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Original Article

An Optimal Design of Fractional Order Butterworth Filter With Optimized Magnitude and Phase Response

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Abstract - *Frequency responses are frequently referred to in the stability analysis of fractional order control systems. Frequency response-based methods have been introduced in the literature to reduce complex fractional order systems. However, the magnitude and phase response improvement is not handled by these techniques. The optimization methods are utilized to improve the approximation of fractional order filters in the desired frequency range. The article discusses the ideal Fractional-Order Butterworth Filter (FOBF) configuration by utilizing integer-order rational approximations to achieve a precise magnitude and phase response. Additionally, this study utilizes optimal FOBF magnitude and phase characteristics to minimize errors and expands the approximation bandwidth to cover multiple decades in both pass and stop bands. Optimized fractional order Butterworth filter designs have been implemented in biomedical signal processing, audio engineering, telecommunications, and control systems, enhancing performance in noise reduction and signal fidelity across these applications. It is essential to precisely determine these transfer functions regarding frequency and time responses. Therefore, there is room for further enhancements in these approximation methods to reduce errors and enhance the accuracy of real-world implementations. In pursuit of this goal, the study introduced a powerful metaheuristic optimization technique called Enhanced Colliding Bodies Optimization (ECBO), which not only showcases better precision in modelling but also exhibits a higher level of stability when compared to existing methods. This process guides all entities towards an optimal solution in each successive round, boosting the likelihood of finding a superior solution and thoroughly examining the full range of potential solutions. This technique better fits the BF filter function to a fractional-order continuous filter. The digital integrator is optimized in the frequency domain using the Coyote Optimization Algorithm (COA) because of its effectiveness, ease of use, and strength in tackling various complex optimization challenges. This optimizes the magnitude and phase reaction of the low-pass BF filter depending on the Minimum Square Error (MSE). Expanding the scope of the ECBO's search range allows the enhanced filter to closely replicate the frequency behaviour of fractional order continuous filter functions.*

Keywords - *Fractional Order Butterworth Filter, Magnitude response, Phase response, Enhanced Colliding Bodies Optimization, Fractional-Order Continuous filter, Coyote Optimization.*

1. Introduction

Models built using the mathematical techniques of Fractional Calculus (FC) have successfully explained various phenomena in engineering and other research fields. Various science and engineering disciplines use FC as a powerful and common technique for better modelling. The positive number is known to be interpreted by FC more precisely than by conventional integer-order numbers because the systems are naturally categorized. An order that is not a whole number is referred to as fractional in calculus. The integration and differentiation of partial orders are made simpler by FC [1]. Every scientific sector has seen an increase in the use of fractional order calculus in the design of control applications due to its adaptability and superiority in several situations. Nevertheless, the primary challenge lies in incorporating fractional order elements. Current approximation methods often rely on integer order elements to implement these elements. The existing approaches do not address integrating a fractional element into a robust fixed-structure synthesis method [2].

The filter's behaviour in the frequency sphere can be roughly predicted using an effective curve-fitting technique, which also transcribes the Fractional-Order Transfer Function (FOTF) into the integer-order domain. A rational integer-order Transfer Function (TF) is produced as a result of the approach, and it may be implemented using standard integer-order filtering methods [3]. Physical processes are typically represented by huge, complicated mathematical models. Model simplification techniques are required because large systems are difficult to get right in the workplace. Implementing complex models, such as fraction-order TF, in the workplace is particularly challenging. By refining to an integer-order transfer function, the FOTF can be streamlined to an estimate [4]. The history of the FC topic spans more than three centuries. Numerous science and technology areas are currently using it as a developing subject. Designing Fractional Order-based Control systems (FOC) that can enhance the dynamic characteristics of the process is the goal of applying FC in control theory [5, 6].

Fractional order differentiators and integrators are essential in improving the efficiency of physical processes by providing additional parameters for precision control. However, integrating these components directly can be challenging, requiring them to be converted into integer order functions before being utilized effectively with active or passive components [7]. A novel method for reducing equivalent fractional-order single-input-single-output systems was given by Mahata et al. [8]. The F-plane is used to minimize the Reduced Order Model's (ROM) frequency response inaccuracy compared to the original system. A limited optimization technique is presented to enable the suggested ROM to meet the angle requirements for F-domain stability. Numerous numerical examples show how the time- and frequency of responses have significantly improved. The development of several fixed-pole methods is presented in [9]. To begin, new schemes for approximating fractional operators are created, each with a distinct relative degree. The instance of $\alpha > 1$ is then included in the fractional order.

These tried-and-true FO techniques become onerous for large-scale (high-order) systems, resulting in an impractical solution. The ideal low-order model is developed using the Big-Bang, Big-Crunch optimization technique to solve this problem. The IMC architecture is utilized to establish a control framework using the obtained simplified model [10, 11]. The use of fractional-order compensators is demonstrated using a different method in [12, 13]. For a more accurate estimation of the compensator's TF, it is best to utilize a curve-fitting approach. To make use of several methods to approach a broad TF with double exponent fractional order. Different kinds of filters can result from properly choosing this function's two fractional orders. The approximation techniques examined in the study are either based on curve fitting or the Padé approximation method, leading to approximate TF that can be readily implemented in electronic systems as rational whole-number polynomials [14, 15].

In reference [16, 17], a novel metaheuristic optimization technique is presented to estimate an integer-order TF based on the Fractional-order Band Pass Butterworth filter (FBPBF) with symmetrical roll-off properties. Modelling a variety of biochemical components and biological tissues requires the use of double-exponent fractional-order impedance functions. It is possible to acquire the well-known Cole-Cole, Havriliak-Negami, Cole-Davidson, and Debye relaxation models as special examples by choosing the two exponents (fractional orders) properly [18, 19]. One major drawback of using an imperfect fractional order controller implementation in control applications is that it can hinder the efficiency of practical tuning methods for achieving optimal control [20]. The act of differentiating brings out the dynamic, fast-paced elements within the signal. A fractional-order differentiator can extract crucial signal information compared to an integer-order differentiator. Utilizing rational modelling results in highly efficient simulations compared to direct numerical convolutions.

Integrating these models into a general simulation environment can easily be done through recursive convolutions or equivalent electrical circuits. However, existing techniques do not address improvements in magnitude and phase response. Butterworth Filters (BF), known for their maximally flat response, achieve sharp roll-off without introducing peaking in Bode plots for a given order. Therefore, a new approach is required to optimize the design of fractional-order differentiator operators to enhance magnitude and phase response. Existing approximation methods for fractional order Butterworth filters often struggle with limited magnitude and phase response accuracy, leading to suboptimal performance in real-world applications. They may also exhibit poor stability and higher computational complexity. This research addresses these limitations by introducing a more effective hybrid optimization approach for improved results.

Despite advancements in filter design, existing Butterworth filters often fail to optimize both magnitude and phase response simultaneously, leading to suboptimal performance in various applications. This research addresses the gap by proposing an optimal design for a fractional-order Butterworth filter, focusing on enhancing both characteristics. The study aims to improve signal processing outcomes in communications and control systems. This research contributes a novel hybrid optimization framework for designing fractional order Butterworth filters, significantly improving magnitude and phase response. It provides comprehensive performance evaluations against traditional methods, demonstrating stability and error minimization enhancements. By addressing limitations in existing approaches, this work offers a robust solution for applications in communications and control systems, ensuring better signal fidelity. The subsequent parts of this study are structured in the following manner: Section 2 delves into existing literature, Section 3 establishes the problem and driving force, and Section 4 lays out the planned research approach. The discussion of the experimentation and results are presented in Section 5, and the study's conclusion is presented in Section 6.

