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Original Article

Simple Fair Power Allocation for NOMA-Based Visible Light Communication Systems Using Fuzzy Order Homomorphism on L-Fuzzy Sets

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Abstract – *VLC, or Visible Light Communication, is a practical high-data rate technology. A technique that looks effective in place of traditional transmission by Radio Frequency (RF). This energy-efficient, secure, and economical VLC method makes use of the current wireless data transmission infrastructure. However, the main barrier to building Light-Emitting Diode (LED) systems is the restricted control bandwidth of high-speed VLC systems, which collapses in the MHz range. Innovative field VLC system issues. In this work, we devised a broad summary of VLC systems centered on PD-NOMA. In the energy industry, Non-Orthogonal Multiple Access, or NOMA, has been viewed as a potentially useful tactic to overcome the capacity constraint of the existing Visible Light Communication (VLC) system. The Distributed Power (PA) and high Signal-to-Noise Ratio (SNR) problems in the NOMA-VLC system are examined in this work. Numerous facets of the Fuzzy order homomorphism on an indefinitely distributive lattice have been studied in this study. In a multi-user scenario, equitable distribution of transmission resources is ensured through the proposal of a Simple Fair Power Allocation (SFPA) approach. Because SFPA requires less Channel State Information (CSI), erroneous channel estimations are less likely to occur. The results show that, in the arrangement under consideration, more reasonable and equitable pricing per user is provided by NOMA with SFPA (up to 79.5% greater) without significantly compromising overall system performance.*

Keywords *- Non-Orthogonal Multiple Access (NOMA), Fuzzy order homomorphism, λ-Level preserving fuzzy sets, Fair power allocation, Visible Light Communication (VLC).*

1. Introduction

One feasible solution to the bandwidth limitations of the current VLC systems uses Non-Orthogonal Multiple Access, or NOMA, as a communication protocol, and researchers have been exploring it recently [1]. When utilising NOMA, users are distributed in the receiving side's power domains, and several user signals have been separated via Successive Interference Cancellation, also known as SIC [2]. The main basis for transmission in VLC systems is the existing illumination sources' intensity modulation mechanism mostly diodes that emit light (LEDs).

Users are outfitted with Photodiodes (PDs) at the communication endpoints, which are responsible for directly detecting the modulated optical signal. Considering that power resources are less rare in NOMA networks than in classic Orthogonal Multiple Access (OMA) techniques, VLC downlinks are believed to be well-suited for replacement by (NOMA-VLC). [3]. Additionally, some research has shown that NOMA is better than OMA in particular VLC conditions [4].

Ensuring the fundamental operation of VLC, which includes broadband communication and ubiquitous connectivity for numerous users, is crucial [5]. Accordingly, when handling many users' simultaneous requests for network access, a suitable Multiple Access (MA) strategy should be used [6]. In the past, VLC systems have employed Techniques for Orthogonal Multiple Access (OMA), including Carrier Sense Multiple Access (CSMA), Code Division Multiple Access (CDMA), and Orthogonal Frequency Division Multiple Access (OFDMA) [7]. Nonetheless, consumers with lower channel yields are always assigned a greater power level in all of the existing projects, which has been shown to improve user fairness and is considered a fundamental principle in NOMA [8].

Through the use of an effective multiple-access technique, VLC can achieve the goals mentioned above for the next wireless networks [9]. One of the most revolutionary multiple-access strategies that have been proposed recently is non-orthogonal NOMA, a multiple-access method with superior spectrum effectiveness [10]. A frequency/time resource block may be shared by multiple users in NOMA, allowing for a high throughput and a large number of connections for modulating is a primary drawback of VLC with just 5–10 MHz in the 3dB band [11]. Intensity Modulating and Immediate Detection (IM/DD) architecture-based solutions have been the subject of extensive study aimed at improving the achievable data rate. These methods include different methods of access [14], improved optical multiplexing [12], channel balance [13], and Multiple-Input Simultaneous-Output (MIMO).

In the fuzzy literature, [15] originally described as fuzzy sets in 1965. Fuzzy function on fuzzes (a fully distributed Structure Fuzzy order homomorphism on L-fuzzy sets and order reversing involution were thoroughly examined [16]. They established the property that fuzzy points are mapped onto fuzzy points under certain conditions by a Fuzzy order homomorphism as well as by a Fuzz function, and they also established the requirements that must be met for a fuzz function to be a function of the Zadeh kind. This article examines a few of the features of the fuzzy order homomorphism that are connected to this attribute and conducts a thorough study of homomorphism in Q-intuitionistic L-fuzzy subrings [17].

The fuzzy sets were proposed for the first time in 1965. The fuzzy function on fuzzes, which are fully distributive lattices, along with fuzzy order homomorphism on L-fuzzy sets and order-reversing involution are also discussed in the fuzzy literature were studied in detail [18]. Homomorphism in Q-intuitionistic L-fuzzy subrings was studied in detail [19].

The prerequisites for a fuzz function, both necessary and sufficient to be a Zadeh type function and the property that fuzzy points are mapped onto fuzzy points under certain conditions by a Fuzzy order homomorphism as well as by a Fuzz function, were established by them. Several facets of the fuzzy order homomorphism associated with this characteristic have been studied here. Also, the essential and sufficient requirements for an ambiguous order homomorphism to be a Zadeh type function have been established with some modifications [20].

