

Original Article

Research on Model Predictive Control for Autonomous Car Assistance Systems Applications

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Abstract - Autonomous vehicles are making significant progress in research and manufacturing, growing closer to becoming a reality. These vehicles function as complicated systems with many various parameters impacting their performance. To guarantee that autonomous cars can navigate properly, maintain a constant pace, and react adequately to their surroundings while keeping passengers secure and comfortable, it is necessary to apply sophisticated control systems to adapt to changing situations. This article describes a technique for driving autonomous cars utilizing Model Predictive Control (MPC) in conjunction with Simulink for model construction. By using script functions and critical constants like vehicle model parameters, controller design parameters, road conditions, and nearby vehicles, the MPC controller can handle various constraints such as speed limits, safe following distances, physical limits of the car, and obstacles that must be avoided. The usefulness of this strategy is proved via simulated situations at various speeds using MATLAB/Simulink. Results show that the MPC controller effectively manages the autonomous vehicle, ensuring safe and efficient navigation in different scenarios. As technology advances, incorporating advanced control systems like MPC will bring autonomous cars closer to widespread adoption.

Keywords - MPC, Autonomous Car, PID, AIS, AUVs.

1. Introduction

Currently, there is a rising trend in the advancement of autonomous driving assistance systems in both theoretical and production phases [1]. Autonomous Intelligence Systems (AIS) are a group of systems that operate or react to the environment without human intervention. Examples include self-driving cars [2] and Unmanned Aerial Vehicles (UAVs) [3]. AIS systems typically encompass the following key functions: collecting sensory data, evaluating the current situation, establishing goals, formulating plans based on past experiences, executing plans, receiving rewards for successful outcomes, and assimilating knowledge from previous encounters [4]. Furthermore, AIS intelligent systems must possess the ability to make sound and logical judgments in unfamiliar environments [5]. The transparency of the decision-making or actions of AIS must be guaranteed to instill the necessary confidence in the driver. Owing to its favorable attributes, this system has attracted significant interest from academia, industry, and governments globally. Its significance lies in its capacity to decrease road accidents, enhance traffic efficiency, and conserve energy. Self-driving vehicles are experiencing rapid expansion and triggering a notable transformation in the automotive and transportation

sectors. This is attributable to the remarkable advancements in computer science, control science, communication technology, and engineering, along with the enforcement of new regulations and policies [6]. This control system needs to navigate the autonomous car according to the required trajectory, which means that the position and driving angle of the autonomous vehicle must be accurate and meet the requirements with the smallest error [7]. That is why the navigator of autonomous vehicles has been using linear control methods such as PID controller [8] and LQR [9]. Nonlinear control methods such as fuzzy [10], neural or hybrid control such as neuron [11]. These control methods have ensured adjustment to meet the position and steering angle of the vehicle. Self-propelled moving in the correct orbit. However, the problem of the navigator having to adapt to the dynamic changes of the autonomous vehicle is an important task. Model Predictive Control (MPC) is an effective optimization approach used for model-based feedback control of a system in this particular scenario. Essentially, the MPC controller performs timely predictions on the system model using various driving techniques. The MPC promptly decides the subsequent control action via optimization. Subsequently, it recommences the optimization



procedure to confirm the subsequent control input [12]. The selection of control inputs, both current and prospective, is aimed at minimizing the disparity between the desired set point and the anticipated output [13]. The MPC exhibits characteristics and capabilities that efficiently fulfill criteria and accomplish optimization tasks. The basic Model Predictive Control (MPC) controller optimizes Linear Programming (LP) problems, enhancing the performance of the conventional Proportional-Integral (PI) controller. In addition, MPC controllers include the inherent capability to manage both soft and harsh limitations effectively. Therefore, the demands dictated by the circumstances under which a system operates may be effectively controlled and defined by the implementation of limitations. However, the implementation of the MPC controller faces obstacles such as significant computing burden and power consumption, which are further compounded by the resource limits inherent in embedded system applications [14 -16].

