding Ranking
Critical Shear Stress Approach	Du Boys	0.822	2
	Meyer-Peter and Muller Equation	0.122	9
	Shield Equation	0.639	3
	Yalin's Equation	0.567	5
	Kalinske's Equation	0.266	8
	Sato, Kikkawa and Ashida Equation	0.598	4
	Van Rijn's Equation	0.005	11
	Roorkee Approach	0.003	13
	Englund and Fredsoe's Equation	0.519	7
Stream Power Approach	Bagnold's Equation	0.009	10
Theory based on Numerical and Regression Analysis	A. Recking's Model	0.843	1
	Samaga et al. Model	0.0035	12
Probabilistic Approach	Wu et al. Model	0.518	6

6. Conclusion

The incipient motion condition of a gravelly bedded stream doesn't obey the basic laws of physics. So, for a particular selected reach where the ambient hydraulic condition is not a function of time, the movement of particles from the selected cross-section is a totally probabilistic phenomenon. As Wu et al.'s theory considers the movement probability of bed particles, it gives a satisfactory result for the present selected reach. While Recking's model has been developed based on a large data set that consists of both

laboratory and field data. Wu et al.'s theory (probabilistic approach) and Recking's approach give better results. In the present study, the discrepancy ratio ranges from 0.73 to 0.87, which can be rated as a satisfactory result as far as the hydraulic uncertainty of a gravel-bedded stream is concerned. Bagnold's equation shows the discrepancy ratio varying from 11 to 13.27. Bagnold's approach is based on the stream power approach, which is based on the work energy principle, but a gravelly bedded stream gives satisfactory results by considering the theory of probability. Samaga et al. and

Meyer-Peter-Muller equations show poor results in the present study; the discrepancy ratio for the Samaga et al. equation varies by an order of 30, while it varies by an order of 111 in the case of the Meyer-Peter-Muller equation. The large variation of the discrepancy ratio may be due to the development of models from steady uniform laboratory flume data.

6.1. Limitations and Future Scope

The present study is based on a relatively limited dataset of 24 bed load samples from two cross-sections of a single stream over a single monsoon season. Although the dataset is sufficient to assess the relative performance of the selected bed load transport equations for the dominant hydraulic conditions, some limitations exist for the general applicability of the results. First, the spatial coverage of the study is restricted. Bed load transport is strongly affected by local channel characteristics, such as bed material composition, channel slope, width to depth ratio, bed forms and hydraulic roughness. Therefore, the performance ranking of the transport equations obtained at a river reach may not be valid at the other reaches of different geomorphic and hydraulic characteristics. The model may perform substantially differently under different sediment gradations, channel morphologies or flow regimes of rivers. Second, there is a single monsoon season in temporal coverage. Bed load transport is highly variable in time due to variations in rainfall intensity, magnitude of discharge, sediment availability, antecedent flow conditions, and seasonal sediment supply.

Hence, the dataset does not capture interannual variability and response of the river to extreme flooding or prolonged low flow conditions, or long-term morphological adjustments. Therefore, the calculated transport rates can be considered as typical only for the period of observation. Another major limitation is that bed load transport is inherently stochastic. Turbulence, particle hiding and exposure effects, armouring and localised bed disturbances lead to an intermittent transport of sediment. Even for nearly identical flow conditions, the instantaneous transport rates can be quite different. Hence, the observed variability is a combination of uncertainty in the models and the inherent randomness of the sediment transport process. To complement experimental and field studies, numerical models have been developed, but these too rely on simplifying assumptions. More recently, Machine Learning (ML) approaches (Random Forest, Extra Trees, Extreme Gradient Boosting, and CatBoost) have been developed to capture complex relationships in data, rather than as direct substitutes for experiments and numerical modelling. To train and validate the models, a large amount of data (shear stress, flow depth, hydraulic radius, water surface slope, water depth, grain size of bed load and bed material, etc.) will be required. Recently, Basumatary et al. (2026) highlighted the potential of ML approaches as trusted alternatives to complex numerical and empirical formulations. In recent years, global warming and the effects of climate change have become sufficiently

serious that extreme hydrographic phenomena are often observed.

Climate change has increased precipitation concentration, volume and intensity, which has greatly affected runoff and soil erosion in many watersheds. Chen et al. (2020) studied the effect of climate change on erosion volume, sediment yield and erosion deposition in a watershed by using a numerical model. The results indicated that, under the climate change scenario, precipitation would significantly increase the volume of soil erosion and subsequently sediment yield. Climate change also increased deposition volume by 2-23% relative to the baseline and 13% relative to the baseline mean.

The deposition mainly occurred at the confluence of rivers, the middle of the river, and the downstream end of the river. Hence, climate change adversely impact to the basin; hence, appropriate conservation measures are suggested [6]. Moreover, the application of remote sensing and high-resolution topographic techniques allows for assessing the impact of sediment transport on river bed morphology, considering in-channel erosion and sedimentation through the difference between pre- and post-flood Digital Elevation Models (DEMs) for each scenario [28].

Abbreviations

q_b	= Bed Load Transport Rate
ρ_s	= Density of sediment kg/m^3
ρ	= Density of flowing fluid
e_b	= Bed load transport rate efficiency
$tana$	= Tangent of the angle of internal friction
τ'_0	= Shear stress due to grain resistance N/m^2
Δ	= $\left(\frac{\rho_s}{\rho} - 1\right)$
τ'_*	= Dimensionless Shear resistance due to grain
Φ	= Dimensionless bed load transport rate
n'	= Manning's Roughness co-efficient due to grain undulations
τ_{ci}	= Critical shear stress corresponding to i^{th} % finer particle
$P_{hi,j}$	= Probability that a particle of size d_j is hidden by d_i
$P_{ei,j}$	= Probability that particle of size d_i is hidden by d_j
W_{tbi}	= Bed load transport rate through i^{th} sub-section $kg s^{-1} m^{-1}$
M_s	= Dry weight of sample
n_s	= Number of samples
L_i	= Length of i^{th} sub-section
q_{bp}	= Predicted bed load transport rate
q_{bm}	= Measured bed load transport rate
x_o^t	= Value of observed variable at selected location
x_m^t	= Value of variable by predictive model at selected location
$\bar{x}_0$	= Mean value of variable at a particular location.

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