The paper's contribution is as follows: This review is recommended for anyone interested in VLCs based on NOMA.

- A comprehensive overview of recent research on VLC systems based on NOMA is given in this study. The reader who is familiar with NOMA-based VLC technology should find this work useful in a variety of ways.
- It provides a comprehensive description of MISO/MIMO techniques for VLC systems based on NOMA. It also explains how more recent technologies, including IRS, PLS, ML, UVLC, OFDM, MIMO, etc., are integrated into NOMA-based VLC.
- Lastly, it clarifies a variety of possible difficulties and future directions for groundbreaking discoveries.

The paper is organized in such a way as, the first Section explains why this project is needed and presents the subject. The Related works are in the second Section. Section Three contains the proposed method of this paper. The fourth Section contains the Results and discussion. Finally, Section Five contains the Conclusion.

2. Related Works

This section provides a review of the NOMA-VLC system studies that have previously been published in the literature. Q. Li an et al. (2019) showcase multiple works that have been carried out using the NOMA-VLC. Lately, NOMA has demonstrated significant potential as a 5G technological enabler. All NOMA customers have access to support for both frequency and time resources, which work together to boost VLC systems' overall rate dependent on NOMA [21]. It has been demonstrated that NOMA is better appropriate for downlink VLC systems (Chen, C. et al. 2018 [22]).

In a similar vein, some research has created multiple users with an LED a NOMA-VLC system's source [8]. A breakthrough from Lin B. et al. (2017) is NOMA-VLC systems. Compared to static power allocation, it improves system efficiency (SPA). Additionally, the researchers found that by modifying the Photodiodes' (PDs') semi-angle and Field Of Vision (FOV), the overall rate of transmitting light could be further increased. The researchers used real indoor VLC channel models as a basis. H. Haas et al. (2015), The authors of the NOMA-based VLC, advise using the Gain Ratio Power Allocation (GRPA) approach. Researchers in [23] discovered that fully networked VLC systems are called Light Fidelity (LiFi) networks. Dogra, T. (2022) found that NOMA frequently increases throughput in various sorts of networks. Moreover, they discovered that NOMA could be able to enhance att cells edge users' performance without noticeably degrading other users [24]. Comparing SFPA computations to other techniques that require knowledge of the CSI of the entire system, they are less susceptible to errors in channel estimate since they only require the exact CSI data for the individual with the optimal channel condition. Comparing the suggested technique to PA systems of comparable computing complexity, numerical simulation results demonstrate that it performs better overall while exhibiting greater transmission rates per user.

3. Materials and Methods

3.1. NOMA for VLC

Although VLC has many great benefits, it also has some significant disadvantages. Specifically, when designing high-data-rate VLC networks, the relatively narrow modulating frequency of LEDs is a major problem. Although VLC allows several whiles allowing multiple users. Consequently, it can be said that NOMA and, more especially A useful multiple access method is PD-NOMA that offers indoor VLC systems enough bandwidth. NOMA can result in significant improvements in throughput as well as a method to boost VLC systems' spectrum efficiency, because it still functions well even in user channels with significant gain differences.

Consequently, it can be said that NOMA, and more especially PD-NOMA, is a good multiple-access strategy that offers indoor VLC systems enough bandwidth. NOMA can result in significant improvements in throughput as well as other performance metrics. Therefore, NOMA can be considered a powerful and effective multiple-access method to increase the VLC systems' spectrum efficiency. It still functions well even in user channels with significant gain differences. Figure 1 depicts a simple VLC system and provides a NOMA-VLC system overview. VLC systems have this property not only because their transmitter is LEDs but also because they can effectively overlay a finite number of users. In enclosed for a restricted quantity of users, the LED sources serve as points of entry. CSI is required at the transmission and recipient sides of the SIC method called NOMA, which stands to allow user power planning, demultiplexing and decoding order must be enabled. With VLC systems, the channel is typically continuous, so this problem does not occur as frequently.