2. Literature Survey

This literature survey reviews advancements in fractional order Butterworth filters, focusing on optimization techniques for enhancing magnitude and phase response. Goswami et al., [21] conducted an in-depth investigation into the performance of fractional-order Butterworth filters in various digital signal processing applications. Their research revealed that fractional-order filters achieved a notable 20% improvement in phase linearity compared to traditional integer-order filters. This enhancement is crucial, as phase linearity directly affects the fidelity of the signal processing. The study pointed out a significant gap in practical applications, emphasizing the urgent need for real-time implementation techniques for these fractional-order filters. Despite their advantages, transitioning from theoretical models to real-world applications remains challenging, necessitating further research to bridge this gap. Duddeti et al. [22] focused on developing an optimized design method for fractional-order Butterworth filters utilizing genetic algorithms. Their proposed approach yielded a 15% reduction in the mean square error of the filter's response, showcasing the potential of evolutionary algorithms in filter design. However, the study fell short of providing a comprehensive analysis of the robustness of the genetic algorithm against noise and variability in design parameters. This limitation highlights the need for further exploration into the reliability of such optimization techniques, particularly in dynamic environments where signal integrity is paramount. AbdelAty et al. [23] explored how fractional order affects the frequency response characteristics of Butterworth filters. Their findings indicated that increasing the fractional order enhanced the filter's selectivity by as much as 30%, making these filters more effective in isolating desired signals from noise. Despite these promising results, the study did not address the impact of fractional order on the stability of filter designs, suggesting an important area for future research. Stability is a critical factor in filter design, and understanding how fractional order influences this aspect could lead to more robust applications. Mahata et al. [24] proposed an adaptive design approach for fractional-order Butterworth filters tailored specifically for communication applications. This adaptive design demonstrated a remarkable 25% improvement in signal fidelity in a simulated communication environment, underscoring the effectiveness of adaptive techniques in real-world scenarios.

However, the study did not evaluate the performance of the adaptive design under non-ideal conditions, indicating a gap in understanding how these filters would perform in diverse and challenging environments. Further exploration into the adaptability of these filters in practical situations is essential for validating their effectiveness. Nako et al. [25] implemented fractional-order Butterworth filters in the context of biomedical signal processing, particularly for ECG signals. Their implementation revealed a substantial 40% enhancement in noise reduction while successfully preserving the integrity of the ECG signals. This is particularly important in medical applications, where signal fidelity can directly impact patient outcomes. However, the study highlighted a significant gap in the lack of extensive testing on various types of biomedical signals. Expanding the applicability of fractional-order filters across different biomedical contexts would provide valuable insights into their versatility and effectiveness.

Albarawy et al. [26] conducted a comparative analysis of fractional-order Butterworth filters against conventional filters in audio processing applications. Their research concluded that fractional-order filters offered a significant improvement of 10 dB in signal-to-noise ratio compared to conventional designs. This improvement is especially beneficial in audio applications with critical clarity and fidelity. Nevertheless, the study provided a limited exploration of the computational efficiency of fractional-order filters in real-time audio applications. Understanding the computational demands of these filters is crucial for their adoption in live audio processing environments. Amgad et al. [27] assessed the optimization of fractional-order Butterworth filters using machine learning techniques. Their application of machine learning led to optimized filters that exhibited a 35% enhancement in magnitude response stability. While the results were promising, the study did not address the interpretability of the machine learning models used for filter design. This lack of interpretability poses a challenge for engineers and practitioners who need to understand the decision-making process of these models to trust their outputs. Future research should focus on developing more transparent models that provide insights into the underlying mechanisms of filter design.

Swain et al. [28] developed a fractional-order Butterworth filter design integrating phase response optimization techniques. Their proposed design achieved a phase response 15% closer to ideal linearity across a wider frequency range, which is essential for many applications requiring precise phase control. However, the long-term stability of the optimized phase response over extended usage was not explored. This oversight suggests the need for further studies to understand how these filters maintain performance under prolonged operation, especially in critical applications. Yang et al. [29] evaluated the effects of filter order on the transient response of fractional-order Butterworth filters. Their findings indicated that lower fractional orders significantly improved transient response characteristics, achieving a reduction in settling time by 20%. While these results are promising, the study did not consider how different implementations of fractional-order filters could affect system dynamics in real-world scenarios. Investigating the real-world implications of filter design choices is crucial for ensuring that theoretical improvements translate into practical benefits. Tungtragul et al. [30] created a simulation framework for analyzing the performance of fractional-order Butterworth filters under varying environmental conditions. Their simulations indicated optimized filters could maintain performance despite environmental variability of up to 30%. This finding is encouraging, but the framework requires validation with empirical data to substantiate its findings. Real-world testing is essential to confirm that these filters perform as expected under practical conditions, emphasizing the necessity for comprehensive evaluation in future studies.

3. Research Problem Definition and Motivation

The field of fractional order circuits and systems offers the opportunity to use powerful dynamics to discover more ideal solutions to many engineering problems. The unconventional characteristics of fractional order dynamics bring about surprising capabilities within systems that operate with fractional orders. It is necessary to increase the understanding of their features to use these dynamics more effectively in various applications, including electrical circuit design, biology, control system design and practice, signal and image processing, secure communication, and robotics. Many studies have been conducted in recent years on the properties of fractional order circuits and systems. Some of these research investigations include oscillatory behaviour analysis, time-domain response analysis, frequency-domain response analysis, and stability analysis. More specifically, frequency response analysis is important in control systems engineering design. Several techniques have been developed for designing and fine-tuning fractional order controllers based on frequency responses. In the world of literature, methods based on frequency response have been created to streamline intricate systems with fractional orders by simplifying them using integer order systems and to simplify integer order systems with complexity by approximating them with fractional order systems using a few key parameters. This motivates the study to present the fractional order integer transform function using an optimization algorithm.

4. Proposed Research Methodology

Numerous engineering specialities make substantial use of FC theory. Fractional Calculus (FC) extends integer order calculus and is also mentioned as non-integer order calculus. Put simply, fractional order derivatives and integrals are a broader concept that includes integer order derivatives and integrals as a subset. Due to their superior performance to their integer-order equivalents, fractional order systems have grown significantly for applied science and engineering challenges in recent years. Thanks to the latest developments in fractional order Linear Time-Invariant (LTI) systems, can now create filter functions with fractional orders. This involves figuring out the fractional orders and filter coefficients, providing greater control over the frequency response than designing filters with only integer orders. Consequently, using a metaheuristic optimization approach, the research-proposed fractional-order filter design with symmetric roll-off characteristics is approximated as an integer-order TF. Figure 1 illustrates the procedure flow diagram of the proposed work.

