

Cascade Adaptive Control for Active Suspension System

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Abstract - This paper presents the active suspension system (ASS) control method using the adaptive cascade control scheme. The control scheme is implemented by two control loops, the inner control loop and outer control loop are designed respectively. The inner control loop use the pole assignment method in order to move the poles of the original system to desired poles respect to the required performance of the suspension system. To design controller in inner loop, the model without the noise caused by road profile and velocity of the car is used. The outer control loop then designed with adaptive mechanism calculates the active control force to compensate the vibrations caused by road profile and velocity of the car. The control force is determined by error between states of reference model and states of suspension systems, the reference model is the model of closed loop with inner control loop without the noise. The simulation results implemented by using the practice data of the road profile show that the capability of oscillation decrease for ASS is quite efficient.

Keywords: Suspension system; Adaptive control; Cascade control; Road excitation; Road profile; Pole assignment method.

I. INTRODUCTION

Main advantage of the ASS is capability of changing the dynamic wheel load (F_{dyn}) in order to improve the ride comfort for passengers by adjusting the characteristic of suspension system like the stiffness, the damping. The electromagnetic, hydraulic actuators mostly are used to generate the active force in the ASS. The control problem for ASS is always more challenging for researching and application in the practice because of some reasons[1, 2]: there are many ASS's states need to be controlled while ASS has only one control input, the controlled active force acts contemporaneously on both sprung mass and sprung mass, the performance bandwidth of actuator is limited, the limitations of time, the response time does not meet the requirement of high velocity, all ASS's state are not measured by sensors, the determining statistic characteristics of road excitation is the practice difficulty.

There are lots of control methods for ASS including: in the material [3], the authors used the self learning neural networks to adjust controller parameters in order to improve the ride comfort, the

complexity of neural networks and capabilities of convergence are the disadvantages of this method.

Using the Skyhook reference model and the adaptive parameter controller based on the Lyapunov method is presented in the [4].

In the [5, 6], authors applied the free-model sliding mode control with time-delay estimation to eliminate the nonlinear, uncertainty and noise acting on the system. The modal feedback method used to control ASS is written in the material [7], this method control independently in each vibration mode of ASS, the controller parameters are turned by optimal control method. Using second order optimal LQG controller for ASS is conducted in the [8], in which the ASS model is rewritten in the linear form. In the article[9], the controller based on back stepping method is used to control ASS, in this method control law is designed back step from inner loop to outer loop by using the virtual control inputs.

The simple decoupling method for ASS is implemented by the authors of [10]. Moreover, there are lots of control method applied to ASS including the Event-triggered control[11], H_∞ control method using the Linear matrix inequality, the fuzzy logic control method[12]etc...

The common feature of the above methods is the noise caused by road profile considered to be the main noise input of the mathematical model, this model is not concern about that noise which is depend on the road profile and velocity of the car. In this paper we use the general model of ASS, in that model the parameters related to the noise caused by road profile and velocity of the car are considered. From the practice test, we see that the faster velocity of the car is the small amplitude of noise is and the faster frequency is. This feature makes the noise characteristic acting to the ASS change. Based on that model of ASS, the control process for ASS is implemented by two control loops. The inner control loop is designed with noises without affecting ASS, this control loop drives the performance of ASS in the transient period. The model of closed loop system with inner control loop is used as reference model for the designing the outer adaptive control loop. The reference model is depend on the velocity of the car. The outer adaptive control loop uses the parameter adaptive mechanism adjusted by the velocity of the car and errors between ASS's states and states of reference model. The adaptive force is generated to compensate the above errors in order to drive that error reaching to zero. The simulation results using

the practice road profile show that this method is efficient to apply to the practice. The remaining part of this paper is structured as follow: Part two mentions the modelling of the suspension system. Part three introduces adaptive control for the active suspension system. Part four shows the simulation results. Finally, part five summarizes some conclusions.

