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*sOriginal Article*

# Numerical Simulation of Crowd-Induced Vibrations Using Duhamel's Integral

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***Abstract*** *- Crowd-induced vibrations in footbridges represent a relevant issue in the design of lightweight structures. The numerical simulation performed in this research, which is based on Duhamel's integral, allows for the estimation of the dynamic response to pedestrian loads. Within this simulation, the structure is initially modeled as a Single-Degree-of-Freedom (SDOF) system that is subjected to ramp, harmonic, and pedestrian-type excitations, which allows for validating the numerical implementation against already known analytical solutions. After this, a simplified modal model of a supported beam is adopted, considering only the first vibration mode. The footfall force is represented through harmonic functions adjusted by walking speed and the Dynamic Load Factor (DLF), including the random distribution of speeds in individual walks and in crowds. The simulations, developed in Python, show that the system responds with increasing amplitudes when the step frequency approaches the natural frequency. In the case of crowds of 200 pedestrians, maximum displacements close to 2.50 mm and amplification of the modal envelope were observed during the minutes of greatest overlap. The model is efficient and applicable in preliminary design stages. Its limitations include structural linearity and the absence of pedestrian–structure interaction, although it provides results consistent with the reviewed literature.*

***Keywords*** *- Duhamel's Integral, Pedestrian Load, Structural Resonance, Numerical Simulation.*

## 1. Introduction

The vibrations in structures induced by pedestrians have gained particular relevance in recent decades because there is an increasing trend for more slender and efficient designs in footbridges, pedestrian bridges, grandstands, and roofs. In spite of the fact that these structures typically meet the strength criteria required by building codes, their dynamic behavior under periodic excitations (like those generated by human walking) can produce surges that affect user comfort and may even lead to serviceability issues during regular use [1, 2].

One of the most paradigmatic cases that can exemplify the issue previously mentioned is the London Millennium Bridge, where the unintentional synchronization of pedestrians generated significant lateral oscillations right after the bridge was opened.

This event put in evidence not only the need to explicitly consider the interaction between pedestrian loading and structural response, but also the necessity to include serviceability criteria related to human comfort from the early stages of design [3].

Another important challenge with regard to bridge dynamics is to find a solution to this dynamic problem that requires the least computational effort because conventional methods, such as numerical solutions, depend heavily on time steps, and these consume a lot of time when solving the problem. In this sense, it has been identified that the Duhamel integral convolution is a more efficient tool when considering computation time for the same result, since it does not depend on the variable at a previous instant in the behavior of the structure [4].

In the Asian region, particularly in China, numerous studies have been developed that focus on Crowd-Structure Interaction (CSI) and vibration control in slender structures. One of those studies evaluated a VMD-STMD system (a semi-active tuned mass damper with variable parameters) that demonstrated a high effectiveness when it was compared to conventional configurations in mitigating random pedestrian vibrations through 100 simulations under probabilistic scenarios [5]. Another previous investigation conducted a comparative analysis of ten international standards on structural vibrational comfort induced by walking; this research revealed the existence of methodological inconsistencies and proposed improvements that were related to the incorporation of stochastic models and multifactorial comfort criteria [6].

Besides, an estimation method based on computer vision for human dynamic loads on floors was also proposed; this method showed accuracy in real-time prediction of collective forces [7]. Additionally, a study about floor dynamics under random crowds integrated a stochastic pedestrian model in order to analyze how their distribution and stiffness affect modal parameters and structural accelerations [8]. Besides, complementary research introduced a new temporal synchronization factor with the purpose of evaluating the coherence of rhythmic crowd movements and their vibrational impact [9], as well as dynamic reliability methods using the Probabilistic Density Evolution Method (PDEM) for long-span structures under synchronized jumping [10].

A comprehensive review on performance-based structural design under human loads addressed topics ranging from the characterization of the human body to probabilistic vibration models and subjective perception [11]. Finally, a passive adaptive tuned mass system (APVS-TMD) with variable stiffness was proposed, outperforming classical TMDs in experimental tests involving walking and running [12].

In Iran, Duhamel’s integral was generalized for MDOF systems with classical viscous damping, deriving a direct matrix-based formulation without the need for modal superposition, numerically validated for arbitrary dynamic loads [13]. In Iraq, another study executed a comparison of three analysis methods under seismic excitations with multiple supports: state-space, Duhamel, and Newmark, which concluded that the first analysis method offers greater computational efficiency in complex structures [14].

In Europe, Italy reported significant advancements on this topic. In this case, a wide range of peer-reviewed research was compiled within the framework of the congress on mechanism dynamics, vibrations, robotics, and structural design, consolidating a multidisciplinary reference for vibratory analysis and structural control [15]. Furthermore, a technique for estimating dynamic loads on grandstands using computer vision and image correlation was validated, confirming its practical applicability even in noisy environments and with low-resolution cameras [16].

In Spain, a framework was developed to quantify and propagate uncertainties in crowd-induced vibrations by integrating dynamic models and human loads under a double-loop analysis (Monte Carlo and optimization) [17]. In this Spanish case, a frequency-domain approach was proposed with the objective to model vertical pedestrian–structure interaction, which was experimentally validated and showed advantages over traditional models [18]. Additionally, another study applied a coupled two-degree-of-freedom model, which was based on experimental data, with the purpose of representing the interaction that happened in cable-stayed footbridges; this last model ended up outperforming models without interaction in terms of accuracy [19].

In France, an analytical model was formulated for vertical vibrations in rails on ballast. This model was analyzed through the application of Floquet’s theorem, and it was necessary to consider the combined response of beams and supports. This permitted the execution of a highly accurate evaluation of resonances and load distribution [20].

In the United Kingdom, the effectiveness of TMDs on lightweight pedestrian bridges under human–structure interaction was analyzed. The outcomes obtained in this analysis lead to the conclusion that although such interaction reduces structural response, TMDs remain effective if properly designed [21].

In Belgium, a simplified single-degree-of-freedom method was proposed in order to include vertical HSI in the comfort analysis. This method was later validated on real bridges and showed good accuracy and conservatism in critical ranges [22].