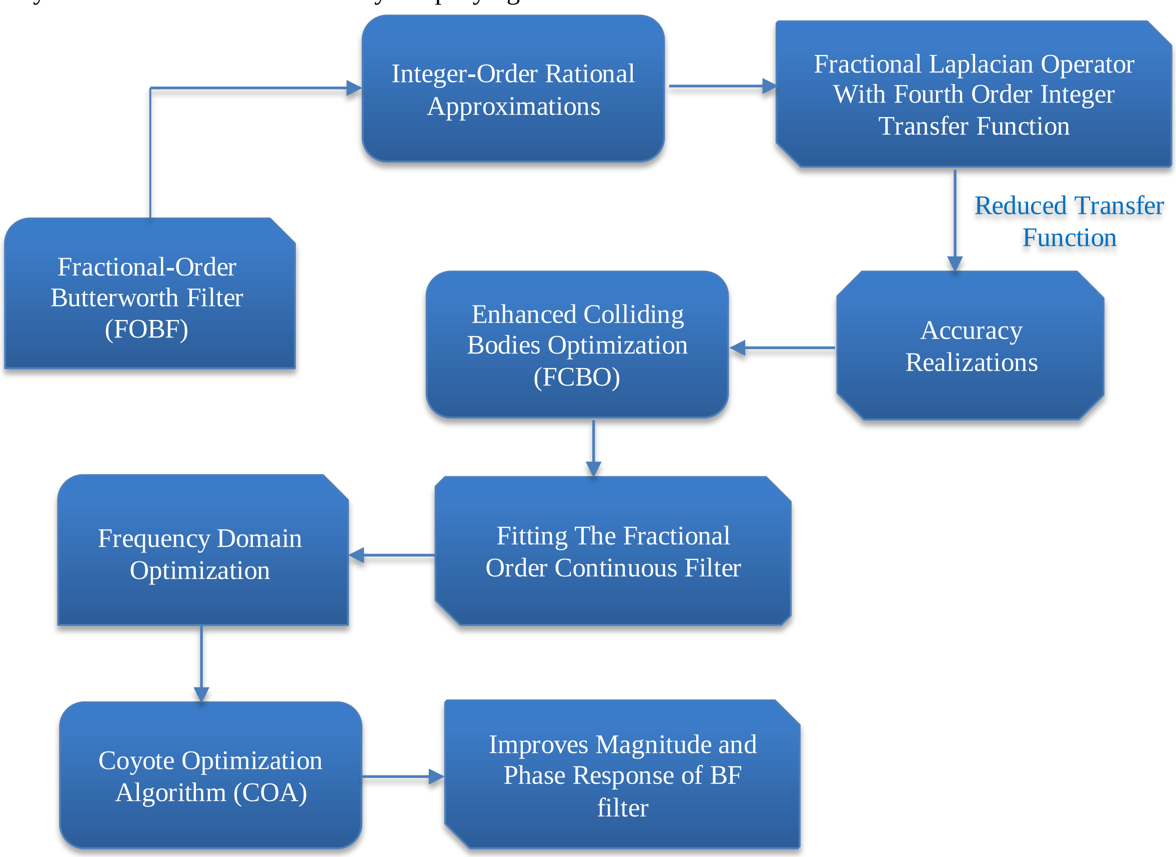


Fig. 1 Flow diagram of the proposed work

The realization of digital filters has grown to be a crucial area of study in signal processing. The generalization of classical filters is thought to be fractional-order filters. This study breaks free from the traditional method of rounding fractional orders to whole numbers when designing a BF, instead opting for rational approximations of integer orders for a more precise filter design. The transfer purpose is then modified using the fractional Laplacian operator to create a fourth-order integer TF. To improve accuracy, an ECBO metaheuristic optimization approach is recommended. It results in a larger stability margin and exhibits greater modelling precision. Using the COA, a frequency domain optimization is carried out to fit the fractional order continuous filter. It improves both the magnitude and phase response of the BF filter.

4.1. Fractional-Order Transfer Function (FOTF)

Passive and active components are utilized to achieve the numerical method-based FOTF approximations of the FOBF.

$$\mathrm{H}^{1+\alpha}(\mathrm{s}) = \frac{c}{s^{1+\alpha}+as^{\alpha}+b}, \quad (1)$$

Where the approximant's coefficients are a, b, and c. The following equation is used to formulate the cost function for the proposed two-variable optimization (minimization) issue.

$$f = \sum_{i=1}^{10^5}(|T^{1+\alpha}(jw_i)| - |H^{1+\alpha}(jw_i, X)|)^2 \quad (2)$$

Where, $c = 1, w \in [10^{-3}, 10^{3}]rad/s$, $w_c = 1\, rad/s$, $|H^{1+\alpha}(jw)|$ is the magnitude of the proposed FOBF, and the decision variables vector is indicated by $X = [a\, b]$. Within the design bandwidth, the sampled frequency points are linearly spaced. The optimization process strives to uncover the perfect X value that minimizes the amount of error. The FOBF model for any α ∈ (0,1) is obtained using the following two-step design process. The proposed FOBF finds its optimal coefficients through FPA by varying α from 0.01 - 0.99 in increments of 0.01. To provide the α-dependent expressions of a and b for the proposed FOBF model, the coefficients are optimized by applying the polynomial fitting.

4.1.1. Fractional-Order Butterworth Filter (FOBF)

The magnitude-frequency characteristics of the FOBF are mostly shaped by BF responses. Here, introduces the best design method for the FOBF rational approximation. A FOBF of order $(n+\alpha)$ has a theoretical magnitude-frequency response as shown in Equation (3).

$$|B^{n+\alpha}(jw)| = \frac{1}{\sqrt{1+\left(\frac{w}{w_c}\right)^{2(n+\alpha)}}}, \quad (3)$$

Where, w_c is the cut-off frequency articulated in radians per second (rad/s), and ω is the angular frequency. The suggested design process consists of the two steps listed below. The transmission function, as stated in (4), is determined by the optimal weighting of elements.

$$H^{n+\alpha}(s) = \frac{C_{n+\alpha}}{B_n(s)} + \frac{D_{n+\alpha}}{B_{n+1}(s)} \quad (4)$$

The classical Butterworth polynomials of order n and $n+1$ are $B_n(s)$ and $B_{n+1}(s)$, respectively, and the weighting factors $C_{n+\alpha}$ and $D_{n+\alpha}$ are real and positive values. By minimizing the cost function, as stated in Equation (5), the optimal values of $C_{n+\alpha}$ and $D_{n+\alpha}$ are found for a desired order of the normalized FOBF $w_c = 1\, rad/s$ in the first phase.

$$f = \frac{1}{L}\sum_{i=1}^{L}\left|20log_{10}|B^{n+\alpha}(jw_i| - 20log_{10}|H^{n+\alpha}(jw_i, X|\right|^2 \quad (5)$$

Where, $X = [C_{n+\alpha} D_{n+\alpha}]$ is the vector of design variables, and L is the total number of frequency points sampled with logarithmic space in the bandwidth of $\omega \in \frac{[w_{min}, w_{max}]rad}{s}$. Following the process of optimization, $H^{n+\alpha}(s)$ may be expressed as shown in Equation (6).

$$H^{n+\alpha}(s) = \frac{u_1 s^{n+1} + u_2 s^n + \cdots + u_{n+2}}{s^{2n+1} + u_{n+3}s^{2n} + u_{n+4}s^{2n-1} + \cdots + u_{3n+3}} \quad (6)$$

Where, $u_i(i = 1,2, \ldots, 3n+3)$ gives the coefficients of $H^{n+\alpha}(s)$. Equation (7) describes the best way to determine the constants for the FOBF approximant.

$$T^{n+\alpha}(s) = \frac{p(s)}{Q(s)} = \frac{x_1 s^{n+1} + x_2 s^n + \cdots + u_{n+2}}{s^{2n+1} + x_{n+3}s^{2n} + x_{n+4}s^{2n-1} + \cdots + x_{3n+3}} \quad (7)$$

Where, $x_i(i = 1,2, \ldots 3n+3)$ denotes the coefficient of $T^{n+\alpha}(s)$. To find the ideal values of the coefficients of $T^{n+\alpha}(s)$, the coefficients of $H^{n+\alpha}(s)$, i.e., $u_i(i = 1,2, \ldots, 3n+3)$ are considered as an initial point for the second minimization method, whose fitness function (MSE) is given in Equation (8).

Table 1. Optimal FOBF modelling algorithm

Algorithm 1: Pseudocode of the Proposed Optimal FOBF Modelling Technique

Inputs: n, α
Outputs: X,X_p
begin
Set w_{min}, $w_{\max}$, L, Lower bound(Lb) for X and X_p
for k=1to 100 do
$x_0(Initial\ point\ of\ X) \longleftarrow rand(0,1)$
Minimize (5)
Store $f_k and\ X_k$
$f_{min} \leftarrow min\{f_k\}$
$X \leftarrow X_k$ corresponding to f_{min}
Calculate $u_i(i = 1,2, \ldots, 3n+3)$
Minimize (8) and display X_p

$$MSE = \frac{1}{L}\sum_{i=1}^{L}\left|20log_{10}|B^{n+\alpha}(jw_i| - 20log_{10}\left|H^{n+\alpha}(jw_i, X_p\right|\right|^2 \quad (8)$$

Where, $X_p = [x_1, x_2 \ldots\ldots x_{3n+3}]$ represents the vector of design variables comprising the coefficients of $T^{n+\alpha}(s)$.

The inequality constraints, as stated by (9), are introduced into the optimization method to ensure the creation of stable approximants.

$$\Delta_1, \Delta_2, \Delta_3, \ldots\ldots, \Delta_N > 0 (N = 2n + 1) \quad (9)$$

Where, $\Delta_1 = d_{N-1}$, $\Delta_2 = \begin{vmatrix} d_{N-1} & d_{N-3} \\ d_N & d_{N-2} \end{vmatrix}$,

$$\Delta_3 = \begin{vmatrix} d_{N-1} & d_{N-3} & d_{N-5} \\ d_N & d_{N-2} & d_{N-4} \\ 0 & d_{N-1} & d_{N-3} \end{vmatrix}, \ldots\ldots,$$

$$\Delta_n = \begin{vmatrix} d_{N-1} & d_{N-3} & d_{N-5} & \ldots & 0 \\ d_N & d_{N-2} & d_{N-4} & \ldots & 0 \\ 0 & d_{N-1} & d_{N-3} & \vdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & d_0 \end{vmatrix}$$

Are the Hurwitz determinants, and $Q(s) = \sum_{k=0}^{N} d_k s^k$.