II. CONTENTS

A. Modelling of the active suspension system

The active suspension depicted in Fig 1 system consists of sprung mass m_s and unsprung mass m_w , the sprung mass and unsprung mass are connected by the suspension part including damping part c_s and spring part d_s in parallel[20]. In case of active suspension, the active actuator (electrical, electromagnetic, hydric devices driven by electrical signal $u(t)$) is added to this part to change the natural characteristics of suspension part in order to get the desired comfort ride. Connection between the unsprung mass and the road surface is the wheel. The wheel is described by wheel damping part c_w and wheel spring part d_w . The states of the suspension system are deflection, velocity and accelerator of sprung mass $z_s, \dot{z}_s, \ddot{z}_s$, unsprung mass $z_w, \dot{z}_w, \ddot{z}_w$, respectively. The z_r is the road excitation supported as the main input noise of the system, z_r depends on the road profile and velocity of the car $v(t)$.

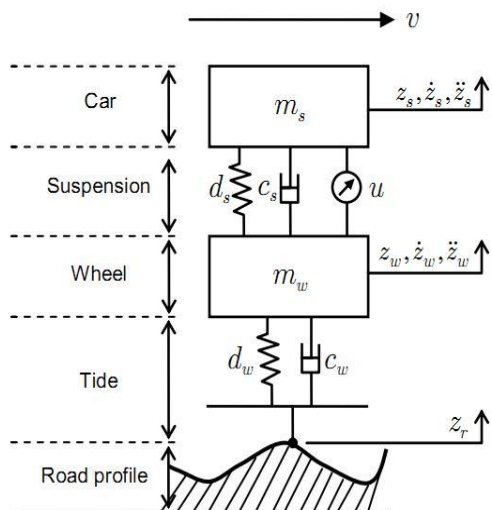


Fig. 1: Physical model of a typical suspension system

The performance quality of the suspension system is reflected upon the comfort ride, this quality is presented by 3 parameters: the accelerations of the car and the wheel \ddot{z}_s, \ddot{z}_w respectively, the dynamic wheel load F_d and the suspension

deflection $z_s - z_w$. The dynamic wheel load F_d is calculated as equation (1) as [13]

$$F_d = c_w (z_r - z_w) + d_w (\dot{z}_r - \dot{z}_w) \text{ [N]} \quad (1)$$

The dynamic wheel load F_d depends on the mass of the car m_c , velocity of the car v and road profile. The smaller values of $\ddot{z}_s, \ddot{z}_w, F_d$ are, the higher comfort rider of the passengers in the car is Fig.1.

The road excitation always is supposed as the main input noise, but we realize that the z_r not only depends on the road profile but also the velocity of the car. For the same road profile, as the velocity of the car changes, the stochastic characteristics of the z_r will be changed.

Figure 2 shows the varying of the z_r in the case velocity of the car changing from 5km/h to 45km/h for the same road profile. The road profile has the stochastic characteristics as: min=-2.319cm, max=2.348cm, mean=0.0131; median=0.0131; variance=0.331678, $\mu = 0.0131, \sigma = 0.576$. From figure 4 we see that the velocity of the car is higher, the amplitude of the variation z_r is lower. The maximum of z_r is 0.07m in the case $v(t) = 5\text{km/h}$, the maximums of z_r are 0.023m and 0.02m in the cases $v(t) = 15\text{km/h}$ and 45km/h , respectively. The changing of stochastic characteristics z_r by the velocity of the car, road profile will affect strongly to the state estimation process.

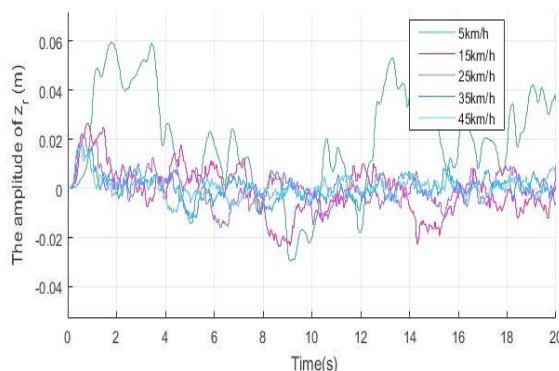


Fig. 2: The varying of the stochastic characteristics for the same road profile by the velocity of the car

The first order model of the road excitation z_r as the function of the road profile and velocity of the car depicted in the Figure 3 can be written by the equation as [14, 15]:

$$\frac{dz_r}{dt} = -\lambda v z_r + w_{zr} \quad (2)$$

Where λ, v are the optional feedback parameter and velocity of the car, respectively, w_{zr} is the input noise caused by the road profile.