In Finland, OpenTorsion was introduced, which is a Python library for torsional vibration analysis, as a tool that was designed as an open-source tool with the purpose of simulating and optimizing rotary systems across various industrial sectors [23].

In the Americas, the United States leads studies on structural monitoring with the employment of sensors. A recursive sparse representation method was developed with the objective of identifying multiple walkers through the usage of floor vibrations, achieving high accuracy even under simultaneous occupancy [24]. Another study proposed a crowd monitoring system for stadiums based on floor vibrations, obtaining results superior to conventional video or audio-based methods [25]. A low-cost system (CHEAP) was also introduced to measure structural vibrations using MEMS accelerometers and Arduino, validated against commercial sensors [26]. Additionally, a probabilistic analysis of footstep-induced vibrations on office slabs demonstrated how Monte Carlo simulation enables estimation of serviceability risks based on stochastic variables such as weight and walking speed [27].

In Peru, operational modal testing and numerical calibration were applied to the By-Pass Bridge on Av. Túpac Amaru, the employment of these tools leads to the achievement of a precise correlation between the model and real-world data [28]. In addition, another study compared the use of TMDs and viscous dampers on the Los Próceres pedestrian bridge; this comparison led to the validation of both strategies to be effective in the absence of national codes [29].

In Colombia, pedestrian bridge comfort under dynamic pedestrian loads was evaluated, which led to the identification of overlaps with critical frequencies and highlighted the need for specific national regulations [30].

In Mexico, experimental tests and numerical analyses were conducted on 33 structural configurations of footbridges, concluding that many structures exhibit serviceability deficiencies due to vibration. The use of simplified models to estimate accelerations was validated, and the inclusion of dynamic criteria in national codes was recommended [31].

Another recent investigation in China [32], used a refined model based on cellular automata incorporating particular attributes of pedestrians into the finite element model and realistic models applied as in the case of the pedestrian bridge near Beijing Jiaotong University, the complexity of this numerical model required input parameters for both the pedestrian (body stiffness, body damping, speed, weight distribution, leg stiffness, leg damping) and for the pedestrian bridge (geometric dimensions, linear mass, modulus of elasticity, Poisson's ratio).

In this regard, the aim is to solve the dynamic problem of obtaining the time response of the damper. The results obtained through computational simulations have demonstrated that the SDOF model with modal projection is effective in representing the dynamic behavior of a footbridge under pedestrian loads. This methodological choice-based on Duhamel’s integral and loads modeled using realistic harmonic functions-is aligned with previous works suggesting that a first-order modal representation can accurately capture the vertical response to human traffic [30, 31], however, this model is applicable to linear cases, although it can be extended to nonlinear models by using Newmark β numerical methods and Newton-Raphson algorithms [33].

Analysed linear systems under arbitrary loads, both through analytical expressions and a simplified numerical implementation. To this end, a simplified model is developed to represent real structures, allowing for the simulation of their behavior under the passage of one or more moving pedestrians, considering factors such as structural mass, walking speed, and excitation frequency. In this context, the objective was to analyze the dynamic behavior under moving loads produced by pedestrian traffic on Single-Degree-of-Freedom (SDOF) structures, using a solution based on the Duhamel integral implemented in Python.

As an illustrative case, Figure 1 shows several suspension bridges located in the town of Izcuchaca, Huancavelica, Peru. These structures, in addition to fulfilling a functional role, have acquired touristic value, resulting in a constant and significant pedestrian flow. Their intensive use makes them representative examples of the type of structure susceptible to crowd-induced vibrations, thereby justifying the need for analysis.

**(a) Suspension Bridge 1** **(b) Suspension Bridge 2** **(c) Suspension Bridge 3**

**Fig. 1 Caption follows**

## 2. Materials and Methods

This study focuses on the numerical simulation of pedestrian-induced dynamic response on footbridge-type structures. The objective is to analyze the vertical vibrations generated by pedestrian traffic through an approach based on Duhamel’s integral. To this end, simplified models were implemented to capture the essential behavior of the structural system, combined with time-dependent functions that simulate the load exerted by pedestrians. All numerical processing and result visualization were programmed in Python.

### 2.1. Structural Model

The structural analysis performed in this research was addressed through two levels of modeling. First, a Single-Degree-of-Freedom (SDOF) system that was composed of a mass, stiffness, and a linear damper. This approach represents the fundamental behavior of lightweight structures like the pedestrian bridges when put under vertical loads. Second, a supported beam model was developed, and in this model, the pedestrian load was projected onto the first vibration mode. This technique of using two levels of modeling extends the validity of the analysis to more realistic structural conditions. Figure 2 shows the conceptual diagram of the SDOF system, while Figure 3 illustrates the beam model under a moving load. These visual representations facilitate the understanding of the vibratory phenomenon induced by human walking.

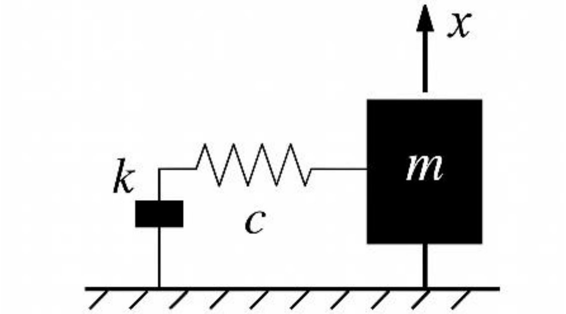


**Fig. 2 Conceptual diagram of the Single-Degree-of-Freedom (SDOF) system**

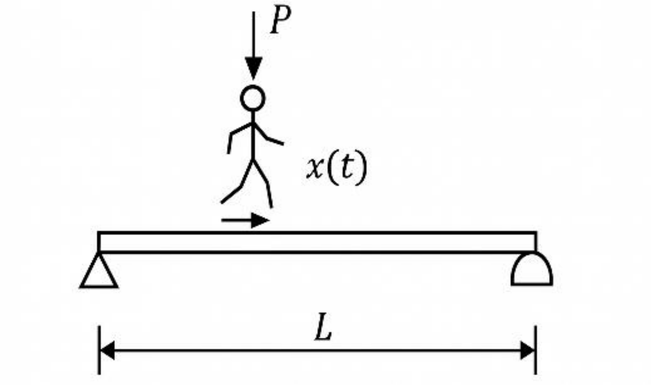


**Fig. 3 Supported beam model under moving pedestrian load.**

#### 2.1.1. Single-Degree-of-Freedom (SDOF) System

A Single-Degree-of-Freedom (SDOF) system model was initially adopted, which is composed of a mass 'm', a stiffness 'k', and a viscous damper 'c'. This system was adopted because it adequately represents the fundamental vertical behavior of lightweight structures like pedestrian bridges [34]. This model also enables the simulation of the response to various types of loads that depend on time. Additionally, the equation of motion for the damped system is given by:

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = F(t) \tag{1}$$

Where u(t) is the displacement, and F(t) is the applied force.