The article is divided into four primary sections. Section 1 outlines the reasons for and importance of researching how to improve the efficiency of self-driving cars. Part 2 develops a kinematic and lateral dynamic model of a self-propelled vehicle with a front axle consisting of two wheels and two rear wheels. This section introduces the design of predictive navigation for autonomous cars, which is based on the lateral dynamics model of the vehicle. Part 4 demonstrates the efficacy of Model Predictive Control (MPC) 's efficacy in autonomously regulating autos' position and steering angle via experimentation. Finally, findings and suggestions for future research aim to enhance the constraints.

Furthermore, Section 3 delves into integrating sensor fusion techniques to enhance perception capabilities for self-driving vehicles. Various sensor modalities such as LiDAR, radar, and cameras are combined to create a robust and comprehensive understanding of the vehicle's surroundings. Fusing data from these sensors enables the autonomous car to make informed decisions in real time, ensuring safe and efficient navigation through complex environments. This section also discusses the challenges associated with sensor fusion, including sensor calibration, data synchronization, and sensor redundancy to ensure the reliability and accuracy of the perception system in autonomous driving scenarios.

2. Lateral Dynamics Model of Autonomous Vehicle

The lateral dynamics model of the autonomous vehicle is shown in Figure 1. Figure 1 depicts the dynamic model of a car's motion with an axle, illustrating the primary forces affecting the vehicle. We consider the oxygen coordinate system, representing the vertical and horizontal directions within the vehicle frame. In contrast, the OXY coordinate system denotes the vertical and horizontal directions in the absolute reference system. Here, ψ signifies the rotation angle of the vehicle body in the OXY reference system.

By applying Newton's Law principle, the differential equations governing the car's motion in Figure 1 can be derived as follows:

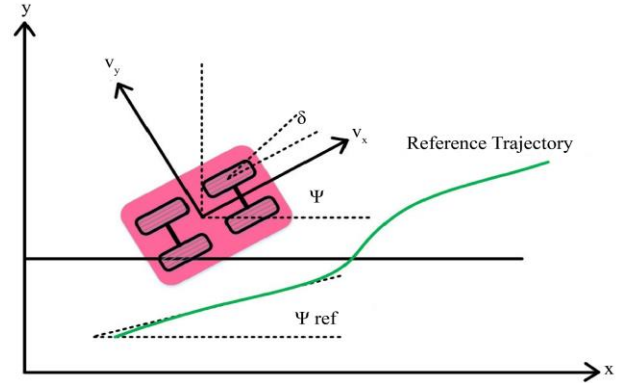


Fig. 1 The lateral dynamics model of the autonomous vehicle

$$\begin{cases} m(\dot{y} + V_x \dot{\psi}_y) = F_{yf} + F_{yr} \\ I_r \ddot{\psi} = I_f F_{xf} - I_f F_{xr} \end{cases} \quad (1)$$

Where:

m and I_r are the vehicle mass and moment of inertia, respectively, and I_r represent the mass and moment of inertia of the vehicle, respectively.

F_{yf}, F_{yr} are the forces acting on the wheels in the x and y directions, respectively.

Empirical evidence suggests that the sideways force exerted by a tire is precisely proportional to the angle at which it slips (for modest slip angles). This relationship is known as the "cornering stiffness" of the tire, and it plays a crucial role in the handling and stability of a vehicle during cornering maneuvers. Manufacturers carefully design tires to optimize this cornering stiffness, balancing factors such as grip, wear, and rolling resistance to achieve the desired performance characteristics. By understanding and manipulating this fundamental property, engineers can fine-tune a vehicle's handling dynamics to provide the best possible combination of grip and control. The slip angle of the tyre is written as Equation 2:

$$a_f = \delta - \theta_{vf} \quad (2)$$

Where: δ is the front tyre steering angle.