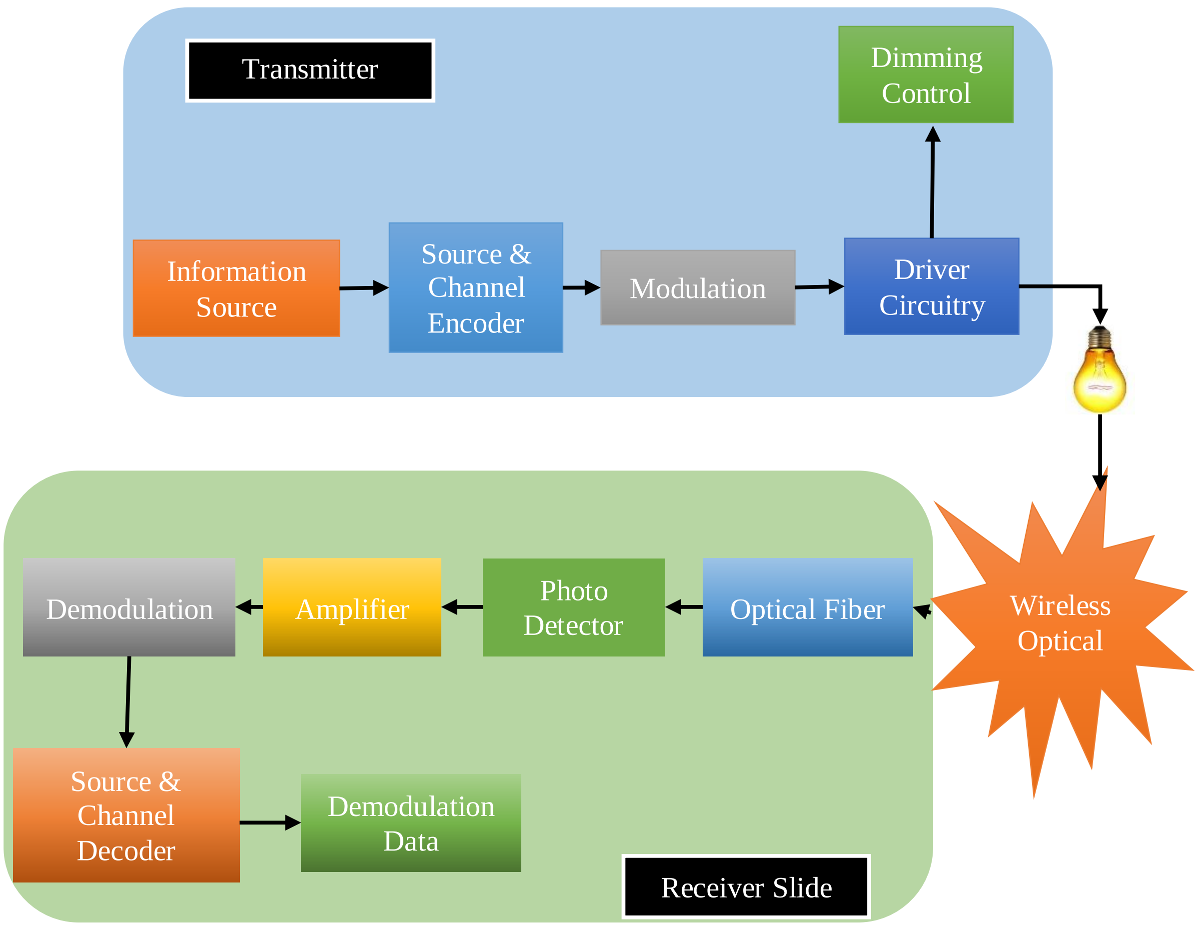

Fig. 1 The outline of a simple VLC system and a conceptual diagram of a NOMA-VLC system

In VLC networks, the proximity of the PD and LED results in higher operational efficiency for NOMA at high SNR levels. In VLC systems, adjusting the FOVs of the PDs and the LEDs' transmitting angles can assist in minimising user-to-user variations in channel gain.

Non-orthogonal, the optimum multiple-access strategy is very low latency (VLC) systems with multiple access (NOMA).

3.2. Proposed NOMA-VLC System Using a Fuzzy-based Demodulator

An LSTM post-equalizer with deep long-short-term memory was created in order to get around the shortcomings of VLC. A multi-coloured VLC uses LEDs that are red, green, and blue. Connection with Deep Learning (DL) assistance. The authors looked at the feasibility of building and implementing a VLC connection for signal demodulation using machine learning.

A fuzzy-based signal demodulator is suggested to lessen the linear and exponential distortions that NOMA-VLC transmissions suffer, as shown in Figure 2. There are two uses for the recorded NOMA signals: (i) Online signal processing and (ii) offline fuzzy training recovery and compensation through the use of a fuzzy-based demodulator. Recall that the free-space channel response is not necessary for the detection of the NOMA signal. Through modelling and experimental data, they demonstrate how the system's functioning would be enhanced by the proposed fuzzy-based demodulator, which would successfully remove both nonlinear as well as linear distortions.