#### 2.1.2. Modal Model of a Supported Beam

With the purpose of representing more realistic structural conditions, a supported beam model subjected to a moving load was employed [35]. The pedestrian load was projected onto the first vibration mode of the beam, which permitted the spatial distribution of the load to be considered through modal analysis. The mode shape for a supported beam is defined by:

$$\phi(x) = \sin\left(\frac{\pi x}{L}\right) \tag{2}$$

Where 'L' is the length of the beam and 'x' is the longitudinal position.

### 2.2. Load Model

To simulate the excitation induced by pedestrian traffic on structures, different types of loads were modeled according to the objective of the analysis. Each type of load allows the study of specific phenomena, such as analytical validation, frequency-domain analysis, or realistic representation of human behavior.

The loads were implemented using time-dependent functions programmed in Python, and their correct representation was graphically verified before being applied to the structural model. Three types of loads are considered, all time-dependent and applicable in different validation and simulation contexts.

#### 2.2.1. Ramp Load

This type of load is defined as a linear function that increases from zero to a maximum value over a period of time, and then remains constant [34]. It is used as a validation case to compare the numerical solution with an exact analytical solution. The expression for the ramp load is:

$$F(t) = \begin{cases} \dfrac{P_0 t}{t_1}, & 0 \le t \le t_1 \\ P_0, & t > t_1 \end{cases} \tag{3}$$

Figure 4 shows the time-dependent behavior of this function.

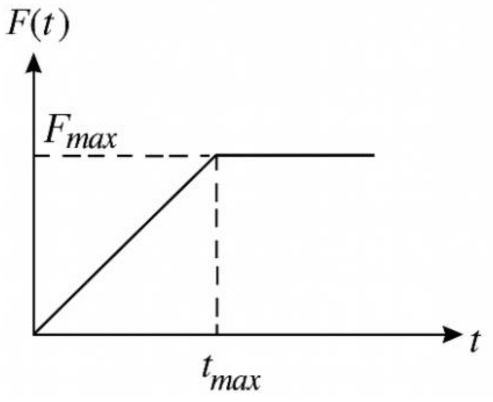


**Fig. 4 Ramp load**

#### *2.2.2. Harmonic Load*

This type of load permits performing a precise analysis of the structural system’s sensitivity when facing periodic excitations; this analysis is fundamental for studying resonance phenomena [34]. Besides, it is modeled as a sinusoidal function of the form:

$$F(t) = P_0 sin(\omega t) \tag{4}$$

Where ‘$P_0$’ is the amplitude and ‘ω’ is the angular frequency of the excitation. This formulation is useful for evaluating the system's response to frequencies near the natural frequency, where large displacement amplitudes may occur. Figure 5 illustrates the typical time-domain behavior of this load.

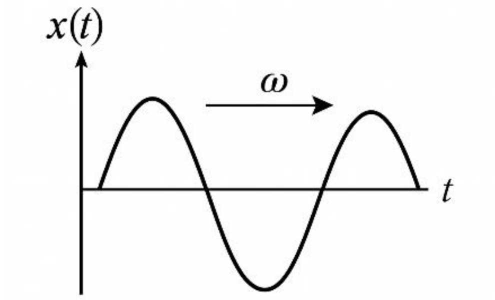


**Fig. 5 Harmonic load**

#### *2.2.3. Pedestrian Load*

The pedestrian load has been modelled by using the concept of Ground Reaction Force (GRF) with a Fourier series–like formulation. The simplified expression of the first harmonic is:

$$F_v(t) = G + \sum_{i=1}^{n} G\, DLF_i \sin(2\pi i f_v t - \phi_1) \tag{5}$$

Where ‘G’ represents the static weight of the pedestrian, ‘$f_v$’ is the stepping frequency, ‘$DLF_i$’ is the dynamic load factor of the i-th harmonic, ‘$\phi_1$’ is the phase of the harmonic, and ‘n’ is the number of harmonics considered. For simplification, only the fundamental harmonic (i=1) is considered in this study:

$$F_v(t) = G + [1 + \mathrm{DLF}\ \sin(2\pi i f_v t)] \tag{6}$$

The stepping frequency, fv, was modeled as a cubic function of the walking speed v, in meters per second, using the empirical relationship proposed by Bruno and Venuti [36]:

$$f_v = 0.35v^3 - 1.59v^2 + 2.93v \tag{7}$$

This relationship is valid for stepping frequencies between 1.0 Hz and 2.8 Hz. Walking speed is considered a normally distributed random variable with a mean of 1.3 m/s and a standard deviation of 0.125 m/s. This probabilistic approach enables pedestrian simulation of uncongested pedestrian flows, with the natural variability of pedestrians being modeled.

In addition, the effective duration of application of the load Fv (t) on the structure depends on the span length L and the walking speed v. This time period is determined as follows:

$$t = \frac{L}{v} \tag{8}$$

Where t is the crossing time of the pedestrian, this expression is basic to the specification of the time interval over which each individual load is acting on the structural system.

### ***2.3. Solution Using Duhamel’s Integral***

The dynamic response was determined with the use of Duhamel's integral, which enables the determination of the displacement u(t) of a damped SDOF system under an arbitrary load F(t) [35].

$$u(t) = \frac{1}{m\omega_d}\int_o^t F(\tau)e^{-\xi\omega_n(t-r)}\,\mathrm{sin}\omega_d(t-r)d\tau \tag{9}$$

Where ωn is the undamped natural frequency of the system, ωd is the damped frequency, and ξ is the damping ratio. It is a formulation that assumes a superposition of responses to infinitesimal impulses.