The forces acting on the wheels in y directions, respectively, for the rear and front tire, are calculated in Equation 3.

$$\begin{cases} F_{yf} = 2C_{af}(\delta - \theta_{vf}) \\ F_{yr} = 2C_{ar}(-\theta_{vr}) \end{cases} \quad (3)$$

Where C_{af}, C_{ar} are cornering stiffness.

And

$$\begin{cases} \tan \theta_{vf} = \frac{v_y + l_f \dot{\psi}}{v_x} \\ \tan \theta_{vr} = \frac{v_y - l_r \dot{\psi}}{v_x} \end{cases} \quad (4)$$

If θ_{vf} & θ_{vr} are small. The θ_{vf} & θ_{vr} are cacuted by Equation 5.

$$\begin{cases} \theta_{vf} = \frac{y+l_f\dot{\psi}}{V_x} \\ \theta_{vr} = \frac{y-l_r\dot{\psi}}{V_x} \end{cases} \quad (5)$$

The forces exerted on the rear and front tires in the vertical direction are computed. Equation 6.

$$\begin{cases} F_{yf} = 2C_{af}(\delta - \frac{y+l_f\dot{\psi}}{V_x}) \\ F_{yr} = 2C_{ar}(-\frac{y-l_r\dot{\psi}}{V_x}) \end{cases} \quad (6)$$

The dynamic model of the autonomous vehicle is rewritten as follows: Equations 7 & 8.

$$\ddot{y} + V_x\dot{\psi} = \frac{2C_{af}\delta}{m} - \frac{2C_{af}(y+l_f\dot{\psi})}{mV_x} - \frac{2C_{ar}(\frac{y-l_r\dot{\psi}}{V_x})}{mV_x} \quad (7)$$

$$\ddot{y} = \frac{l_f}{l_r}(2C_{af}\delta - \frac{2C_{af}(y+l_f\dot{\psi})}{V_x}) + \frac{l_f}{l_r}\frac{2C_{ar}\delta(y+l_r\dot{\psi})}{V_x} \quad (8)$$

Equations (7) and (8) are rewritten as Equations (9) and (10):

$$\ddot{y} = \frac{2C_{af}\delta}{m} - \frac{2(C_{af}+C_{ar})}{mV_x}\dot{y} - (V_x + \frac{2(C_{af}l_f-C_{ar}l_r)}{mV_x})\dot{\psi} \quad (9)$$

$$\ddot{y} = \frac{l_f 2C_{af}\delta}{l_r} - \frac{2(C_{af}l_f-C_{ar}l_r)}{l_r V_x}\dot{y} - \frac{2(C_{af}l_f^2-C_{ar}l_r^2)}{l_r V_x}\dot{\psi} \quad (10)$$

The dynamic state-space model of the autonomous vehicle is rewritten as follows Equation 11.

$$\frac{d}{dt} \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{2(C_{af}+C_{ar})}{mV_x} & 0 & -\frac{2(C_{af}l_f-C_{ar}l_r)}{mV_x} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2(C_{af}l_f-C_{ar}l_r)}{l_r V_x} & 0 & -\frac{2(C_{af}l_f^2+C_{ar}l_r^2)}{l_r V_x} \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2C_{af}\delta}{m} \\ 0 \\ \frac{2l_f C_{af}\delta}{l_r} \end{bmatrix} \delta \quad (11)$$