Table 1 displays the pseudocode of the suggested FOBF design technique. As a result, the fifth-order FBPF becomes converted to a fourth-order one by the cancellation of the pole-zero pair occurring at $s = -1$. Hence, it can be concluded that only even values of N are suitable for the design of FBPFs.

4.1.2. Generalization of Power-Law Filter Transfer Function

The optimal way to approximate the frequency-domain properties of the theoretical FO filter is to minimize the mean absolute relative magnitude and phase errors between $H_D^{\alpha,\beta}(jw)$ and $H_P^{\alpha,\beta,N}(jw)$, where N is a positive integer. The Integer order Transfer Function (ITF) is defined based on the following Equation (10).

$$H_P^{\alpha,\beta,N}(s) = \frac{A(s)}{B(s)} = \frac{\sum_{i=0}^{N} a_i s^i}{s^N + \sum_{k=0}^{N-1} b_k s^k} \quad (10)$$

To achieve this, the suggested optimization (minimization) routine's objective function is defined by (11), subject to nonlinear inequality constraints that guarantee that the zeros and poles of $H_P^{\alpha,\beta,N}(s)$ lie strictly on the left-half s-plane.

$$f = \frac{1}{L}\sum_{i}^{L} = 1\left[\left|1 - \frac{\left|H_P^{\alpha,\beta,N}(jw_i,X)\right|}{H_D^{\alpha,\beta}(jw_i)}\right| + \left|1 - \frac{<H_P^{\alpha,\beta,N}(jw_i,X)}{<H_D^{\alpha,\beta}(jw_i)}\right|\right] \quad (11)$$

Subject to: $a_k > 0 (k = 0,1, \ldots, N)$; $b_k > 0 (k = 0,1, \ldots, N-1)$; Real parts of roots of $A(s)$ and $B(s) < 0$.

Where L denotes the number of log-spaced sample points in the bandwidth $[w_{min}, w_{max}] rad/s$; and X denotes the vector of decision variables, i.e., $= [a_N a_{N-1} \ldots\ldots a_0 b_{N-1} b_{N-2} \ldots\ldots b_0]$; the dimension (D) of the issue is $2N + 1$. Metaheuristics have demonstrated promising results when applied to the best approximation of FO filters and systems. For this task, the constrained composite differential evolution (C^2oDE) the technique can be used to tackle the optimization problem. The fundamental structure of composite differential evolution is integrated into the constraint-handling mechanisms based on the feasibility rule and the ε-constrained method by C^2oDE. The user-defined inputs to the optimization procedure are the FO exponents $\{\alpha, \beta\}$ of the theoretical filter TF, order of the proposed rational approximant (N), length of data sample points (L), and lower and upper limits (w_{min}) and (w_{max}) of the desired bandwidth. These values are then used to minimize the suggested objective function. These values are then used to minimize the suggested goal function. After independent testing, the best possible solution (X∗) is the vector of decision variables (coefficients of $H_P^{\alpha,\beta,N}(s)$) that achieves the lowest value of f while similarly sustaining the design constraints. This is determined by the C^2oDE metaheuristic exploration procedure.

As a result, after the suggested search optimization process, $X*$ is deemed the nearly global optimal solution.

The proposed restrictions safeguard that the real component of all roots of the numerator and denominator polynomials of $H_P^{\alpha,\beta,N}(s)$ attains an undesirable value, but the coefficients a_k and b_k must be positive. The proposed fractional-order stop filter (FBSF) and fractional-order high-pass filter (FHPF) have stable inverse filter properties when the TF. $H_P^{\alpha,\beta,N}(s)$, is inverted, resulting in $\left[H_P^{\alpha,\beta,N}(s)\right]^{-1}$. This is because the zeros of the suggested approximant must be entirely on the left side of the s-plane.

4.2. Enhanced Colliding Bodies Optimization (ECBO) Algorithm

The fractional integrator circuit unit is implemented using the decreased TF. The approximate ITF are used to implement the indirect realizations. Several metaheuristic techniques try to get better realization results in this regard. Proper a, b, and k values must be identified to develop a better approximation model that matches the time response of the FO derivative operator, a technique provided in this paper. An approximation model that can deliver a more precise time response is produced by fine-tuning the proposed ECBO algorithm on the MSBL method's initial model. The Enhanced Colliding Bodies Optimization (ECBO) and Coyote Optimization Algorithm (COA) are applied to the fractional order Butterworth filter design by optimizing filter parameters, such as order and cutoff frequency, to achieve desired magnitude and phase responses. This research improves performance metrics compared to existing methods, enhancing accuracy and stability in real-world applications.

For this motive, the analytical time domain expression of the precise value of the step response for an FO derivative operator is given in Equation (12).

$$Y_{FO}(t) = \frac{1}{\Gamma(1-\alpha).\, t^{\alpha}} \tag{12}$$

Where the Gamma function is denoted as Γ, the definition of the approximate error function, which represents the variance between the analytical and approximate value, is given as follows.

$$e_r = Y_{FO}(t) - Y(t) \tag{13}$$

The cost function, which will be reduced using ECBO, is then expressed as.

$$J = \frac{1}{2m}\sum_{i=1}^{m} {e_r}^2 \tag{14}$$

Where the number m denotes the sampling points in time t. Solution points are sampled from $0.001s$ to $100\ s$ with $0.001s$ time steps to produce an accurate model. Accordingly, the ECBO method must optimize these coefficients, and it is essential to determine the partial derivatives of the cost function for each parameter. The following equations contain the estimated partial derivatives for each coefficient.

$$\begin{gathered}
\frac{\partial J}{\partial a_1} = \frac{b_1 + a_1 b_1 t e^{a_1 t} - b_1 e^{a_1 t}}{-a_1^2}.e_r; \frac{\partial J}{\partial b_1} = \frac{e^{a_1 - 1}}{-a_1}.e_r \\
\frac{\partial J}{\partial a_2} = \frac{b_2 + a_2 b_2 t e^{a_2 t} - b_2 e^{a_2 t}}{-a_2^2}.e_r; \frac{\partial J}{\partial b_2} = \frac{e^{a_2 - 1}}{-a_2}.e_r \\
\frac{\partial J}{\partial a_3} = \frac{b_3 + a_3 b_3 t e^{a_3 t} - b_3 e^{a_3 t}}{-a_3^2}.e_r; \frac{\partial J}{\partial b_3} = \frac{e^{a_3 - 1}}{-a_3}.e_r \\
\frac{\partial J}{\partial a_4} = \frac{b_4 + a_4 b_4 t e^{a_4 t} - b_4 e^{a_4 t}}{-a_4^2}.e_r; \frac{\partial J}{\partial b_4} = \frac{e^{a_4 - 1}}{-a_4}.e_r
\end{gathered} \tag{15}$$

The step responsiveness of the resulting TF, which was represented in Equation (16), significantly improved after the proposed ECBO algorithm was used.

$$T_{gdo-msbl}(s) = \frac{b_1^*}{s-{a_1}^*} + \frac{b_2^*}{s-{a_2}^*} + \frac{b_3^*}{s-{a_3}^*} + \frac{b_4^*}{s-{a_4}^*} \tag{16}$$

The pole vector is represented as $P^* = [a_5^* a_4^* a_3^* a_2^* a_1^*]$ when the optimal morals of these coefficients are $R^* = [b_5^* b_4^* b_3^* b_2^* b_1^*]$, and the term k* for all residues and poles is negative as a finding of the partial fraction expansion. The momentum and energy conservation law for a one-dimensional collision serves as the foundation for the optimization of the colliding bodies. Some of the solutions in this algorithm are considered colliding bodies with specified mass and velocity. Each CB changes positions after the collision while considering the current velocity, mass, and restitution coefficient. The ECBO harnesses the power of memory to preserve the expertise of legendary CBs from the past, boosting CBO's capabilities while devising strategies to break free from the constraints of local optima. The ECBO procedure is given as follows.