The solution of the integral was obtained numerically using the trapezoidal rule to ensure the accuracy and efficiency of the evaluation of the displacements under complex loads. Additionally, the algorithm was programmed in Python and validated through the comparison of it with the already known analytical response.

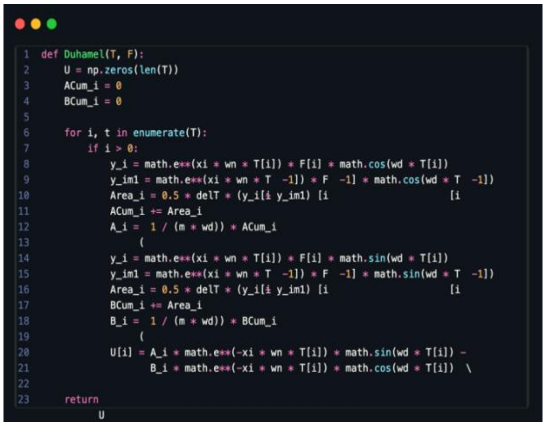
### ***2.4. Computational Simulation***

The numerical simulations were done completely in Python, which includes vector classes from NumPy [37] and graphical visualization from Matplotlib [38]. The structural model was a footbridge, 37.5 m long, and the linear mass was 960 kg/m, which means that the first-mode modal mass was 18,000 kg. Regarding the natural frequency, an average value was chosen within the critical states for vibrations in pedestrian walkways established in Eurocode 0, which sets it between 1.25 Hz and 4.60 Hz. A natural frequency of 2.5 hertz and a relative damping ratio of 5 percent were assumed. Furthermore, it establishes that when the frequency is within this range, a specific dynamic analysis is necessary [39]. The corresponding modal stiffness was calculated based on these parameters.

With the objective of representing the pedestrian action, each pedestrian was modeled with a mass of 80 kilograms; this assumption generated an average vertical load of 784.8 newtons. In addition, the stepping frequency was estimated considering a walking speed of 1.3 meters per second, and the dynamic load factor was obtained from empirical expressions found in the literature. The harmonic load, which is associated with human stepping, was applied with a characteristic duration of 0.8 seconds and a time discretization of 0.05 seconds.

Regarding the crowd simulation, a flow of 200 pedestrians was considered, which were randomly distributed within a 10-minute simulation window, with an additional margin of 60 seconds in order to capture transient effects after the group had crossed. Each pedestrian was assigned a random entry time as well as a random walking speed; also, their dynamic influence was evaluated individually. Subsequently, the individual structural responses were superimposed with the purpose of obtaining the global response of the structure subjected to collective pedestrian traffic.

The function to solve Duhamel's integral, the heart of the numerical procedure developed, is presented in Figure 6, so that the displacement response of the structural system can be computed as a function of any load applied that is defined by a set of times. This function was designed as a modular approach to facilitate the re-use of the function and validation for various loading scenarios. In addition, Table 1 shows the summary of the input parameters for the simulation in the simplified model.



```python
def Duhamel(T, F):
    U = np.zeros(len(T))
    ACum_i = 0
    BCum_i = 0

    for i, t in enumerate(T):
        if i > 0:
            y_i = math.e**(xi * wn * T[i]) * F[i] * math.cos(wd * T[i])
            y_im1 = math.e**(xi * wn * T  -1]) * F  -1] * math.cos(wd * T  -1])
            Area_i = 0.5 * delT * (y_i[i y_im1) [i                      [i
            ACum_i += Area_i
            A_i =  1 / (m * wd)) * ACum_i
                 (
            y_i = math.e**(xi * wn * T[i]) * F[i] * math.sin(wd * T[i])
            y_im1 = math.e**(xi * wn * T  -1]) * F  -1] * math.sin(wd * T  -1])
            Area_i = 0.5 * delT * (y_i[i y_im1) [i                      [i
            BCum_i += Area_i
            B_i =  1 / (m * wd)) * BCum_i
                 (
            U[i] = A_i * math.e**(-xi * wn * T[i]) * math.sin(wd * T[i]) -
                   B_i * math.e**(-xi * wn * T[i]) * math.cos(wd * T[i])  \

    return
          U
```

**Fig. 6 Python function for solving Duhamel's integral using numerical integration**

**Table 1. Input parameters for the simplified model**

| **Pedestrian bridge** | |
|---|---|
| L (m) = | 37.5 |
| ΔT (s) = | 0.005 |
| m (kg/m) = | 960 |
| $\zeta$ = | 0.05 |
| $f_1$ (Hz) = | 2.5 |
| **Pedestrian** | |
| m (kg) = | 80 |
| $T_{route}$ (s) = | 600 |
| $T_{free}$ (s) = | 200 |
| Number of pedestrians = | 200 |

## 3. Results

This section presents the main findings obtained through the computational implementation of the SDOF model and the evaluation of pedestrian loads using Duhamel's integral. This section also covers basic validations with idealized loads, as well as the analyses of individual walking and crowd simulations. Besides, structural and load parameters are not presented as unique values, but rather explored across multiple representative scenarios with results that are organized in tables to facilitate quantitative comparison.

### *3.1. Numerical Model Validation*

The validation of the computational scheme was carried out using a ramp load; this was chosen because of its exact analytical formulation that permits a direct comparison between the numerical and theoretical solutions. This load, characterized by a linear increase up to a constant value, is commonly employed with the objective of verifying the integration of the algorithms in dynamic systems. The outcomes obtained in the analysis depicted a precise match between both responses; this validates the correct implementation of Duhamel's integral. Besides, this validation ensures the reliability of the model under arbitrary loads and confirms the stability of the procedure under appropriate time discretization. Figure 7 presents the superposition of the analytical and simulated responses, highlighting the accuracy of the developed algorithm for solving the transient response of SDOF systems.