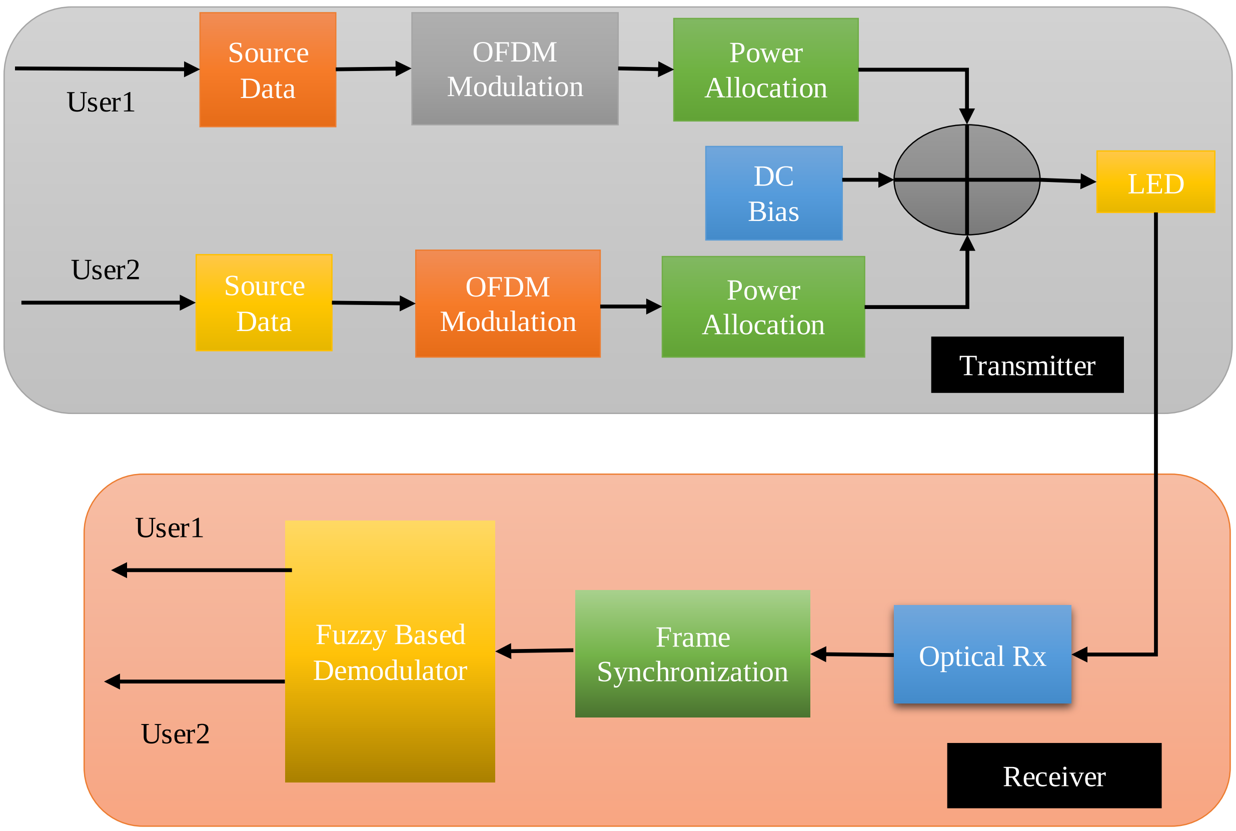

Fig. 2 Proposed diagram NOMA-VLC system using a fuzzy-based demodulator

Table 1. Research parameter

	Parameter	Value
Room	Size	9x9x3 m
Transmitter	Type	LED
	The number of LED bulbs	1
	Power Transmitted	5 Watt
	Model Channel	LOS
	Bandwidth	20 MHZ
Receiver	Detector Area	1 cm^2
	Field of View	70
	Responsivity	0.55 A/W

The subsequent user instantly uses its threshold hard decoding to decode upon the completion of the cancellation at the following user signal to acquire the original, owned information.

As indicated in Table 1, this study took into account a few crucial factors influencing each research situation.

3.3. System Model

We examine a MIMO-VLC system with K users and I LEDs, as shown in Figure 3. Every user has access to Inside are J Photodetectors (PDs). The centre of the cell is home to U_1, and the radius of the cell's area of coverage is R. The symbol r represents the distance between the centre and edge users. The symbol symbolises the expression of that distance. U_k is indicated as r_k. We define L_i as the i_{th}LED transmitter, and D_j as the j_{ith} PD. The signal is always subjected to a DC bias IDC in order to produce the non-negative waveform. i_{th} to drive L_i, which can be expressed by

$$x_i = \sum_{K=1}^{K} \sqrt{\mu_i, kP_t s_i, k} + I_{DC} \quad (1)$$

WhereP_t is the emitter′s electrical power, μ_{ik} stands for the power allocation factor that has been normalised at the i_{th} LED transmitter for U_k and S_{ik} indicates the OOK modulated signal with zero means that it is ready for U_k at L_i. In order to ensure a consistent overall electrical power output, the power distribution factors must be met.

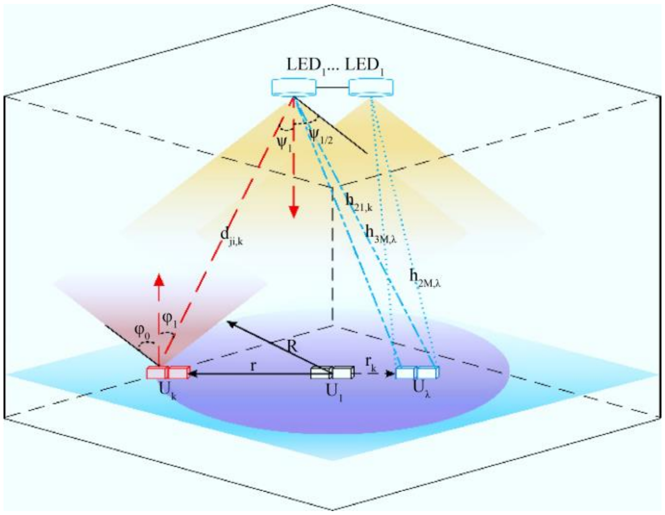

Fig. 3 Indoor NOMA-based MIMO-VLC system with K users and I transmitters; every receiver is equipped with a J PD

$$\sum_{k=1}^{k} \mu_i, K = 1 \qquad (2)$$

We take a look at a MIMO-VLC setup for a K user and 100 LEDs. Every user has access to J Photodetectors (PDs) within. The DC signal is always removed from the received signal prior to demodulation because it does not contain any meaningful information. Consequently, the signal received for U_k can be given by

$$y_k = \gamma_{oe} P_o \zeta H_k x + n_k, \qquad (3)$$

Where γ_{oe} is the photodiodes' optical-electric responsiveness, Po is the optical power output of the emitter, ζ is the modulation index, and the channel matrix.