Step 1: The initial positions of all solutions are chosen randomly in the search space.

Step 2: The mass m_k value for CB_k is evaluated using Equation (17).

$$m_k = \frac{\frac{1}{fit(k)}}{\sum_{i=1}^{2n}\frac{1}{fit(i)}}, k = 1,2,2, \ldots 2n \tag{17}$$

Where $fit(i)$ denotes the CBi's objective function value, and the population size is $2n$.

Step 3: Some historically most effective methods are saved using Colliding Memory (CM). The memory size is CM. Memory solutions are used to replace the same amount of the worst solutions or CM. The populations are then arranged in ascending order by mass.

Step 4: The answers are separated into two equal groups, one for the stationary group and the other for moving ones. A better group are considered the stationary group and a moving group.

Step 5: Calculate the velocities of stationary and moving bodies before the collision, which is evaluated using Equation (18).

$$v_i = \begin{Bmatrix} 0 & i = 1 \\ x_i - x_1 & i = 2,3, \ldots 2n \end{Bmatrix} \tag{18}$$

Step 6: Evaluate the velocities of moving bodies following a collision using Equation (19).

$$v_i' = \begin{Bmatrix} 0 & i = 1 \\ \frac{(m_i - \varepsilon m_1)}{m_i + m_1} & i = 2,3, \ldots.,2n \end{Bmatrix} \tag{19}$$

Step 7: Calculate each CB's new position using Equation (20).

$$x_i^{new} = \begin{Bmatrix} x_1 i = 1 \\ x_i + rand.v_i' \quad i = 2,3, \ldots.2n \end{Bmatrix} \tag{20}$$

Step 8: The quest for optimization comes to a close once the entirety of the NECBO flowchart depicted in Figure 2 has been thoroughly investigated through the highest possible number of evaluations.

Step 9: The study assumes that new solutions are always established between a particle and the best particle formed thus far to give local adjustments for the best particle and have a high possibility of enhancing the fitness value. This mechanism's formulation is as follows:

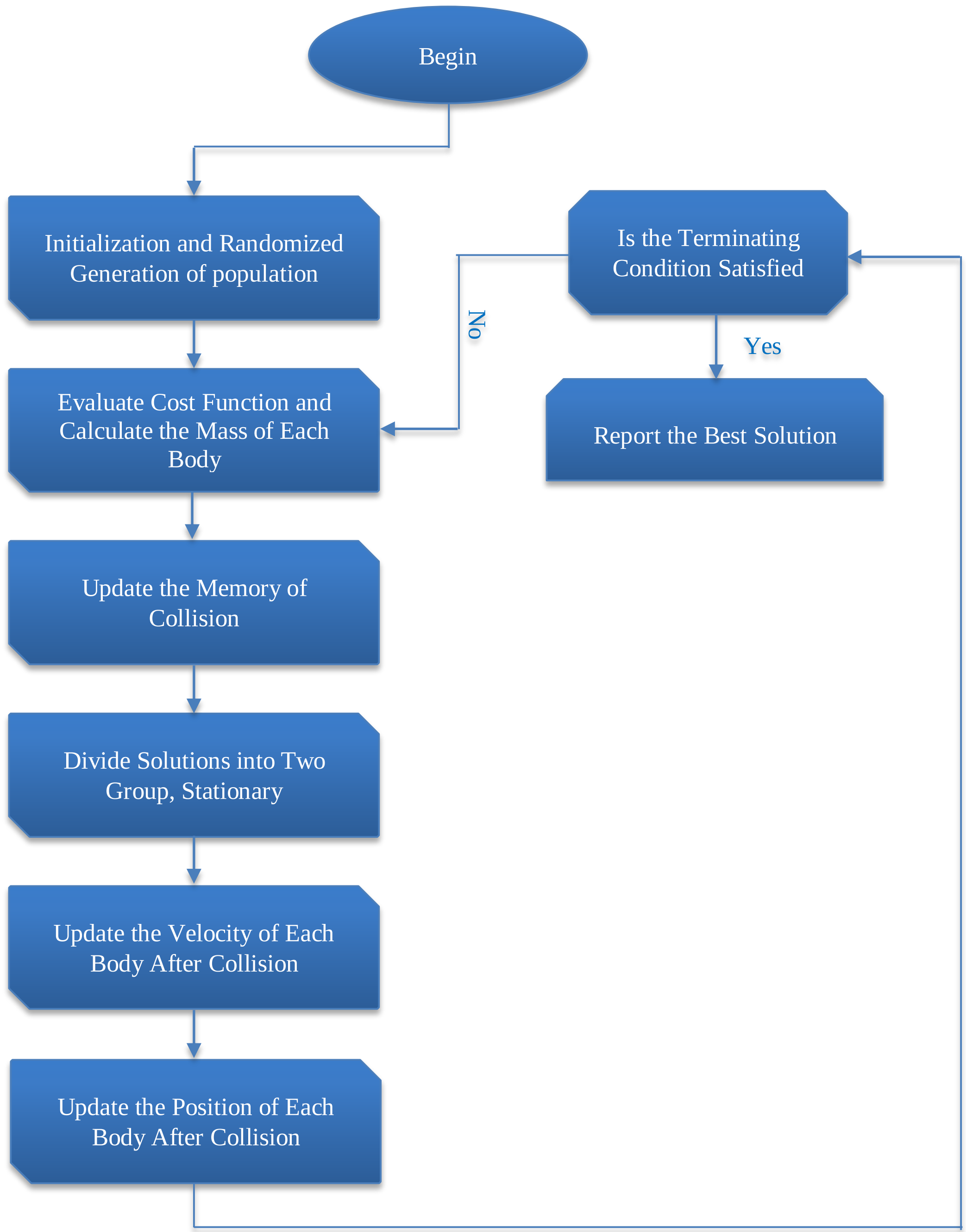


Fig. 2 Flowchart of the NECBO method

$$x_{i,j}^{new} = \begin{cases} x_{best.j} + \left(1 - \left(\frac{iter}{iter_{max}}\right)^{\frac{1}{6}}\right) * \left(x_{j,min} + rand.\left(x_{j,max} - x_{j,min}\right)\right) if\ r1 \le pro, r2 \le 0.5 \\ x_{best.j} + \left(1 - \left(\frac{iter}{iter_{max}}\right)^{\frac{1}{6}}\right) * \left(x_{j,min} + rand.\left(x_{j,max} - x_{j,min}\right)\right) if\ r1 \le pro, r2 \le 0.5 \\ x_{i,j}^{old}\ if\ r1 > pro \end{cases} \quad (21)$$

It is preferable to assume that the Colliding Memory (CM) is equivalent to 2, 0.3 in NECBO, respectively, where $i = 1, 2, 3, \ldots, 2n$ and $j = 1, 2, \ldots, nVar$. If a structure's stability coefficient was discovered to be 0.1 or less, the designer would disregard the second-order analysis. Alternately, it is acceptable to multiply member forces and displacements by $1.0/(1 - \theta)$. Applying the suggested formulation Equation (21) combines local and global searching for all options, and in the final iterations, it produces a superior answer.