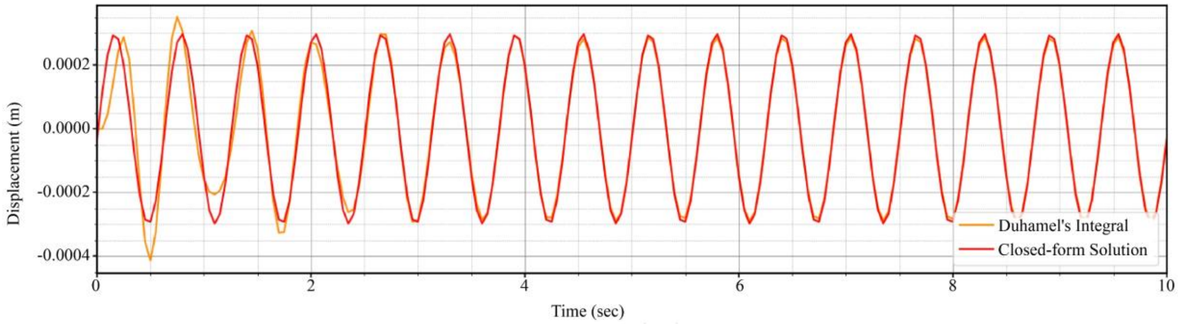


**Fig. 7 Comparison between analytical and numerical response of the SDOF system under ramp load**

### *3.2. Individual Walking Analysis*

With the aim of analyzing the dynamic effects of human walking on footbridge-type structures, the pace of a pedestrian moving at constant speed over a supported beam was simulated. Additionally, the vertical ground reaction force generated by each step was projected onto the first vibration mode of the structure; this permitted the modal load to be obtained as the product of both the instantaneous force and the mode shape, which were evaluated at the pedestrian's position. The evolution of this modal load is illustrated in Figure 8, where its increase can be perceived as the pedestrian approaches the center of the span.

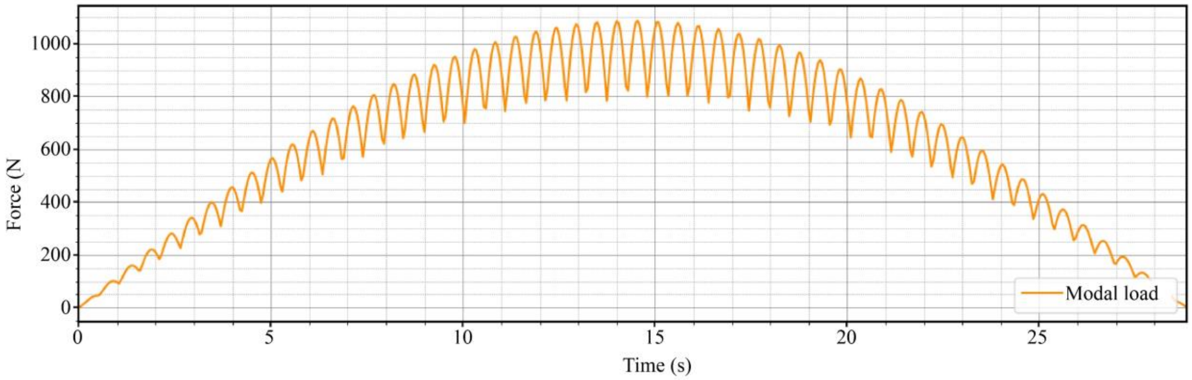


**Fig. 8 Evolution of the modal load induced by a pedestrian in longitudinal motion**

With regards to the dynamic response, it was solved through the usage of Duhamel's integral by integrating the contribution of the applied load over time. This procedure was applied separately to the static and dynamic components of the load, and subsequently to their combination. Figure 9 depicts the individual responses for both components, while Figure 10 presents the combined total response. These findings exhibit an oscillatory behavior that is superimposed on the deflection induced by the pedestrian's constant weight.