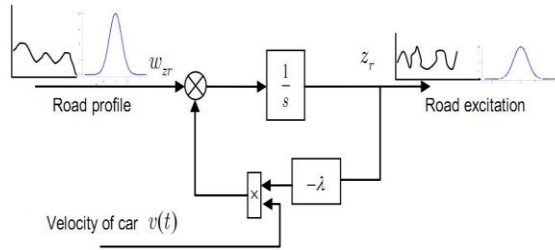


Fig. 3: The first order model of road excitation z_r as the function of the road profile and velocity of the car

Let consider the state vector of the suspension system described in the Figure 1 as

$$z_1 = z_s, z_2 = \dot{z}_s, z_3 = z_w, z_4 = \dot{z}_w, z_5 = z_r, z_6 = \dot{z}_r$$

$$\underline{z} = [z_1 \ z_2 \ z_3 \ z_4 \ z_5 \ z_6]^T \tag{3}$$

Suppose $u(t)$ as the active force, if $u(t) = 0$ system is called the passive suspension system. The model of the suspension system is written by following equation.

$$\frac{dz}{dt} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-c_s}{m_s} & \frac{-d_s}{m_s} & \frac{c_s}{m_s} & \frac{d_s}{m_s} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{c_s}{m_w} & \frac{d_s}{m_w} & \frac{-c_w - c_s}{m_w} & \frac{-d_w - d_s}{m_w} & \frac{c_w}{m_w} & \frac{d_w \lambda w(t)}{m_w} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\lambda w(t) & 0 \end{bmatrix}}_{\mathbf{A}(v(t))} \underline{z} + \underbrace{\begin{bmatrix} 0 \\ 1 \\ \frac{-1}{m_w} \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{B}} u(t) + \underbrace{\begin{bmatrix} w_{zs} \\ w_{dzs} \\ w_{zw} \\ w_{dzw} \\ w_{zr} \\ w_{d zr} \end{bmatrix}}_{\mathbf{w}(t)} \tag{4}$$

The equation **Error! Reference source not found.** is written briefly as the equation **Error! Reference source not found.**

$$\frac{dz}{dt} = \mathbf{A}(v(t)) \underline{z} + \mathbf{B}u(t) + \mathbf{w}(t) \tag{5}$$

By modifying the equation (5) as

$$\frac{dz}{dt} = \mathbf{A}(v(t)) \underline{z} + \underbrace{(\mathbf{B} \ I_n)}_{\tilde{\mathbf{B}}} \begin{bmatrix} u(t) \\ \mathbf{w}(t) \end{bmatrix}, \quad n = 6$$

$$\frac{dz}{dt} = \mathbf{A}(v(t)) \underline{z} + \tilde{\mathbf{B}} \left[\begin{bmatrix} u(t) \\ \mathbf{0}_n \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{w}(t) \end{bmatrix} \right] \tag{6}$$

$$= \mathbf{A}(v(t)) \underline{z} + \tilde{\mathbf{B}} [\underline{v}(t) + \underline{d}(t)]$$

the system has the input $\underline{u}(t)$

$$\underline{v}(t) = \begin{pmatrix} u(t) \\ \mathbf{0}_n \end{pmatrix} \Rightarrow u(t) = \begin{pmatrix} 1 & \mathbf{0}_n^T \end{pmatrix} \underline{v}(t)$$

and noise vector is $\underline{d}(t) = \begin{pmatrix} \mathbf{0} \\ \mathbf{w}(t) \end{pmatrix}$

The matrix $\mathbf{A}(v(t))$ has six eigenvalues located in the left of imaginary axis which make 03 pairs of eigenvalues that is symmetric with respect to the imaginary axis. Two first of them are fixed and have the negative real part, the last pair is depend on the velocity of the car. The velocity of the car increasing makes the imaginary part of third pair of eigenvalues move along the imaginary axis far from the root as described in the Fig. 4.