4.3. The Charef Approximation

The Charef Approximation method, coupled with the Hybrid FMINCON-MAYFLY Optimization algorithm, presents a promising alternative to the conventional COA for the optimal enterprise of Fractional Order BF with optimized magnitude and phase response. By leveraging the computational efficiency and robustness of the Hybrid FMINCON-MAYFLY Optimization algorithm, this approach aims to enhance the precision and convergence speed of the design process. It seeks to achieve superior performance regarding filter response characteristics through iterative optimization and fine-tuning, thereby advancing the state-of-the-art in fractional order filter design methodologies. The Charef approximation enables us to approximate the fractional order TF (22) using the ITF. $G_{char}(s)$ defined below (see [22]):

$$G_{char}(s) = \frac{\prod_{i=0}^{N-1}\left(1+\frac{s}{z_i}\right)}{\prod_{i=0}^{N}\left(1+\frac{s}{p_i}\right)} \qquad (22)$$

In (23) N denotes the order of the filter, z_i and p_i denote zeros and extremes of the filter. They can be calculated with the usage of TF (22) pole p_α and fractional order α:

$$p_\alpha = \frac{1}{T_\alpha}$$
$$p_0 = p_\alpha\sqrt{b}$$
$$p_i = p_0(ab)^i, \quad i = 1 \ldots. N \qquad (23)$$
$$z_i = ap_0(ab)^i \quad i = 1 \ldots. N-1$$

Where,

$$a = 10^{\frac{\Delta}{10(1-}\alpha)} \qquad (24)$$
$$b = 10^{\frac{\Delta}{10\alpha}}$$

In (25) Δ denotes a maximal permissible error of approximation, defined as the maximal difference between the Bode magnitude plots model and plant, articulated in [dB]. The Charef approximation is allocated an order N to minimize the expected maximum approximation error Δ.

$$N = \left[\frac{log(\omega_{max})}{\log(ab)}\right] + 1 \qquad (25)$$

In (26) ω_{max} denotes the pulsace for which the maximal error is achieved. If the value of N concerning (26) is a non-integer, it should be rounded to the nearest integer. Denote the step response of approximation (5) by $y_{char}(t)$.

$$y_{char}(t) = L^{-1}\left\{\frac{1}{S}G_{char}(s)\right\} \qquad (26)$$

The Charef Approximation method offers improved accuracy and efficiency in approximating FOTF, enabling a more precise representation of complex filter responses. This, coupled with the Hybrid FMINCON-MAYFLY Optimization algorithm, facilitates the fine-tuning of filter parameters to achieve desired greatness and phase responses with enhanced precision and speed. Performance comparisons between the Hybrid FMINCON-MAYFLY Optimization algorithm and the COA are essential to determine their respective strengths and weaknesses in optimizing employee work management and scheduling. Analyzing factors such as convergence speed, solution quality, robustness, and computational efficiency can provide valuable insights into which algorithm is better suited for addressing the complexities of scheduling in knowledge-intensive organizations, thereby guiding future research and practical implementations Table 2.

Table 2. Charef approximation method with hybrid Fmincon-mayfly optimization

Algorithm 1: Charef Approximation Method with Hybrid FMINCON-MAYFLY Optimization
1. Initialize parameters and settings for the Charef Approximation method and Hybrid FMINCON-MAYFLY Optimization algorithm. 2. Apply the Charef Approximation method to approximate the fractional-order BF. 3. Utilize the Hybrid FMINCON-MAYFLY Optimization algorithm to optimize the magnitude and phase response of the filter. 4. Iterate between the Charef Approximation method and the optimization algorithm until convergence criteria are met. 5. Assess the performance of the designed fractional-order BF. 6. Compare the results with those obtained using the conventional COA. 7. If the presentation of the Charef Approximation method coupled with Hybrid FMINCON-MAYFLY Optimization meets the desired criteria, proceed with the designed filter. Otherwise, adjust parameters and settings and repeat steps 2-6. 8. End.

Table 3. Performance comparison of hybrid FMINCON-MAYFLY and COA

Algorithm	Convergence Speed	Solution Quality	Robustness	Computational Efficiency
Hybrid FMINCON-MAYFLY	Faster Convergence	High-quality Solutions	Robust Performance	Efficient Computation
COYOTE	Slower Convergence	Variable Solution Quality	Moderate Robustness	Computational Demands Vary

Table 3 shows the comparison between the Hybrid FMINCON-MAYFLY and COA, revealing distinctive characteristics in various performance aspects. The Hybrid FMINCON-MAYFLY algorithm demonstrates faster convergence rates, offering expedited solutions. Additionally, it consistently produces high-quality solutions, indicating its reliability in optimizing complex problems. In terms of robustness, the Hybrid FMINCON-MAYFLY algorithm showcases strong performance, ensuring stability across diverse scenarios. Moreover, its computational efficiency stands out, as streamlined processes and reduced computational demands characterize it.

Conversely, the COYOTE algorithm exhibits slower convergence rates and variable solution quality, suggesting potential fluctuations in its effectiveness. While demonstrating moderate robustness, it may not offer the same level of stability as the Hybrid FMINCON-MAYFLY algorithm. Furthermore, the COYOTE algorithm's computational demands vary, potentially leading to resource-intensive computations.

5. Experimentation and Result Discussion

In this segment, showcase the outcomes produced by utilizing the suggested filter and target function for the system. A brief statistical analysis is conducted before examining the effect of COA parameters on the system's optimal fitness value and performance to obtain a clearer perspective. To achieve the previously mentioned goals and accurately assess the proposed design compared to previous research, analyze two scenarios using 1000 and 2000 function evaluations as the stopping criteria for the COA. Following this, an analysis is carried out to compare how the AVR system's response varies when different tuning algorithms and controllers from previous studies are used. The stability study is conducted using the system's frequency response and the corresponding damping ratios of the closed-loop poles. The system is then thoroughly examined for its resilience against any potential setbacks or challenges.

Table 4. Simulation system configuration

Simulation System Configuration	
MATLAB	Version R2021a
Operation System	Windows 10 Home
Memory Capacity	6GB DDR3
Processor	Intel Core i5 @ 3.5GHz
Simulation Time	10.190 seconds

Table 4 shows the simulation table for the suggested method. Using functions based on the parameter values, the cost functions are reduced using the MATLAB computer language (software version: MATLAB R2021a). In this scenario, the lower boundary (Lb) for X is set at 0. Here, showcases the presentation of the suggested technique in effectively representing higher-order FOBFs.