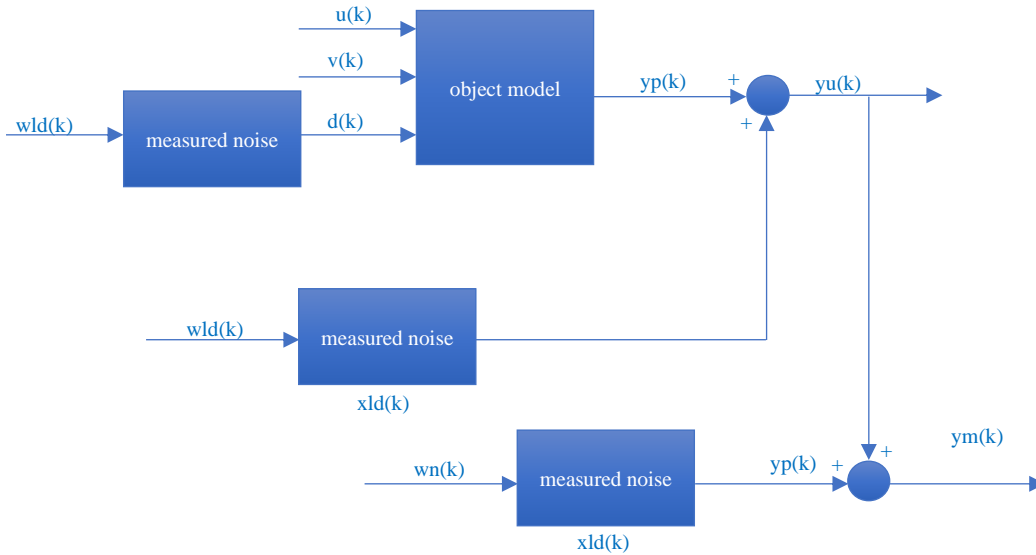


Fig. 2 The MPC controller architecture [1]

3. Designing a Model Predictive Navigation Controller

Figure 2 illustrates the model structure used in the MPC controller. The model prediction controller calculates the optimal control input by minimizing a cost function that penalizes deviation from the desired state trajectory. Afterwards, the expected state is used to dynamically adjust the control input, allowing the controller to track the desired trajectory accurately.

The car state model is written as the Equation (12):

$$\begin{aligned} x_p(k+1) &= A_p x_p(k) + B S_i u_p(k) \\ y_p(k) &= S_0^{-1} C x_p(k) + S_0^{-1} D S_i u_p(k) \end{aligned} \quad (12)$$

Where:

x_p, y_p is the input and output variable of the object.

A_p, B, C are state space matrices with constant zero delay

S_i is the input diagonal matrix.

S_0 is the output diagonal matrix.

The diagonal matrix of output scale factors x_p is the state vector that includes all delay states.

u_p is a vector of input variables consisting of manipulated variables, measured noise, and unmeasured input noise.

y_p is a vector of output variables.

State model Equation (8) does not include input and output noise. So, the car state model is rewritten as the Eqs. (13) & (14):

$$x_p(k+1) = A_p x_p(k) + B_{pu}(k) + B_{pv}(k) + B_{pd}(k) \quad (13)$$

$$y_p(k) = C_p x_p(k) + D_{pu}(k) + D_{pv}(k) + D_{pd}(k) \quad (14)$$

Where

$C_p = S_0^{-1} C, B_{pu}, B_{pv}, B_{pd}$ is a parameter of $B S_i$.

D_{pu}, D_{pv}, D_{pd} is a parameter of $S_0^{-1} D S_i$; $(k), v(k), d(k)$ are the measured and unmeasured input noises.

The MPC controller is limited, so $D_{pu} = 0$ means that the MPC controller does not allow direct transmission from any controlled variable to any output of the control object. Matrix A , B , C and D are determined as follows.

$$A = \begin{bmatrix} A_p & B_{pd}C_{id} & 0 & 0 \\ 0 & A_{id} & 0 & 0 \\ 0 & 0 & A_{od} & 0 \\ 0 & 0 & 0 & A_n \end{bmatrix}$$

$$B = \begin{bmatrix} B_{pu} & B_{pv} & B_{pd}D_{id} & 0 & 0 \\ 0 & 0 & B_{id} & 0 & 0 \\ 0 & 0 & 0 & B_{od} & 0 \\ 0 & 0 & 0 & 0 & B_n \end{bmatrix}$$

$$C = [C_p \quad D_{pd}C_{id} \quad C_{od}] \begin{bmatrix} C_n \\ 0 \end{bmatrix}$$

$$D = 0 \quad D_{pv} \quad D_{pd}D_{id} \quad D_{od} \begin{bmatrix} D_n \\ 0 \end{bmatrix}$$