$$\sigma_{noise}^2 = \sigma_{shot}^2 + \sigma_{thermal}^2 \qquad (4)$$

$$\sigma_{noise}^2 = \left(2qI_{bg}I_2B + 2q\gamma_{oe}P_oBH\right) + (\frac{8\pi KT}{G}\eta ApDI_2B^2 + \frac{16\pi KTT}{g_m}\eta^2A^2pDI_3B^3 \qquad (5)$$

Let p_i indicate the corresponding electrical power coefficient to the i_{th} user k's received signal is satisfied when the power-domain superposed user in LED l is

$$yk = p_o \sum_{l=1}^{L} hl,k \sum_{i=1}^{k} \sqrt{pI,i}\ \ xI,i + z_k \qquad (6)$$

Next, let us connect the SINRK (that is, the user's SINR in optimal channel circumstances) to the lower bound of the SINR in order to ensure a just distribution of resources.

$$SINR_K = (\frac{1-\mu}{\mu})\beta \qquad (7)$$

Where it is convenient to set β ≥ 1 to give user K an excess rate. That is, to manage the ratio of the strongest user's rate to that of the weakest user.

3.4. Fuzzy Order Homomorphism on L-Fuzzy Sets

A mapping $f : {L_1}^X \to {L_2}^Y$ is said to be a fuzzy order homomorphism if

1) $f(0)=0; f^{-1}(0)=0$
2) f and f^{-1} are union-preserving.

If $L_1 and L_2$ are fuzzes, then $f : {L_1}^X \to {L_2}^Y$ is called a Fuzz function, if

3) $f(0)=0$
4) f is union preserving and
5) For each B ∈ ${L_2}^Y$, $f^{-1}(B') = (f^{-1}(B))'$

Let f : X→Y be a mapping. Then the mapping, $f: L^X \to L^Y$ defined by

$f(A)(y) = \begin{cases} \vee\{A(x): f(x) = y\}, & if\ y\epsilon f(X) \\ 0, & elsewhere \end{cases}$, is called Zadeh type function

Proposition 1. Let: ${L_1}^X \to {L_2}^Y$ be a fuzzy order homomorphism where L_1 is infinitely distributive and $L_2\ is$ regular. Then f is supported preserving if and only if L_1 is regular.

Proof: Suppose f is support preserving. Then by Yang Le Cheng[5] for each x ∈X ,∃ a unique y∈ Y , such that f (x_λ)=y_μ hold for all λ's and corresponding μ's. Defining y to be the image of x, we obtain mappings, $\tilde{f}$:X→Y and f_x: $L_1 \to L_2$ defined by $\tilde{f}(x)$=y and $f_x(\lambda)=\mu$ if and only if $f(x_\lambda)=y_\mu$

Let $\lambda_1, \lambda_2 \in L_1$, where $\lambda_1, \lambda_2 \neq 0$

Let A and B have the same support and let x be an element of support of A, which also belongs to the support of B.

Then $x_{\lambda_1} \neq 0$, $x_{\lambda_2} \neq 0$

Let $f(x_{\lambda_1})=y_{\mu_1}$ and $f(x_{\lambda_2})= y_{\mu_2}$, since f is support preserving, $f(x_{\lambda_1})$and $f(x_{\lambda_2})$ have the same support.

Here $\mu_1, \mu_2 \neq 0$

Let $f(x_{\lambda_1 \wedge \lambda_2}) = y_\mu$, where $\mu = f_x(\lambda_1 \wedge \lambda_2)$

Since f_x: $L_1 \to L_2$ is order-preserving and L_1 is infinitely distributive, by Liu Ying Ming [2], f_x is finitely meet preserving.

Therefore $\mu = f_x(\lambda_1 \wedge \lambda_2) = f_x(\lambda_1) \wedge f_x(\lambda_2) = \mu_1 \wedge \mu_2$ $\neq 0$, since $L_2\ is$ regular.

$\Rightarrow \mu \neq 0$
$\Rightarrow y_\mu \neq 0$
$\Rightarrow f(x_{\lambda_1 \wedge \lambda_2}) \neq 0$
$\Rightarrow x_{\lambda_1 \wedge \lambda_2} \neq 0$

That is, if $\lambda_1, \lambda_2 \neq 0$, then $\lambda_1 \wedge \lambda_2) \neq 0$

$\Rightarrow L_1$ is regular.

Conversely, suppose L_1 is regular.

Let $A, B \in {L_1}^X$ and with the same support.