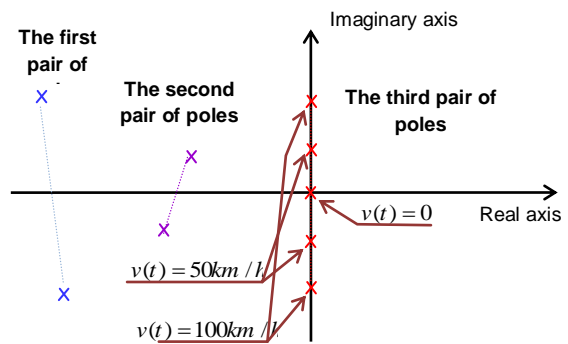


Fig. 4: The locations of pole pairs of the ASS

B. Adaptive control for the active suspension system

The adaptive control for ASS is designed by two steps as follow:

Step 1: Design the state feedback controller in order to keep closed loop system without noise $\underline{d}(t) = 0$ having the desired dynamic.

The pole placement is the technique which is used to place the closed loop poles of the system in pre determined locations. Based on the desired dynamic wheel load and the desired dynamic of z_w, \dot{z}_w the pre determined locations of the poles are calculated. The results of step 1 is the state feedback controller which have the gain depending on the velocity of the car and the road profile. This is inner control loop.

Step 2: Design the adaptive controller in the outer control loop to attenuate the noise caused by road profile and velocity of the car. This control loop use the reference model to be the model of inner loop without noise $\underline{d}(t)$, the dynamic of the reference model is depend on the velocity of the car and road profile. Using state error between the ASS and reference model, the adaptive mechanism calculates the active force to compensate the noise caused by road profile and velocity of the car. The adaptive mechanism gain is varied road condition and velocity of the car.

1) *Designing the inner control loop*

To design the controller in the inner control loop we suppose that velocity of the car is zero, so $\mathbf{w}(t) = \underline{0}_n$ or $\underline{d}(t) = \underline{0}_{n+1}$. Consider that all states of ASS $\underline{z}(t)$ are measurable. Therefore the model of ASS can be written as equation (7):

$$\frac{d\underline{z}}{dt} = \mathbf{A}(v(t))\underline{z} + \tilde{\mathbf{B}}\underline{v}(t) \quad (7)$$

So the designing is to determine the matrix R to guarantee that $\underline{v}(t) = \underline{\eta}(t) - R\underline{z}$ makes the model of closed loop system become

$$\frac{d\underline{z}}{dt} = (\mathbf{A}(v(t)) - \tilde{\mathbf{B}}R)\underline{z} + \tilde{\mathbf{B}}\underline{\eta}(t) = \tilde{\mathbf{A}}(v(t))\underline{z} + \tilde{\mathbf{B}}\underline{\eta}(t) \quad (8)$$

and the poles of dynamic system (8) s_1, s_2, \dots, s_n are pre determined locations with respect to the desired performance of the ASS [16, 17]. So the equation (9) needs to be solved.

$$\det(sI - \mathbf{A}(v(t)) + \tilde{\mathbf{B}}R) = (s - s_1)(s - s_2) \dots (s - s_n) \quad (9)$$

Note that the matrix R is depend on the velocity of the car $v(t)$. One of the methods to solve the equation (9) is modal method in the material[18]. This method uses the regression calculation process. If m is rank of matrix $\tilde{\mathbf{B}}$ and $\lambda_1, \lambda_2, \dots, \lambda_n$ are eigenvalues of matrix \mathbf{A} then the regression calculation process is implemented by n/m steps, on each step the matrix R_k is calculated to move m poles of \mathbf{A} among $\lambda_1, \lambda_2, \dots, \lambda_n$ to m new poles among s_1, s_2, \dots, s_n of matrix $\tilde{\mathbf{A}} = \mathbf{A} - \tilde{\mathbf{B}}R$. The matrix R then is calculated by sumaring matrices R_k

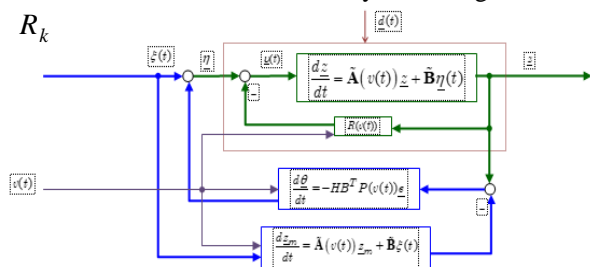


Fig. 5: The diagram of two control loops for ASS: inner control loop (green line), outer adaptive control loop (blue line).