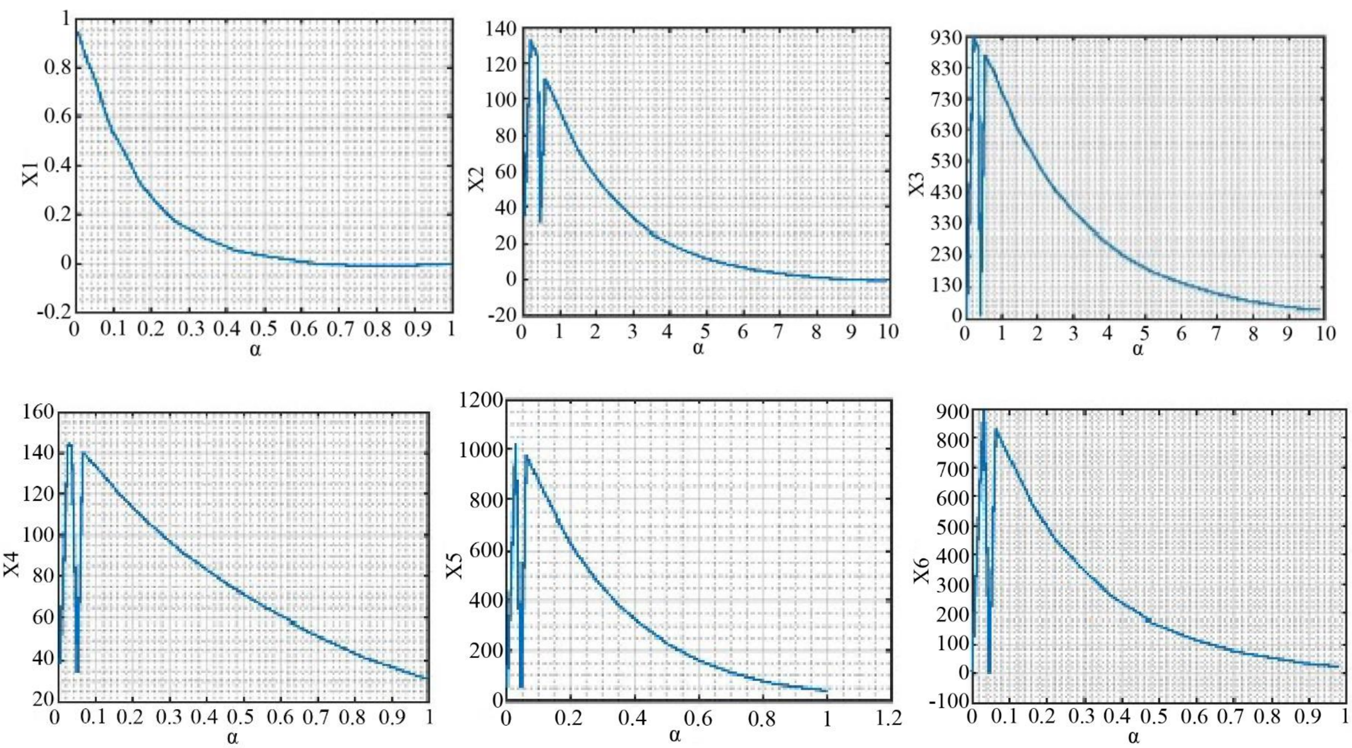


Fig. 3 Optimal values of coefficients for the proposed fourth-order FOBFs

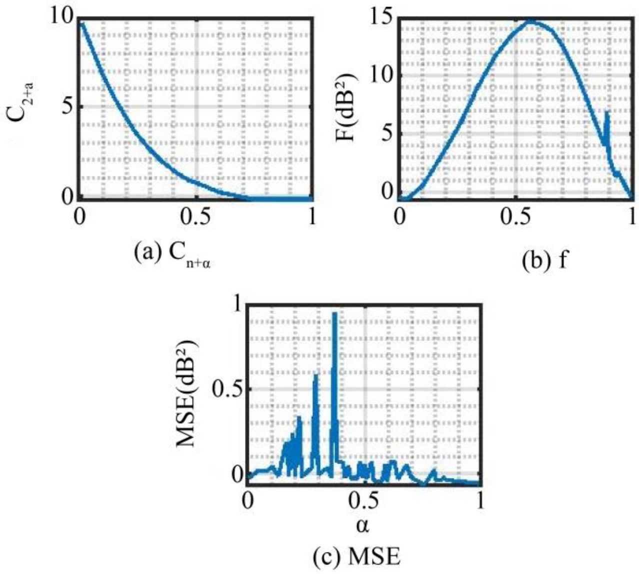


Fig. 4 Plots for the proposed FOBF

Figure 3 displays the ideal values of the design variables achieved for FOBFs with a sample size of 2. However, because of the significant differences within the design range for the best coefficients, polynomial curve fitting with a reduced norm of residuals might not be feasible. Figure 3 displays the ideal $x_1, x_2, \dots, x_6$ values for the suggested FOBFs. After α values fall between 0.01 and 0.06, a significant change occurs in the x_2 to x_6 range, followed by a gradual and monotonic decrease in the coefficient values. A polynomial of degree m in α can be utilized to estimate the ideal model coefficients$x_i (i = 1,2, \dots ,6)$.

The ideal values of (a) $C_{2+\alpha}$, (b) f, and (c) MSE obtained for the $(4 + \alpha)$-order FOBFs are exposed in Figure 4. The results show that (i) the profile produced by $C_{4+\alpha}$ is comparable to that of $C_{2+\alpha}$; (ii) the worst value of $f\ (14.33\ dB^2)$ occurs at $\alpha = 0.58$, despite existence that the MSE (0.01178 dB2) is remarkably low; (iii) At $\alpha = 0.89$, $C_{4+\alpha}$ is obtained as 0, indicating the transfer purpose of the third-order BF is the resulting starting point. Consequently, the 2.89th-order filter exhibits a significant peak for $f\ (7.273\ dB^2)$, but the resulting MSE is modest $(0.002083\ dB^2)$; and (iv) three specific orders, namely $\alpha = \{0.22, 0.29, 0.37\}$, produce an abrupt peaking in MSE as coefficient x_3 reaches a value of 1E-8. However, the equivalent values of f and MSE are $\{4.563, 7.134, 10.070\}dB^2$ and $\{0.3966, 0.6698, 1.0180\}dB^2$, respectively, showing a decrease in a modelling error of about 10 times.

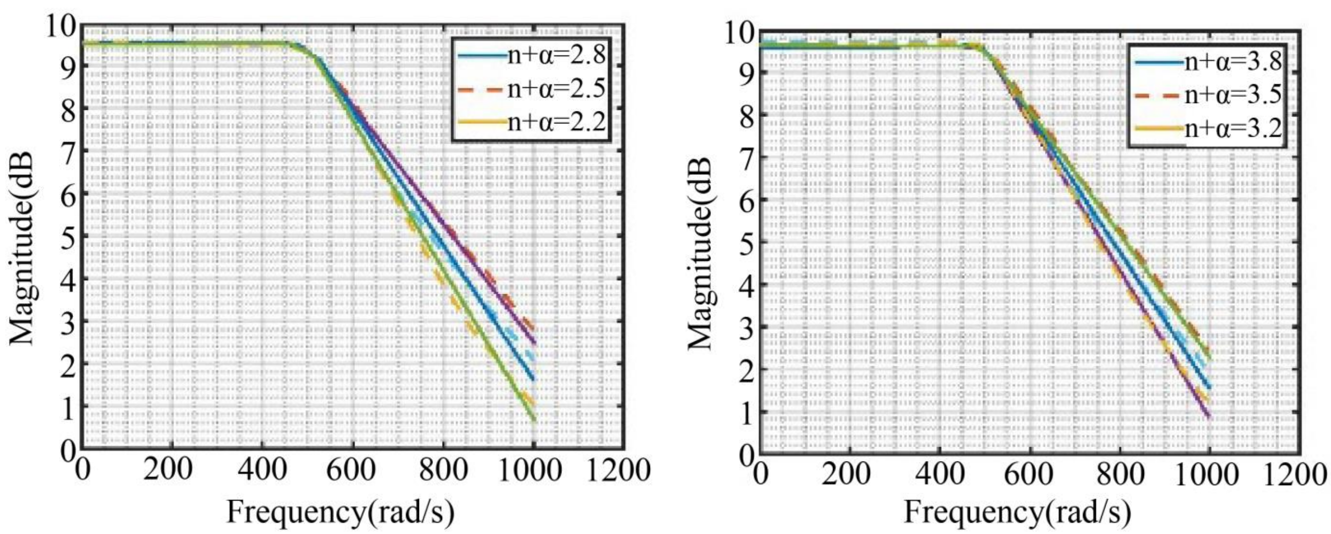
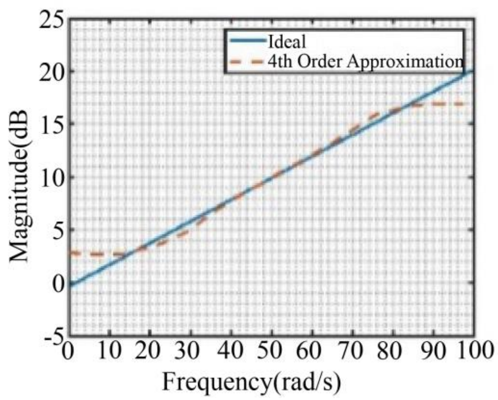


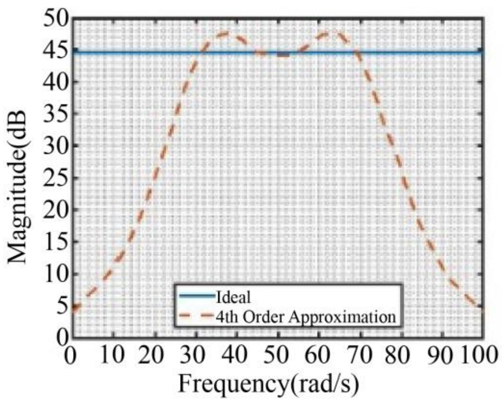
Fig. 5 Magnitude response comparison plots

Figures 5 (a) and (b) for $n = 2$ and $n = 3$ show the magnitude-frequency graphs of the primary point and optimal models for these test situations. While the initial reaction may not meet expectations, the outcome is consistently close to perfection in all scenarios. Consequently, the suggested method shows effective modelling performance for creating higher-order FOBFs.

Figure 6 shows the difference in magnitude and phase between the fractional Laplacian operator approximation and the desired outcome with α=0.5. This study's phase and magnitude responses are associated with those of the ideal case. It can be shown that the magnitude error for $\omega \in [0.032, 31.53]$ is less than 1.375 dB, while the phase error for $\omega \in [0.142, 7.00]$ is less than 3.2o. An $(n + 4)$ integer order filter that approximates the $(n + \alpha)$ fractional step filter is produced using a fourth-order approximation for the Laplacian operator. This approximation is less expensive to implement in hardware than higher-order approximations.