$A = \underset{x \in supp\ A}{\vee} x_{A(x)}$ and $B = \underset{x \in supp\ B}{\vee} x_{B(x)}$

$\Rightarrow f(A) = \underset{x \in supp\ A}{\vee} f(x_{A(x)})$ and $f(B) = \underset{x \in supp\ B}{\vee} f(x_{B(x)})$, Where supp A = supp B

Since L_1 is regular, by proposition 2 of Liu- Ying- Ming [12], $f(x_\lambda)=y_\mu$, where $\tilde{f}(x)$=y and $f_x(\lambda)=\mu$

$\Rightarrow f(A) = \vee_{x\in supp\ A=supp\ B}\ y_\mu$, where $\tilde{f}(x)$=y and $f_x(A(x))=\mu$, and

$f(B)= \vee_{x\in supp\ B=supp\ A}\ y_{\mu'}$, where $\tilde{f}(x)$=y and $f_x(B(x))=\mu'$

$\Rightarrow f(A)$ and $f(B)$ have the same support.

Proposition 2. Let $f: {L_1}^X \to {L_2}^Y$ be a fuzzy order homomorphism, where L_1 and L_2 are completely distributive lattices. Then f is fuzzy point preserving if and only if f is support preserving.

Proof: Let f be fuzzy point preserving. Let $f(x_1)= y_s$

Then for $0< \lambda \leq 1$, $f(x_\lambda) \leq f(x_1)= y_s$, as f is order-preserving.

$\Rightarrow f(x_\lambda) \leq y_s$

$\Rightarrow f(x_\lambda) = y_\mu$, where $\mu \leq s$

As $f(x_\lambda)$ and $f(x_{\lambda'})$ have the same support, let $f(x_\lambda) = y_\mu$ and $f(x_{\lambda'})= y_{\mu'}$.

Let A and B have the same support.

$A = \vee_{x\in supp\ A}\ x_{A(x)}$ and $B = \vee_{x\in supp\ B}\ x_{B(x)}$

$\Rightarrow f(A)= \vee_{x\in supp\ A=supp\ B}\ f(x_{A(x)})$ and

$f(B)= \vee_{x\in supp\ A=supp\ B}\ f(x_{B(x)})$

$\Rightarrow f(A)$ and $f(B)$ have same support.

$\Rightarrow f$ is support preserving.

Conversely, suppose that: ${L_1}^X \to {L_2}^Y$ be a support preserving fuzzy order homomorphism. Let $x_\lambda \in {L_1}^X$ be a fuzzy point. Then f(x_1) is a fuzzy point in ${L_2}^Y$ [15].

x_λ and x_1 have the same support{x}.

$\Rightarrow f(x_\lambda)$ and $f(x_1)$ have the same support.

Since $f(x_1)$ is a fuzzy point, and its support is a singleton set.

$\Rightarrow f(x_\lambda)$ also has the support as the singleton set.

$\Rightarrow f(x_\lambda)$ is a fuzzy point.

A necessary and sufficient condition for a fuzzy order homomorphism on an infinitely distributive lattice to be a Zadeh type function, which is a modification of the condition given by Liu Ying-Ming [12], is as follows.

Proposition 3. Let $f: L^X \to L^Y$ be a fuzzy order homomorphism, where L is infinitely distributive. Then f is a Zadeh type function if and only if f is fuzzy point preserving and f_x is the identity mapping.

Proof: Let x_λ be a fuzzy point and let f be a Zadeh-type function induced by $g: X \to Y$.

Let $g(x)$ = y

$f(x_\lambda)$(y)=∨{ x_λ(z):$g(z)$=y} = λ

If $t \neq$ y, $f(x_\lambda)\, t$ =∨{ x_λ(z) : $g(z)= t$ }

Here x_λ(z)= λ only if $z = x$, that is only if $g(z)= g(x)$, that is only if t = y.

Here $t \neq$ y

$\Rightarrow f(x_\lambda)(t)$ =0, if $t \neq$ y

Therefore $\Rightarrow f(x_\lambda)$(y)= λ and $f(x_\lambda)(t)$=0, if $t \neq$ y

$\Rightarrow f(x_\lambda)=y_\lambda$

$\Rightarrow f$ is fuzzy point preserving. Also $f_x(\lambda)= \lambda$

$\Rightarrow f_x$ is the identity mapping.

Conversely, suppose that f is fuzzy point preserving and f_x is the identity mapping. Here f is support preserving.Then for each, $x \in X$, ∃ y∈ Y such that $f(x_\lambda)=y_\mu$ holds for all λ's and corresponding μ's. Define y to be the image of x; we get the associated crisp mapping. $\tilde{f}: X \to Y$,and a mapping, $f_x: L \to L$, called restriction of f at x , defined by, $f_x(\lambda) =\mu$ if and only if f$(x_\lambda)=y_\mu$, where $\tilde{f}(x)$= y

Then $f(A)= \vee_{x\in supp\ A}$ f $(x_{A(x)}) = \vee_{x\in supp\ A}\ \tilde{f}(x)_{f_x(A(x))}$

$\Rightarrow f(A)(t) = [\vee_{x\in supp\ A}\ \tilde{f}(x)_{f_x(A(x))}](t)$

$\Rightarrow f(A)(t) = \vee_{x\in supp\ A}\ [y_{f_x(A(x))}](t)$, where y = $\tilde{f}(x)$

= ∨ $f_x(A(x))$, when t =y= $\tilde{f}(x)$ and =0, otherwise.