2) *Designing the outer adaptive control loop*

Back to the ASS in presence of noise $\mathbf{w}(t)$ caused by road profile and velocity of the car (6), with the inner control loop the ASS model becomes

$$\frac{d\underline{z}}{dt} = \tilde{\mathbf{A}}(v(t))\underline{z} + \tilde{\mathbf{B}}[\underline{\eta}(t) + \underline{d}(t)] \quad (10)$$

in which $\tilde{\mathbf{A}}$ is stable matrix having desired poles s_1, s_2, \dots, s_n . We have the Lyapunov equation (11)

$$\tilde{\mathbf{A}}(v(t))^T P + P \tilde{\mathbf{A}}(v(t)) = -Q \quad (2)$$

If symmetric matrix Q is positive definite, the equation (11) always exists a unique positive definite symmetric matrix $P(v(t))$ [19].

We use the reference model described by equation (12)

$$\frac{d\underline{z}_m}{dt} = \tilde{\mathbf{A}}(v(t))\underline{z}_m + \tilde{\mathbf{B}}\underline{\xi}(t) \quad (3)$$

Designing the outer adaptive control loop is to guarantee the system (10) tracking to the system (12). Suppose $\underline{d}(t) = \underline{\theta}$, we have

$$\frac{d\underline{z}}{dt} = \tilde{\mathbf{A}}(v(t))\underline{z} + \tilde{\mathbf{B}}(\underline{\xi}(t) + \underline{\theta}) \quad (4)$$

So the task of noise attenuation adaptive mechanism is to adjust $\underline{\theta}$ in order to make the error $\underline{e} = \underline{z} - \underline{z}_m$ tend to zero, in which

$$\frac{d\underline{e}}{dt} = \tilde{\mathbf{A}}(v(t))\underline{e} + \tilde{\mathbf{B}}\underline{\theta} \quad (5)$$

The diagram of two control loops for ASS is shown in the Figure 6.

Use the positive-definite Lyapunov candidate function as follow

$$V(\underline{e}, \underline{\theta}) = \underline{e}^T P \underline{e} + \underline{\theta}^T H^{-1} \underline{\theta} \quad (6)$$

in which matrix P is root of Lyapunov function (15) and H is the positive-definite symmetric matrix chosen arbitrarily. So to keep $dV/dt < 0$ or $\underline{e} \rightarrow \underline{0}$ for all $\underline{e} \neq \underline{0}$ the vector $\underline{\theta}$ needs to be satisfy equation (16)

$$\left[\tilde{\mathbf{B}}^T P \underline{e} + H^{-1} \frac{d\underline{\theta}}{dt} \right] = \underline{0} \quad (16)$$

Therefore the noise attenuation adaptive mechanism is

$$\frac{d\underline{\theta}}{dt} = -H \tilde{\mathbf{B}}^T P(v(t)) \underline{e} \quad (7)$$

Some remarks

- Function dV/dt is negative-definite with respect to \underline{e} therefore $\underline{\theta} \neq \underline{d}$. In another word, adaptive mechanism (19) is the noise attenuation mechanism, it is not identification function of noise caused by road profile and velocity of the car.

- The velocity of tending to zero of error \underline{e} is depend on the positive -definite matrix Q .

- Larger the norm of matrix H is, the faster error \underline{e} tend to zero does. Larger eigenvalues of matrix H are, the larger amplitude of $\underline{\theta}$ is, this causes the oscillation in the system

C. *The simulation results using the practical road profile data*

The simulation results below is conducted in two cases of that are even road profile and stochastic practice road profile. The physical parameters of ASS are

$$\lambda = 0.2, c_c = 8399[N/m]; c_w = 151176[N/m],$$

$$d_c = 6665[N \text{ sec}/m], d_w = 500[N \text{ sec}/m]$$

$$m_c = 194.2[kg], m_w = 49[kg]$$

1) The simulation results for even road profile

The initial condition of states is [0;0.2;0;0.2], the amplitude of event road profile is 0.02m as described in the Fig. 6.