4. Results of Simulation and Assessment

4.1. Build a Reference Trajectory for Vehicle Position and Autonomous Driving Angle

Within the research scope of the topic, students establish a reference trajectory for the position and angle of autonomous driving using the Driving Scenario Design tool from the Automated Driving Toolbox of MATLAB software. Build a reference trajectory for the vehicle position and autonomous driving angle as the code program below. Build a road with a width of 6m, 2 lanes, using a self-driving ego car.

4.2. Simulation Parameters

The MPC and adaptive MPC predictive control models simulate the following parameters:

4.3. Simulation Scenario

The MATLAB simulation structure of MPC navigation used for autonomous cars is shown in Figure 3.

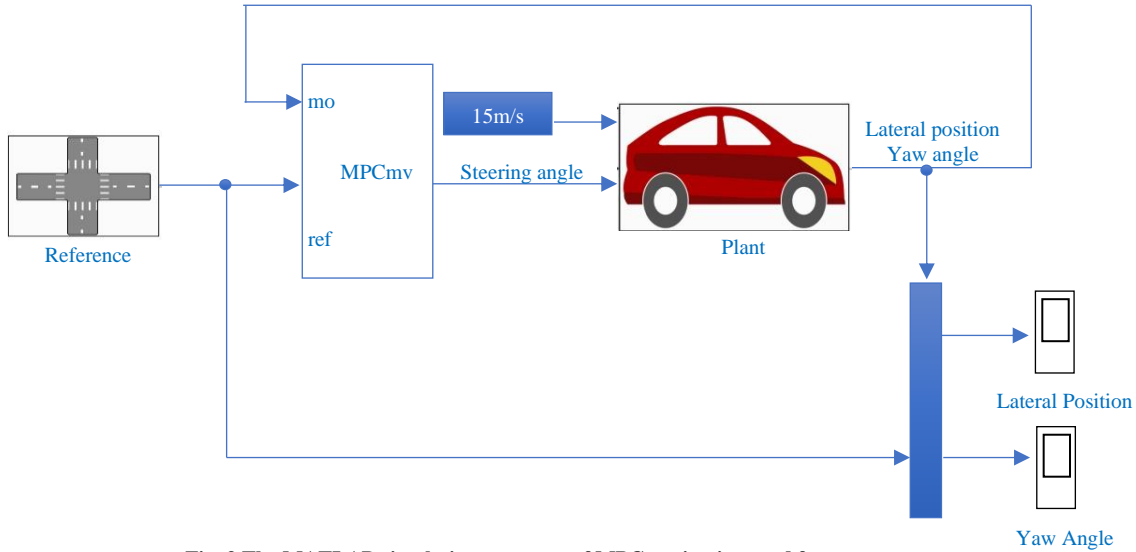


Fig. 3 The MATLAB simulation structure of MPC navigation used for autonomous cars

Table 1. Autonomous vehicle parameters

Parameter	Value
m is the vehicle mass (kg).	1575
J is the moment of inertia of the self-driving car (kgm ²).	2875
L_f : vertical distance from center of gravity to the front wheel (m).	1.2
L_r is the vertical distance from the center of gravity to the rear wheel (m).	1.6
C_f is the hardness when entering the front wheel (N/rad).	19000
C_r is the cornering stiffness of the rear wheel (N/rad).	33000

Table 2. MPC controller parameters

Parameter	Value
Sample extraction time T_s	0.1 (s)
Forecast (P)	15 (s)
Predictive control	4 (s)
Constraints for autonomous vehicles	
Fixed steering angle	(-0.5 to 0.5) rad