=∨ $A(x)$, when $t = \tilde{f}(x)$, as f_x is the identity mapping

If there exists an element z ∈ X , such that z ∉ supp A, but $\tilde{f}$(z)= y, then A (z)=0

$\Rightarrow f_x(A$ (z)) =0

Therefore, $f(A)$(t)= = ∨ $A(x)$, where $t= \tilde{f}(x)$

⇒ f is a Zadeh type function induced by the mapping $\tilde{f}$.

Proposition 4. Let $f: L^X \to L^Y$ be a bijective Zadeh type function, where L is infinitely distributive and regular, and X, Y are groups. Then, the corresponding crisp function $\tilde{f}$: $X \to Y$ (given by $\tilde{f}(x)$=y if and only if $f(x_\lambda)=y_\mu$)is a homomorphism from $X \to Y$,if and only if $f(A.B) = f(A).f(B)$

Proof: Let $f : L^X \to L^Y$ be a bijective Zadeh type function. Since Zadeh type functions are fuzzy order homomorphisms and by [12] Liu Ying–Ming, the restriction of f at x, namely f_x is the identity mapping. Also, by [12], since $f$is bijective, $\tilde{f}$ and f_x are bijective.

Here $f: L^X \to L^Y$ is induced by $\tilde{f}$ and $f_x(\lambda)=\lambda$

To prove that $\tilde{f}(l.m)= \tilde{f}(l).\tilde{f}(m)$, for all l, m in X.

Claim: $f[(l.m)_\lambda] = \tilde{f}(l).\tilde{f}(m)]_\lambda$

$f[(l.m)_\lambda] = f(l_\lambda.m_\lambda)=f(l_\lambda).f(m_\lambda)=n_\lambda.v_\lambda = (n.v)_\lambda$ $=(\tilde{f}(l).\tilde{f}(m))_\lambda$, where, $f(l_\lambda) = n_\lambda$ and $f(m_\lambda) = v_\lambda$ so that $\tilde{f}(l) = n$ and $\tilde{f}(m)= v$

$\Rightarrow \tilde{f}(l.m)= \tilde{f}(l).\tilde{f}(m)$

$\Rightarrow \tilde{f}$ is a homomorphism.

Conversely, suppose that $\tilde{f}$ is a homomorphism

$\Rightarrow \tilde{f}(l.m)= \tilde{f}(l).\tilde{f}(m)$ for all l, m in X

$f(A.B)(y) = \sup_{x\in f^{-1}(y)} (A.B)(x) = (A.B)(x)$, where $\tilde{f}(x)$=y = $\sup\min_{lm=x} (A(l),B(m))$ (1)

Also, $(f(A).f(B))(y) = \sup\min_{pq=y} (f(A)(p).f(B)(q))$

$= \sup\min_{pq=y} (\sup_{\tilde{f}(l)=p} A(l), \sup_{\tilde{f}(m)=q} B(m))$

$= \sup\min_{pq=y} (A(l),B(m))$, since $\tilde{f}$ is bijective.

$= \sup\min_{pq=y} (A(l),B(m))$.

$= \sup\min_{\tilde{f}(l).\tilde{f}(m)=y} (A(l),B(m)) = \sup\min_{\tilde{f}(l.m)=y} (A(l),B(m))$ (2)

From (1) and (2), $f(A.B) = f(A).f(B)$

4. Result and Discussion

This section assesses the effectiveness, taking into consideration a 3 x 3 m2 room with two collaborating LEDs employing SIC decoding order and a commonly designed superposition coding scheme using NOMA. Users were arranging themselves vertically and at random. In Table 1, the configuration and particular parameters used are compiled. It should be mentioned that, in this case, the LEDs' coverage zones overlap in every imaginable way. As a result, a high SNR is obtained overall. Through the use of the tight capacity lower bound, achievable rates were estimated.

The mean sum rate obtained using the GRPA, SFPA, FPA, NGDPA, and EAP techniques is displayed in Figure 4. When it comes to the average sum rate, NGDPA performs better in this case than any other strategy, with SFPA having the lowest score. In the worst scenario, the reduction in the total rate shown with SFPA is 12.8%, in contrast to the optimal plan that supports EPA fairly.

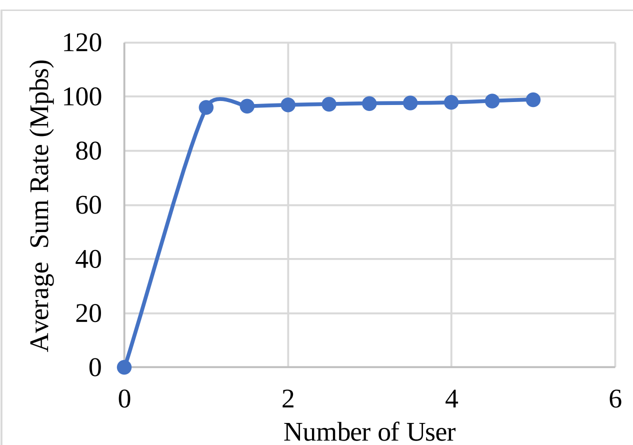

Fig. 4 Average total rate for five distinct PA techniques with a growing user base that uses NOMA

Instance (K = 8). As can be seen below, a considerably more equitable allocation of transmission capacity more than makes up for the overall system performance loss of SFPA.