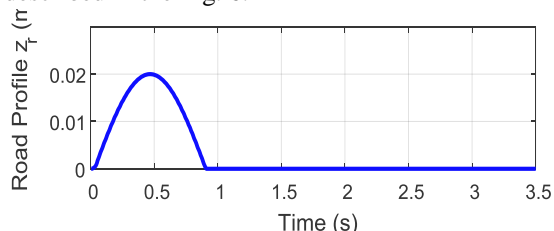


Fig. 6: Event road profile

The varying of position, velocity of sprung mass and unsprung mass is depicted in the Figure 7. In this figure the passive case, the pole assign case and the adaptive case are compared with. In the case of pole assign, the number of oscillations of position and velocity of sprung mass and unsprung mass is decrease but the amplitude of oscillation is increase in comparing to passive case. In the case of adaptive control, the amplitude and number of oscillations are both decrease.

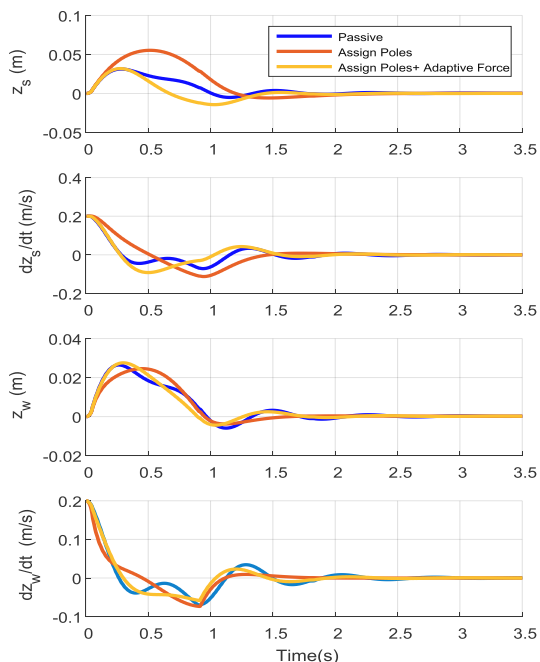


Figure 7: Response of position, velocity of the sprung mass and unsprung mass in the case of event road profile

2) The simulation results for stochastic practice road profile

Probability density function of the stochastic practice road profile is min = -2.319cm, max=2.348cm, mean=0.0131cm; meadian=0.013cm; variance =0.331678cm.

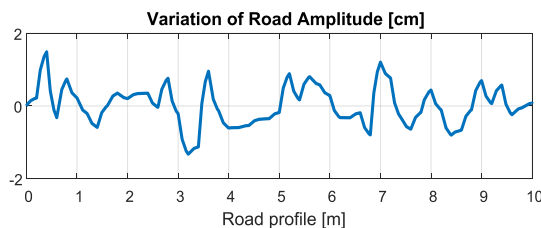


Fig.8: Stochastic practice road profile

The simulation results are conducted for three velocities of the car 5km/h, 40km/h and 80km/h. Figure 9 and Figure 10 are depicted the simulation oscillation responses of ASS, the dynamic wheel load and active force, respectively. In these figure, the active control responses is compared to the passive case. Similarly, Figure 11 and Figure 12 show the simulation results the case that velocity of the car is 40km/h, Figure 13 and Figure 14 plot the results for 80km/h.

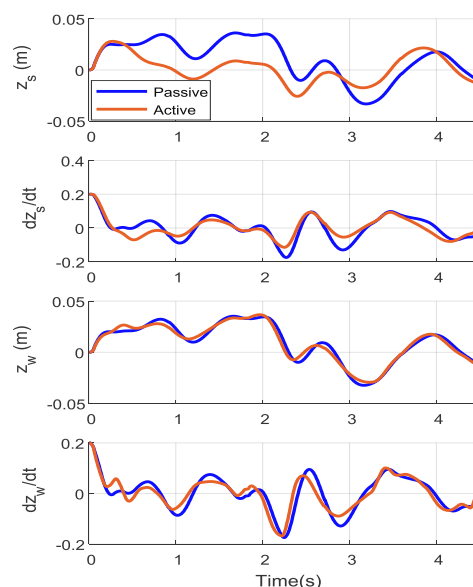


Fig. 9: Position and velocity of sprung mass and unsprung mass for cases: passive and adaptive control at 5km/h (velocity of the car)