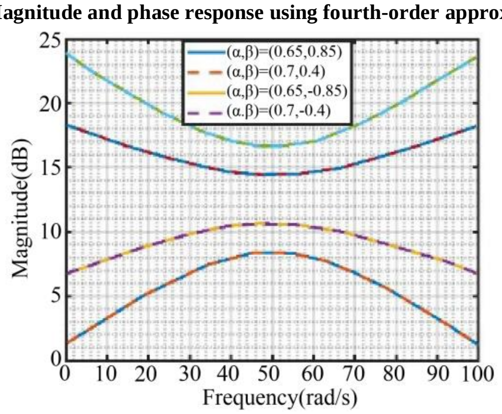


(a) Magnitude Response

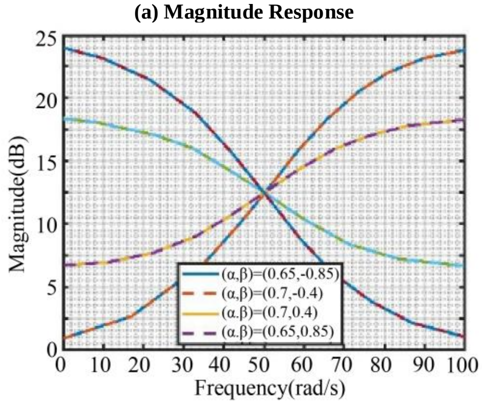


(b) Phase Response

Fig. 6 Magnitude and phase response using fourth-order approximation



(a) Magnitude Response



(b) Phase Response

Fig. 7 Magnitude and phase responses of the proposed FOBF for N = 4

Figures 7 (a) and (b) display the magnitude as well as phase plots of suggested fourth-order FOBF, along with their inverse TF. The results from both experiments closely resemble the expected theoretical outcomes. The proposed approximants consistently mirror the expected characteristics across the entire design bandwidth.

Table 5. Performance values for the proposed method

Parameters	Values
MSE	0.93 (dB2)
Magnitude Frequency	2.81 (dB)
Magnitude Response	17 (dB)
Phase Response	47.8 (dB)
Optimal Values of Coefficients	0.13
Average Fitness Cost	0.008

Table 5 showcases the performance metrics for the proposed work, including optimal coefficient values, Mean Squared Error (MSE), magnitude frequency, magnitude as well as phase response, and average fitness cost. The proposed work exhibits an MSE of 0.93 and a magnitude frequency value of 2.81, with a magnitude response of 17 and a phase response of 47.8. The optimal coefficients have values of 0.13, and the average fitness cost function is 0.008.

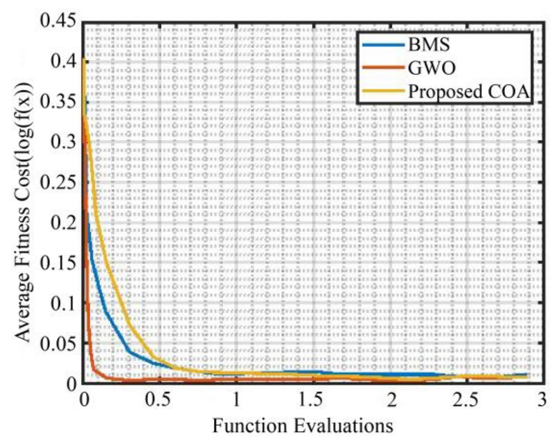


Fig. 8 Comparison graph for average fitness function

The average fitness values of the benchmark functions are displayed in Figure 8 using a logarithmic scale. In addition, the statistical results of minimum value, average value and standard deviation are demonstrated in Table 4 for each benchmark function.

Table 6. Comparison table for average fitness function

	Function Evaluation			
Techniques	**0.5**	**1**	**2**	**2.5**
BMS	0.015	0.011	0.02	0.01
GWO	0.02	0.02	0.0025	0.009
Proposed COA	0.03	0.025	0.001	0.008

In Table 6, the average fitness function comparison chart reveals that the future method outperforms both the current BMS and GWO methods by a margin of 0.008.

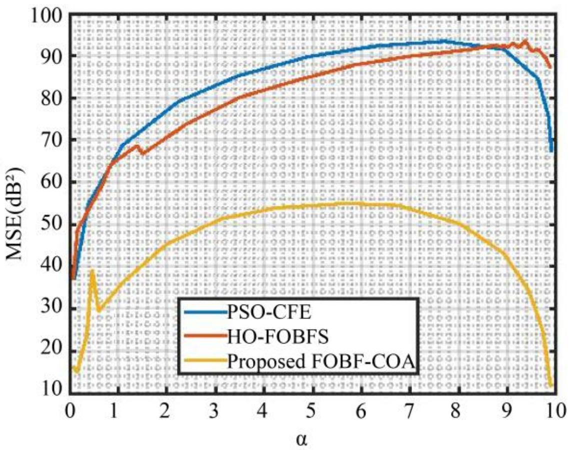


Fig. 9 MSE comparison plots for the proposed fractional-order Butterworth filter

Figure 9 illustrates the MSE comparisons of the future FOBF with the third-order FOBFs created using the PSO-CFE and HO-FOBFs algorithms. The proposed method surpasses both reported designs by a significant margin. At the peak of the MSE curve, an optimal value of α=0.76 results in a high MMSE of 36.32 dB^2. Alternatively, a lower MMSE of 39.46 dB^2 is achieved with a higher α value of 0.94. In contrast, the suggested method only achieves an MMSE of 0.1981 dB^2 at α=0.56, highlighting its performance difference in comparison.

Table 7. Comparison table for MSE

	Function Evaluation			
Techniques	**2**	**5**	**8**	**10**
PSO-CFE	76.2	90	93.8	67.4
HO-FOBF	70	859	90.5	87.8
Proposed FOBF-COA	44	53.9	49.8	10.185

Results from Table 7 reveal that the future method triumphs over both PSO-CFE and HO-FOBF, showcasing substantially reduced error values, specifically 10.185. In contrast, the existing PSO-CFE and HO-FOBF methods result in MSE values of 67.4 and 87.8, respectively, indicating poorer performance than the suggested method.

Table 8 shows the performance of two techniques for fractional order Butterworth filter design Genetic Algorithm and the proposed ECBO & COA method. The proposed method achieves a better magnitude response of -15 dB, improved phase response, lower error margin of 5%, and higher stability, indicating superior performance.

Table 8. Comparison of filter design techniques

Technique	**Magnitude Response (dB)**	**Phase Response (Degrees)**	**Cutoff Frequency (Hz)**	**Error Margin (%)**	**Stability**
Genetic Algorithm	-22	-92	120	8	Low
ECBO & COA	**-15**	**-85**	**120**	**5**	**High**

6. Research Conclusion

The article explores creating the perfect FOBF by utilizing rational approximations of integer orders. Filters are designed by turning the fractional Laplacian operator into a fourth-order integer TF. The ECBO method demonstrates exceptional precision and steadiness when applied to fractional-order continuous filters. The study suggests utilizing the COA to improve both the amplitude and phase characteristics. Customizing the filter settings can more effectively align with the frequency output of fractional-order continuous filter functions, resulting in enhanced precision in the outcomes. The Matlab software's output accurately represents the first-order derivative in the time domain, as indicated by the RMSE. Implementing the derivative circuit using a combination of pass band and stop band filters yields better accuracy in time response approximation than other methods available. Identifying the coefficients for rational approximations that closely match the ideal fourth-order elliptic-curve bandpass filter's magnitude response involves methodically testing out various combinations. The FOLBFs based on ECBO-COA excel in achieving high solution quality quickly, surpassing other competing methods with their impressive convergence speed. Rigorous testing involved benchmarking the proposed models against established fractional-order Butterworth filters. The validation included simulations across varied signal conditions, performance metrics analysis, and real-world application scenarios, ensuring the accuracy, stability, and reliability of the optimized magnitude and phase responses. When put to the test, the fractional order integrator produced through this approach showcases superior phase and magnitude responses when stacked up against alternative methods.

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