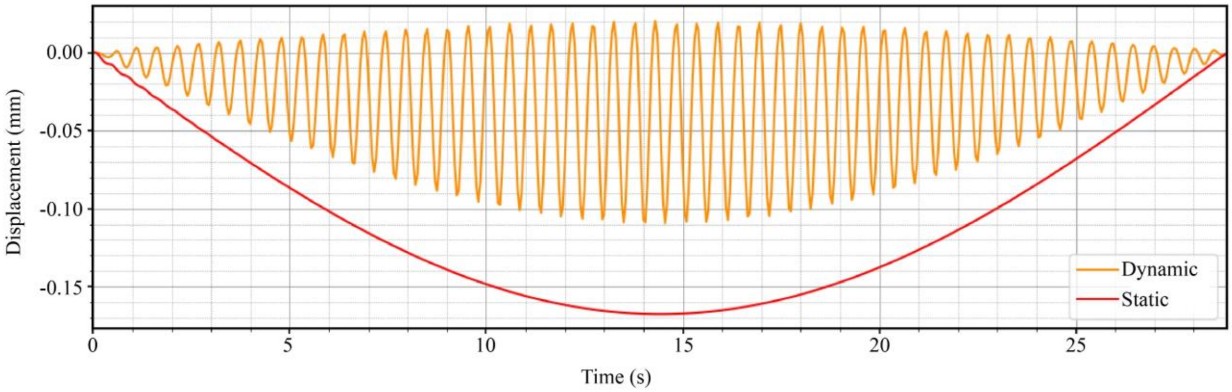


**Fig. 9 Dynamic response separated into static and dynamic components**

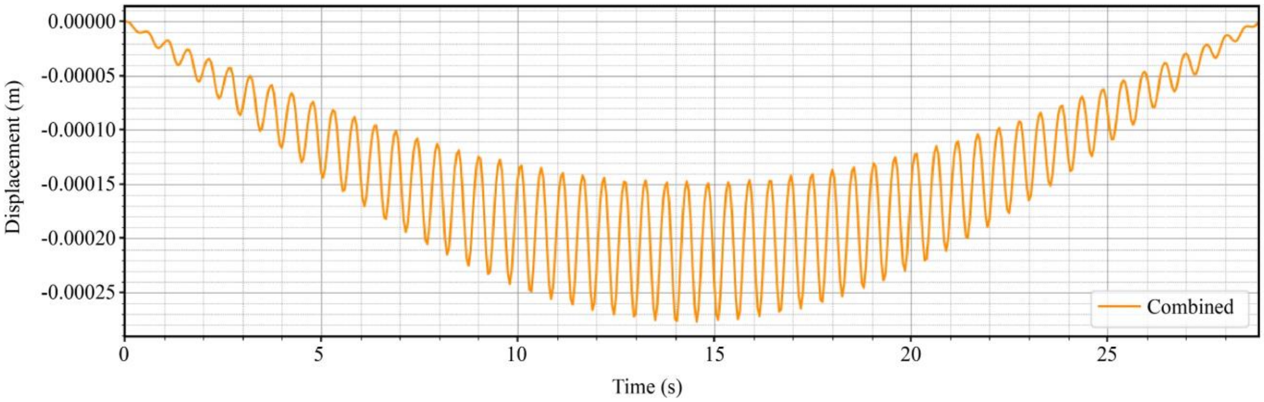


**Fig. 10 Combined response of the structural system under pedestrian load**

For better interpretation, the envelope of maximum displacements was plotted, obtained from the response peaks recorded over time. This visualization, presented in Figure 11, clearly identifies the regions of greatest amplitude, which are usually located when the pedestrian is crossing the central area of the span.

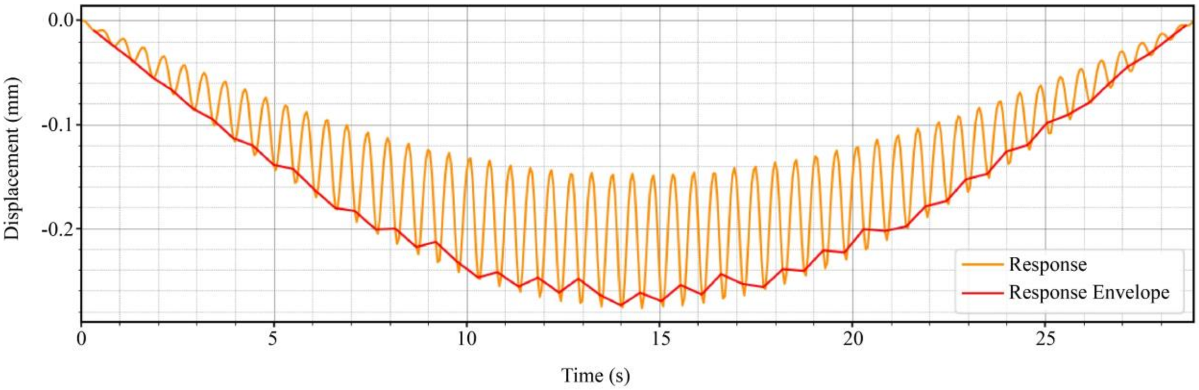


**Fig. 11 Vertical displacement envelope during pedestrian crossing**

An oscillatory response is observed; at the same time, it is superimposed on a quasi-static deflection associated with the constant weight of the moving pedestrian. It was also noticed that the amplitude of the oscillations increases as the pedestrian approaches the midspan (where the mode shape reaches its maximum value) and subsequently decreases as they move toward the opposite end. This asymmetry is the result of the spatial projection of the load onto the first vibration mode.

The envelope highlights the displacement peaks and provides a clear view of the global evolution of the vibratory phenomenon. The maximum displacement is detected to be about 0.275 mm occurring near the mid crossing, which is in good agreement with the quasi-resonant condition caused by the approach of the stepping frequency and the natural frequency of the structure.

### *3.3. Crowd Simulation*

The individual dynamic contribution of each pedestrian was then characterized by randomly assigning each of the 200 pedestrians a walking speed (normally distributed) and a uniformly distributed entry time over a 10-minute period while the pedestrian crossed the footbridge. Figure 12 illustrates the magnitude of the modal loads applied by each pedestrian to the first vibration mode as a function of time.

This shows a progressive superimposition of vertical impulses as pedestrian density on the deck is increased; each of these curves is an individual load trajectory. This visualization of the graph allows the identification of the zones of highest modal overlap since they are necessary to explain the noticed structural response peaks in subsequent stages of the analysis.

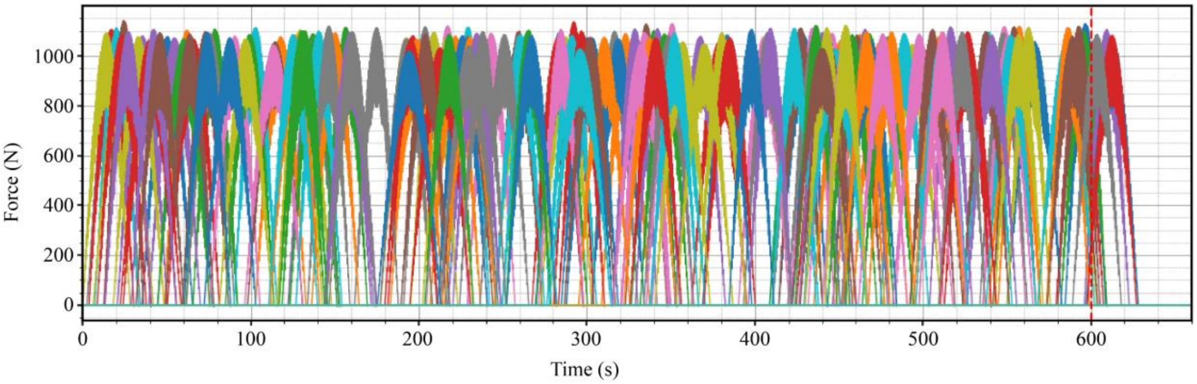


**Fig. 12 Individual modal forces generated by a crowd of pedestrians in motion**

The time evolution of the induced structural displacement at each pedestrian is shown in Figure 13, where the pedestrian is modeled as a harmonic load moving on the first vibration mode. The curves are the individual modal responses corresponding to a range of pedestrian speed and entry time randomly assigned to each of the crossing pedestrians. The colored lines show how the individual responses overlap over time, causing more and more displacements to be experienced.