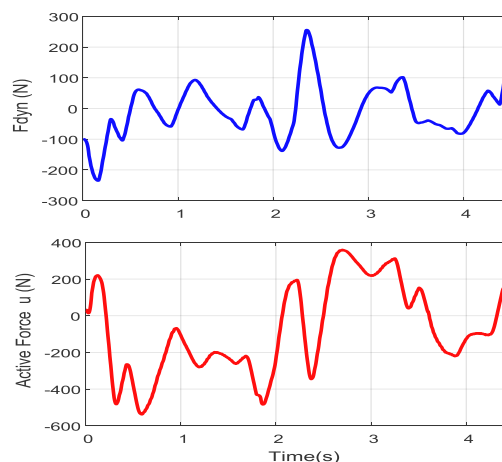


Fig.10: The dynamic wheel load and adaptive control force at 5km/h (velocity of the car)

At 5km/h of velocity of the car, we see that the oscillation and amplitude of oscillation the sprung mass is effectively decrease as shown in the Figure 9. The active control force acts to the system clearly at 0.5s from beginning. Efficiency of the oscillation decrease of the car is better than the wheel.

Reducing the car oscillation is more efficient than reducing the wheel oscillation. The dynamic load varies in range $\pm 300\text{N}$, the active control force ranges from -600N to 400N .

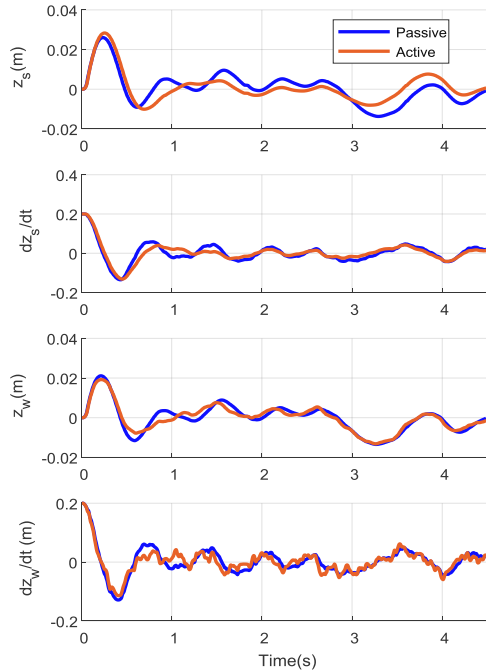


Fig.11: Position and velocity of sprung mass and unsprung mass for cases: passive and adaptive control at 40km/h (velocity of the car)

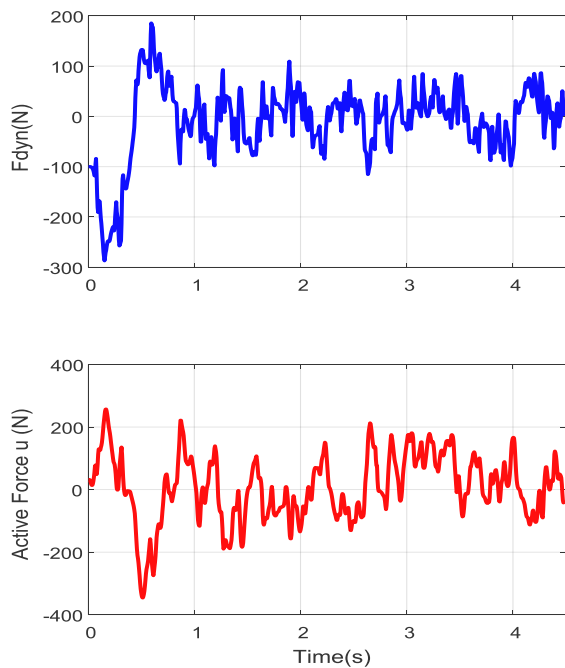


Fig.12: The dynamic wheel load and adaptive control force at 40km/h (velocity of the car)