Steering angle during long speed changes.	(-0.25 to 0.25) rad
Self-driving car location.	[-2 6]
Steering angle.	[-0.2 0.2]
Input and output signals of the MPC controller	
Steering angle during long speed changes	(-0.1 to 0.1) rad
Self-driving car location	1

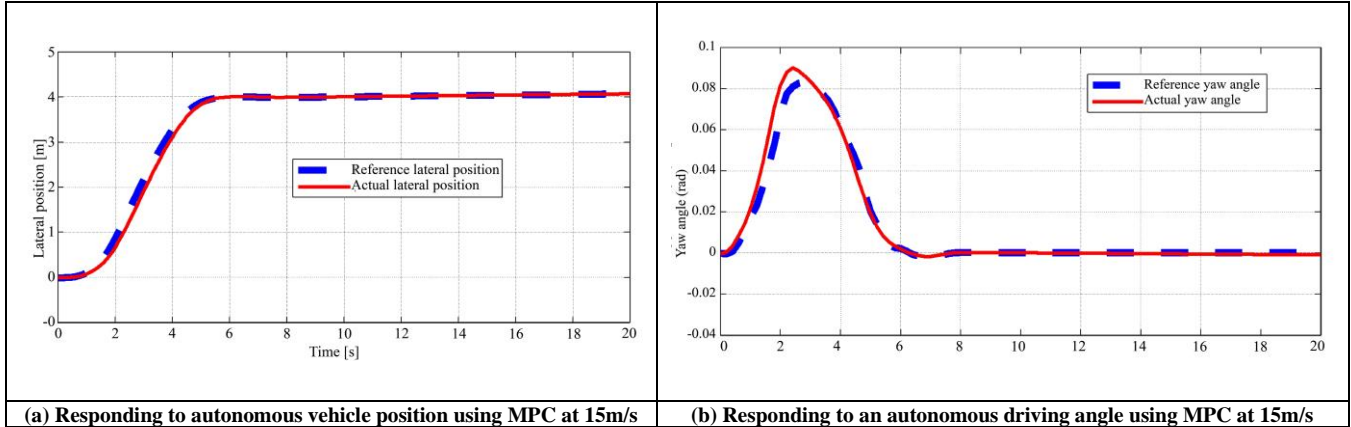


Fig. 4 Response to position and angle of autonomous driving using MPC 15m/s

Case 1: let the self-propelled vehicle move at a long speed of 15m/s. The position and tilt angle response of the autonomous vehicle using MPC control is shown in Figure 4. Figure 4 shows that the MPC controller’s performance at a long speed of 15m/s is constant. The self-propelled vehicle quickly follows the set trajectory with small deviation angle errors. Thus, the results obtained in Figure 4 show that the MPC controller achieves satisfactory performance under constant operating conditions when the autonomous vehicle moves at a long speed of 15m/s. In summary, the consistent performance of the MPC controller at a speed of 15m/s demonstrates its effectiveness in maintaining trajectory tracking with minimal errors. This stability and precision are

crucial for the reliable operation of autonomous vehicles, especially in scenarios where maintaining a specific speed is essential for optimal performance.

The results depicted in Figure 4 underscore the capability of the MPC controller to handle varying conditions and deliver reliable performance, making it a valuable component in the development of autonomous driving systems.

Case 2: let the self-propelled vehicle move at a long speed of 35m/s.

The autonomous driving position and angle response using MPC control are shown in Figure 5.