The strength of the modal excitation is evident during the entire simulation period, between the red vertical lines corresponding to the beginning and end of the analysis window (600 seconds). The independent responses show a common trend that can cause large vibrations when the stepping frequency is in partial coincidence with the resonance frequency of the structure. This analysis is crucial in achieving a deeper understanding of the dynamics of multiple asynchronous excitations and their combination on lightweight pedestrian structures.

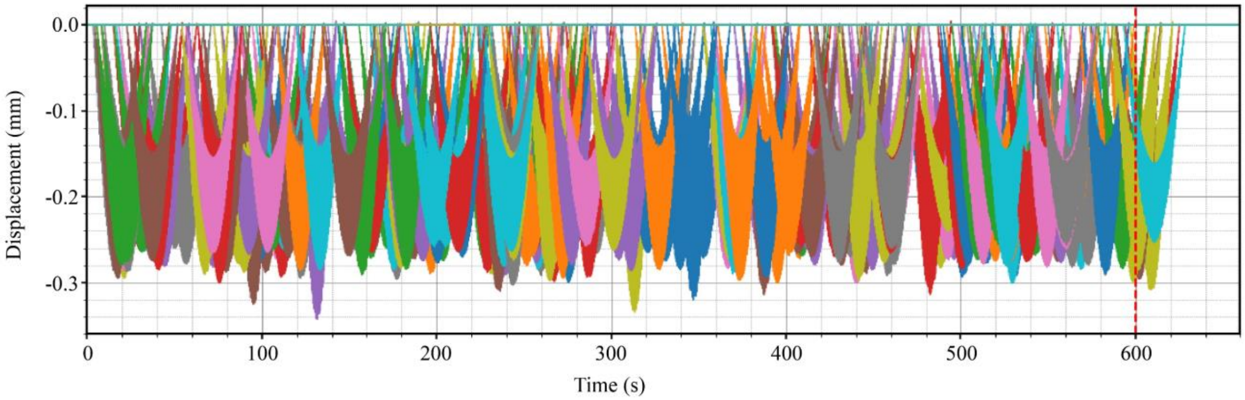


**Fig. 13 Individual modal displacement generated by multiple pedestrians crossing the footbridge**

The evolution of the total modal force induced by a pedestrian crowd was analysed by decomposing the individual forces into the first mode of vibration of the structure. This allowed for the generation of realistic pedestrian flow conditions on bridges, each pedestrian being represented by a harmonic load, with randomly assigned stepping frequency, walking speed, and initial phase.

A non-periodic but structured response was seen during the main simulation interval of 600 seconds (corresponding to the collective crossing window), as peaks in force were seen to occur at peaks in the number of pedestrians crossing the deck simultaneously. The maximum value of the total modal force was found at about the minutes, indicating a critical condition of dynamic superposition. After the crowd had passed, the response decreased rapidly; this outcome indicated that the structural system had recovered once the load was removed.

The results obtained enable quantitative assessment of the cumulative response of a lightweight structure to human dynamic actions and highlight the relevance of addressing phenomena of partial synchronization and pedestrian densification during the design process. This total modal force will be presented as it evolves over time in Figure 14.

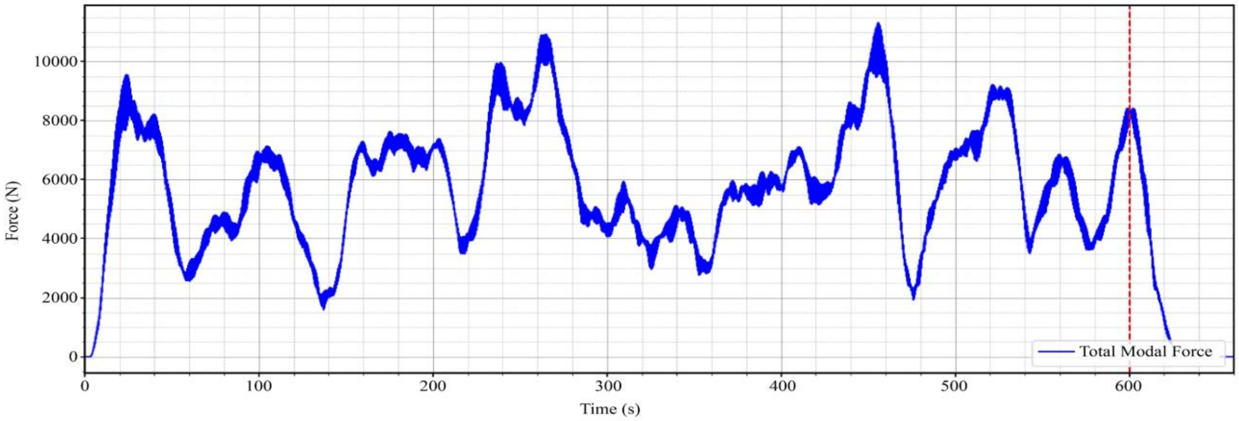


**Fig. 14 Temporal evolution of the total modal force induced by a pedestrian crowd**

Lastly, the overall dynamic response of the structural system due to the combined passage of 200 pedestrians was investigated, shown in Figure 15, and modeled by the summation of individual modal forces. Both the global modal response of the system and its envelope were shown, and this observation allowed for an easy assessment of the maximum amplitude at each point in time. Moreover, the answer has a complicated oscillatory structure, and the time dependence of this structure is directly affected by the random distribution of pedestrian speed and entry time.

In the first phase, the greater the number of pedestrians on the footbridge, the stronger the response. Between 300 and 500 seconds, several dynamic peaks were identified, corresponding to local quasi-resonant conditions, when the coincidence of the stepping frequency and the structural frequency boosted the modal response; moreover, in Figure 14, the occurrence of the critical instant that yielded the modal load during the time interval 300 to 500 seconds was already shown in the displacement response for the approximate instant of 458 s. The highest amplitude recorded is over 2.50 mm.