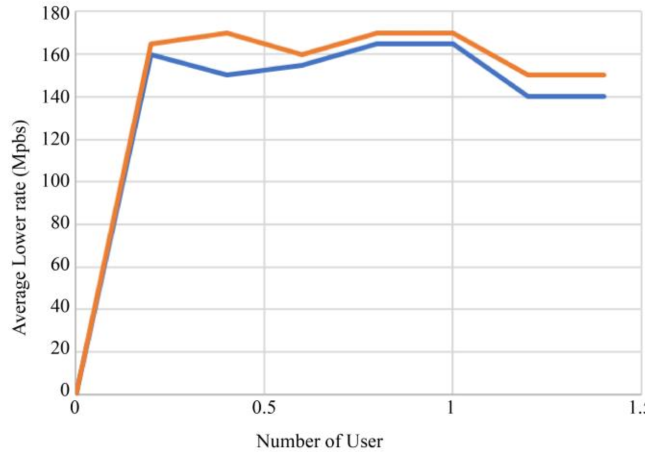

Fig. 5 Minimum rate of users in a two LED environment with strong SNR perceived while utilising NOMA with various PA techniques.

For example, when there are exactly two users, the maximum average cumulative rate that can be reached using NGDPA is 177 Mbps. The least favoured user's average rate (FPA with μ = 0.2) is 17 Mbps under the best of circumstances. as depicted in Figure 5.

Minimum rate as reported by users of NOMA with various PA and Table 2. Displays the Simulation Parameter techniques in a situation with two LEDs and high SNR SFPA results in a 168 Mbps as the average cumulative rate (a 5.1% reduction) based on an approximate 83 Mbps average rate perceived per user (a 79.5% increment).

Table 2. Simulation parameter

Transmitter Specification	Power 1: Watt Viewing Angle:60
Receiver Specification	Diameter of aperture: 5cm,180-degree field of view
Link Range(m)	20
Depth (m)	45
Coefficients of absorption, scattering, and extinction (m^{-1})	0.0508,0.2116,0.2624
Phase Function of Scattering	OTHG
The average angle of dispersion Angle s(g)	0.9470

The fairness index for each of the five PA techniques is analysed, calculated, and displayed in Figure 6. As anticipated, the order is reversed from the average total rate ranking, with the approach that provides the most equity in the transmission capacity distribution being SFPA, furthermore, regardless of the number of connected users. Table 3 presents a comparison of research papers pertaining to the design of caching strategies and performance analysis for IAB networks with cache support.

Table 3. Comparison of research on caching strategy and performance analysis

Objective	Methodology	Network
Interference mitigation.	Heuristic Algorithm.	VLC and NOMA
Power Consumption minimization.	Projected Sub gradient and stochastic analysis.	VLC
SINR analysis Throughput maximization.	Lagrange dual decomposition.	FSO
In a multi-user scenario, equitable distribution of transmission resources is ensured through the proposal of a simple, fair power allocation approach (SFPA).	Fuzzy Order Homomorphism on L-Fuzzy Sets.	VLC-NOMA Fuzzy-based demodulator

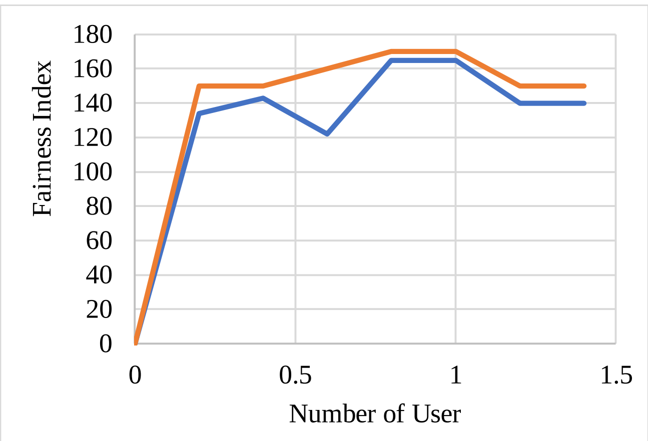

Fig. 6 Fairness index with five distinct PA techniques utilising NOMA

5. Conclusion

For NOMA-VLC systems, a novel PA method was suggested in this letter. In contrast to alternative techniques with minimal computing complexity, the suggested approach enables equitable resource allocation, irrespective of the quantity of contending users. A high transmission SNR regime is required for the suggested SFPA approach, which is a common situation for VLC applications. Additionally, the stronger user's CIS is all that is needed for the computation of the SFPA. This reduces the likelihood of estimating mistakes, in contrast to techniques that benefit from full knowledge of channel circumstances. Aspects of the endlessly distributive fuzzy order homomorphism have been examined. Fuzzy order homomorphisms that preserve fuzzy points and fuzzy supports are connected, and this has led to a modification of the requirements that must be met in order for a fuzzy order homomorphism to be considered as a function of the Zadeh type. According to the investigated setup, it is possible to divide the gearbox capacity fairly without materially lowering the performance of the system as a whole. Future NOMA-VLC multi-user systems. In order to guarantee high (in our research, up to 79.5% better) and uniform transmission rates, this implies that the SFPA approach is a good choice.

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