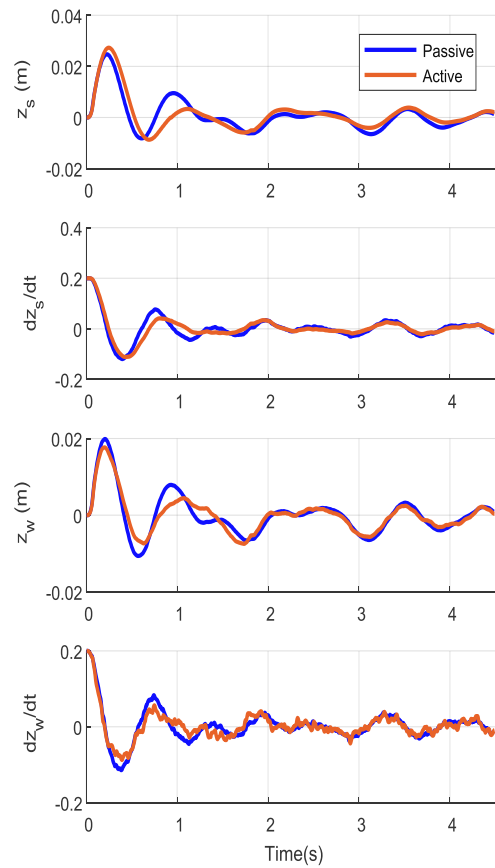


Fig.13: Position and velocity of sprung mass and unsprung mass for cases: passive and adaptive control at 80km/h (velocity of the car)

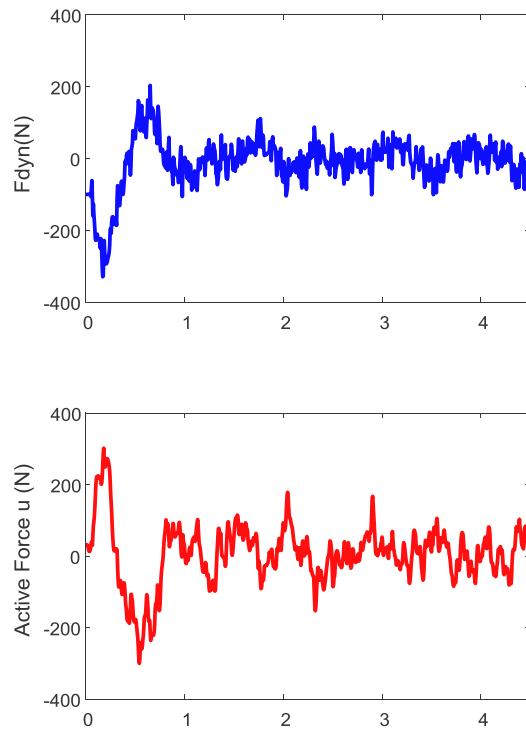


Fig.14: The dynamic wheel load and adaptive control force at 80km/h (velocity of the car)

At car speed of 40km/h with active adaptive control, the car oscillation decrease is quite good in comparison to the passive case. For first 0.5 seconds of the adaptive control process, the dynamic wheel load varies in the range -300N to 200N, and after that the dynamic wheel load decreases, it ranges in ± 100 N. The active control force varies from 400N to 400N for first 0.5 seconds and it varies in the range ± 200 N for after that. At car speed of 80km/h, the simulation results are the same at 40km/h. Therefore, by using the cascade adaptive control for active suspension system, the oscillations of the car and the wheel are decreased effectively, it is the same case with the dynamic load.

III. CONCLUSIONS

The paper introduces the cascade adaptive control method for active suspension system. The inner control loop uses the pole placement technique which is used to place the closed loop poles of the system without noise in pre determined locations in order to change the dynamic characteristics of ASS to desired performance requirements. The outer control loop uses the adaptive control strategy with the adaptive mechanism to compensate the oscillation of the car caused by road profile and velocity of the car acting to ASS

The simulation results using even road profile and stochastic practice road profile show that the capability of oscillation decrease for the car and the wheel of ASS is quietly efficient. The dynamic wheel load is decreased so the ride comfort of passengers is better.

Funding Data

This research is funded financially by the Vietnam ministry of education and training under grant No. B2016-TNA-06References (Size 10 & Bold)

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