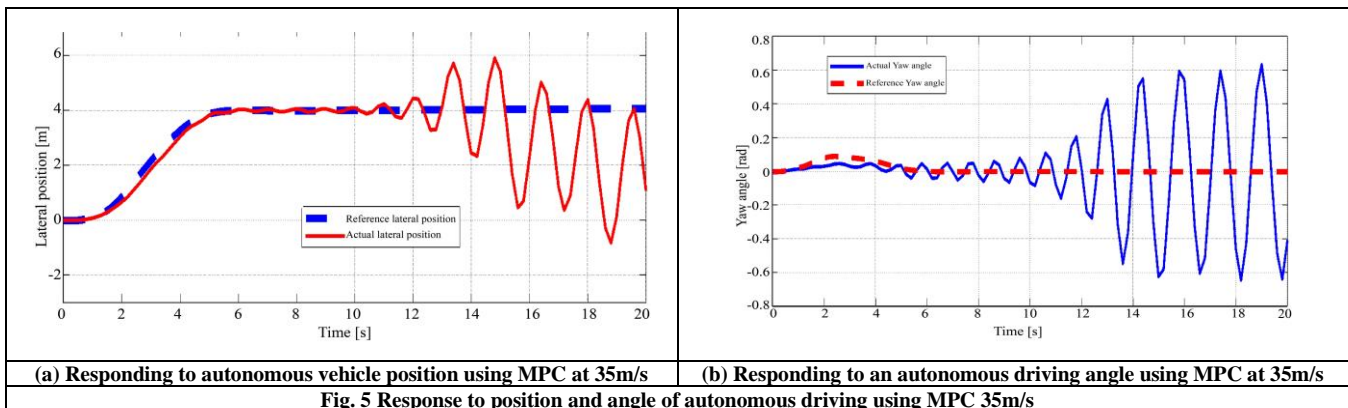


Fig. 5 Response to position and angle of autonomous driving using MPC 35m/s

The simulation results in Figure 5 indicate a significant change in the performance of the MPC controller when the vehicle accelerates to 35m/s from 0s to 10s. During this acceleration phase, the autonomous vehicle exhibits unstable movement, with a large error (60%) between the actual path

and the set value. The autonomous driving angle deviates from the required trajectory between 2s and 10s, highlighting an inability of the MPC controller to adjust to evolving system dynamics. The findings in Figure 5 suggest that traditional MPC controllers are ineffective in managing diverse dynamics

due to their reliance on a linear, parameter-invariant model. To address this limitation, researchers have proposed various advanced MPC techniques that incorporate more sophisticated models capable of capturing non-linear and time-varying system behaviors. By utilizing these advanced MPC strategies, such as nonlinear MPC or adaptive MPC, it may be possible to enhance the controller's ability to handle complex and changing dynamics effectively. These approaches offer the potential to improve the performance and robustness of autonomous vehicle control systems, enabling more precise trajectory tracking and stability across a wide range of operating conditions. Further research and development in this area could lead to significant advancements in autonomous driving technology, making it more reliable and adaptable in real-world scenarios.

5. Conclusion

The simulation results demonstrate that using the MPC controller for autonomous vehicle navigation is only suitable at low and medium speeds. At this speed range, the lateral dynamic model of the autonomous car does not change. However, there needs to be a solution to improve the MPC

controller so that the autonomous vehicle's navigator adjusts the vehicle's position and steering angle according to the set value. Therefore, an adaptive MPC control solution will be proposed for the upcoming research of this scientific work. Adaptive MPC control solution hopes to bring high performance to the control system and limit the disadvantages of MPC controllers.

The adaptive MPC control solution aims to enhance the autonomous vehicle's navigation capabilities by continuously adjusting the control parameters based on real-time feedback and varying operating conditions. By incorporating adaptability into the MPC framework, the system will be able to handle unpredictable scenarios and changes in the vehicle's dynamics more effectively. This adaptive approach is expected to improve the overall performance and robustness of the autonomous navigation system, making it suitable for a wider range of speeds and driving conditions.