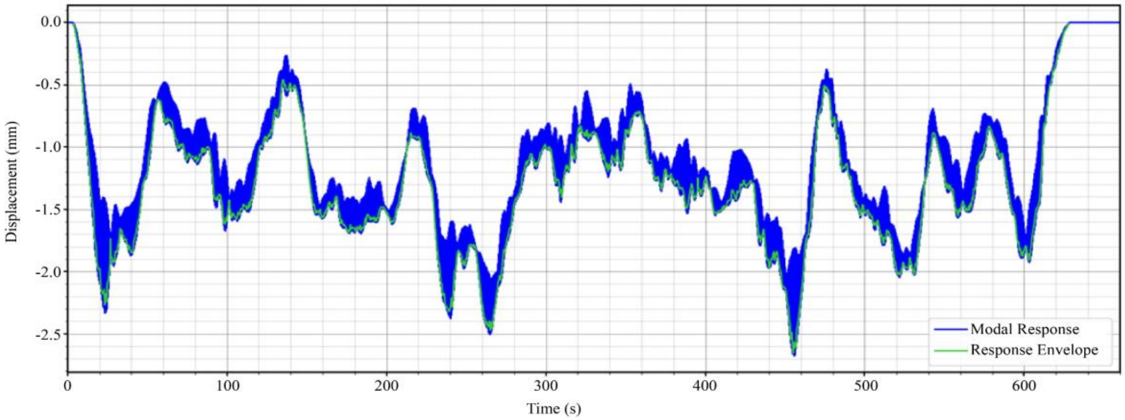


**Fig. 15 Total modal response and displacement envelope induced by a pedestrian crowd**

## 4. Discussions

Besides, this research [40] shows that the first fundamental bending mode is more important for resonant cases of structure, due to the similar smoothing under the passage of the moving load that is attained through the participation of the second and third bending modes.

In the individual walking analysis, the results showed that the interaction between the stepping frequency and the natural frequency of the structure is the critical factor governing the magnitude of vertical displacement. This is consistent with studies conducted in Colombia and Mexico, which report that the proximity of these frequencies can trigger quasi-resonance conditions, even in structures that meet static stiffness requirements [30, 31].

The present model allows this resonance to be observed without requiring the complexity of a full MDOF system, thereby validating the modal reduction approach used. Therefore, these simplified models can be easily extended to multiple degrees of freedom by decoupling and applying modal superposition, as in the research [41], where a semi-analytical method based on the formulation of modal superposition for multiple spans in bridges was used.

Regarding crowd simulations, it was verified that the progressive accumulation of loads leads to significant amplification of the modal response, reaching displacement values greater than 2.5 mm. This finding is consistent with observations from European and Asian research, where partial synchronization among multiple pedestrians has been shown to exacerbate vibrational amplitudes in slender structures, [5, 7, 21]. In particular, the Probabilistic Density Evolution Method (PDEM) model proposed in China [9] reinforces the importance of treating multiple pedestrians as a collective stochastic system, which was addressed here through the random distribution of walking speeds and entry times.

Another case study [32] conducted in China, where refined finite element models were developed considering the novel bidirectional mechanical model for simulating vertical and lateral movements, as well as experimental models on pedestrian bridges in Beijing, found that the impact frequency of pedestrians between 1.6 and 2.4 Hz tends to increase the structure's response. This phenomenon was also evident in Figure 13, where the critical frequency was 2.5 Hz. However, when pedestrian density exceeds the critical density threshold, the phenomenon of "mutual repulsion" occurs, which consists of the readjustment of the synchronization between pedestrians and the structure, thus attenuating the vibrational response.

The displacement graphs above also indicate that the response does not immediately fade after the peak pedestrian demand; this behavior has also been observed in models of dynamic crowd–structure interaction in Asia and Europe, [6, 8, 17].

Realistic loads derived from Ground Reaction Forces (GRF) and Dynamic Load Factors (DLF) adjusted by empirical formulas [1] allowed the creation of the pedestrian force curves in accordance with the field measurements. This is comparable to computer vision–based methodologies developed in Italy and China, [6, 15], although in this work, synthetic pedestrian generation was sufficient to evaluate global effects.

Moreover, the strategy of modeling the modal force and then solving through direct integration allowed for the identification of the influence of structural parameters such as modal mass and stiffness. This aligns with analyses conducted in Peru on the calibration of structural models through modal testing [27] and with studies on passive control in pedestrian bridges [28].

Finally, it is important to note that, although the present study does not incorporate bidirectional pedestrian–structure interaction or lateral synchronization, its approach adequately reproduces the main vertical dynamic effects of interest. This is compatible with simplified approaches validated in Belgium for comfort evaluation [21] and with uncertainty propagation methodologies developed in Spain [16], particularly in the early stages of structural design or numerical model validation.

## 5. Conclusion

This study has demonstrated the effectiveness of a simplified structural model (based on a Single-Degree-of-Freedom (SDOF) system and Duhamel's integral) for accurately simulating the vertical dynamic response induced by pedestrians on lightweight footbridges. Realistic load functions derived from Ground Reaction Forces (GRF) were implemented and validated through comparison with analytical responses under ramp-type loads. The temporal discretization and numerical implementation in Python allowed the capture of both transient and steady-state effects under conditions of varied loads.

In the individual walking analysis, it was verified that the maximum structural response occurs when the pedestrian's stepping frequency approaches the natural frequency of the structure. This circumstance generates quasi-resonant conditions. The effect of the harmonic part of the load was observed and showed good segregation between the static and dynamic components, which allowed the observation of the amplifying effect of the harmonic part of the load.

The simulation results of a pedestrian crowd with 200 randomly assigned walking speeds and pedestrian entry points resulted in the observation of a progressive build-up of modal effects in the scenarios of collective walking. The largest displacements in the system, up to 2.50 mm, were reached, and the envelopes of those displacements were clearly dominated by the superposition of partially synchronized harmonic loads. The maximum response happened around the 9-minute mark on the simulation; this observation agrees with the maximum pedestrian population that occurs on the structure.

These results have validated the results of the previous study in that the presence of multiple walking loads can create high levels of vibration in structures designed outside of the typical resonance range. Therefore, dynamic assessment, based on realistic pedestrian loading models, is necessary to ensure comfort and structural serviceability, particularly for long and flexible footbridges.

The method outlined in this study is a powerful and effective tool for preliminary investigation or conceptual design and allows the use of probabilistic parameters without resorting to complicated pedestrian–structure interaction models.

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