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References

- [1] Adile Akpunar, and Serdar Iplikci, "Runge-Kutta Model Predictive Speed Control for Permanent Magnet Synchronous Motors," *Energies*, vol. 13, no. 5, pp. 1-17, 2020. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [2] Hussain Bassi, and Youssef Mobarak, "State-Space Modeling and Performance Analysis of Variable-Speed Wind Turbine Based on a Model Predictive Control Approach," *Engineering, Technology & Applied Science Research*, vol. 7, no. 2, pp. 1436-1443, 2017. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [3] Do Trong Tu, "Enhancing Road Holding and Vehicle Comfort for an Active Suspension System Utilizing Model Predictive Control and Deep Learning," *Engineering, Technology & Applied Science Research*, vol. 14, no. 1, pp. 12931-12936, 2024. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [4] Patrick Weyers, Alexander Barth, and Anton Kummert, "Driver State Monitoring with Hierarchical Classification," *2018 21st International Conference on Intelligent Transportation Systems (ITSC)*, Maui, HI, USA, pp. 3239-3244, 2018. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [5] Vo Thanh Ha, "Torque Control of an In-Wheel Axial Flux Permanent Magnet Synchronous Motor using a Fuzzy Logic Controller for Electric Vehicles," *Engineering, Technology & Applied Science Research*, vol. 13, no. 2, pp. 10357-10362, 2023. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [6] Andrea Minari, Aurelio Piazza, and Alessandro Costalunga, "Polynomial Interpolation for Inversion-Based Control," *European Journal of Control*, vol. 56, pp. 62-72, 2020. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [7] Abdeslam Boularias et al., "Learning Qualitative Spatial Relations for Robotic Navigation," *Proceedings of 25th International Joint Conference on Artificial Intelligence (IJCAI-16)*, New York, USA, pp. 4130-4134, 2016. [[Google Scholar](#)] [[Publisher Link](#)]
- [8] Ahmed Abdelmoniem et al., "A Path-Tracking Algorithm Using Predictive Stanley Lateral Controller," *International Journal of Advanced Robotic Systems*, vol. 17, no. 6, pp. 1-11, 2020. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [9] Akinobu Goto, Takashi Fukushige, and Takeshi Kimura, "Real-Time Trajectory Planning for Autonomous Driving in Urban Area Based on Dynamic Programming," *Society of Automotive Engineers of Japan*, vol. 52, no. 3, pp. 639-644, 2021. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [10] Noor Hafizah Amer et al., "Modelling and Control Strategies in Path Tracking Control for Autonomous Ground Vehicles: A Review of State of the Art and Challenges," *Journal of Intelligent & Robotic Systems*, vol. 86, pp. 225-254, 2017. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [11] Anouer Bennajeh et al., "Bi-Level Decision-Making Modeling for an Autonomous Driver Agent: Application in the Car-Following Driving Behavior," *Applied Artificial Intelligence*, vol. 33, no. 13, pp. 1157-1178, 2019. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]

- [12] Bandana Barman, Ria Kanjilal, and Anirban Mukhopadhyay, “Neuro-Fuzzy Controller Design to Navigate Unmanned Vehicle with Construction of Traffic Rules to Avoid Obstacles,” *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, vol. 24, no. 3, pp. 433-449, 2016. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [13] Pedro Bautista-Camino et al., “Local Path Planning for Autonomous Vehicles Based on the Natural Behavior of the Biological Action-Perception Motion,” *Energies*, vol. 15, no. 5, pp. 1-23, 2022. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [14] Boliang Yi et al., “Real Time Integrated Vehicle Dynamics Control and Trajectory Planning with MPC for Critical Maneuvers,” *2016 IEEE Intelligent Vehicles Symposium (IV)*, Gothenburg, Sweden, pp. 584-589, 2016. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [15] Yong Chen et al., “Accurate and Efficient Approximation of Clothoids Using Bézier Curves for Path Planning,” *IEEE Transactions on Robotics*, vol. 33, no. 5, pp. 1242-1247, 2017. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [16] Enrique Herrera-Viedma et al., “Special Issue on Intelligent Decision Support Systems Based on Soft Computing and Their Applications in Real-World Problems,” *Applied Soft Computing*, vol. 67, pp. 610-612, 2018. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]