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Original Article

Applying Second-Order Padé Approximation in Optimal Control Problem for a Distributed Parameter System with Delayed-Time

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***Abstract** - This paper gives a solution of an optimal control problem for a distributed parameter system with delayed-time (DPSDT), governed by a heat-conduction equation, using the numerical method. In which, the delayed object $e^{-\tau s}$ is replaced by using second-order Padé approximation model (Padé-2). The system is also applied to a specific one-sided heat-conduction system in a heating furnace to control temperature for the objects which have flat-slab shape following the most accurate burning standards [2,6,9,10,11,12]. The aim of problem is also to find an optimal control signal (optimal voltage) so that the error between the distribution of real temperature of the object and the desired temperature is minimum after a given period of time t_f [2,6,9,10,11,12].*

To verify the solution of the problem, the author have proceeded to run the simulation programs on a flat-slab of Carbon steel and a flat-slab of Diatomite.

Keywords - *Optimal control, Distributed parameter systems, Delay, Numerical method, Padé approximation*

I. INTRODUCTION

Theoretically, Padé approximation model has been studied for a long time and mainly applied in finding solutions of differential algebraic equations.

Padé approximaton model can offer a function approximation having more advantages than Taylor approximation, especially with the objects having large time delay compared to its time constant [5], [9].

The paper will continue to be developed in number of previous papers as in [9], [12]. In this paper, author will replace a delayed object $e^{-\tau s}$ by using Padé-2 approximation model in order to solve the problem of optimal control for a DPSDT, typically a controlled object is also described by heat transfer equation, which is one of the physical processes with distributed parameters.

II. THE PROBLEM OF OPTIMAL CONTROL

A. The object model

As a typical distributed parameter system, the one-sided heat conduction system is considered. The process of one-sided heating of the objects which have flat-slab shape in a furnace is described by the parabolic-type partial differential equation, as follows in [2,6,9,10,11,12]:

$$a\frac{\partial^2 q(x,t)}{\partial x^2} = \frac{\partial q(x,t)}{\partial t} \tag{1}$$

where *q(x,t)*, the temperature distribution in the object, is the output needing to be controlled, depending on the spatial coordinate x with $0 \le x \le L$ and the time t with $0 \le t \le t_f$, a is the temperature-conducting factor (m^2/s), L is the thickness of object (m), t_f is the allowed burning time (s)

The initial and boundary conditions are given in [2,6,9,10,11,12]:

$$q(x,0) = q_0(x) = const \tag{2}$$

$$\lambda\frac{\partial q(x,t)}{\partial x}\bigg|_{x=0} = \alpha\left[q(0,t) - v(t)\right] \tag{3}$$

$$\frac{\partial q(x,t)}{\partial x}\bigg|_{x=L} = 0 \tag{4}$$

with α as the heat-transfer coefficient between the furnace space and the object ($W/m^2.^0C$), λ as the heat-conducting coefficient of material ($W/m.^0C$), and *v(t)* as the temperature of the furnace respectively (^0C).

The temperature *v(t)* of the furnace is controlled by voltage *u(t)*, the temperature distribution *q(x,t)* in the object is controlled by means of the fuel flow *v(t)*, this temperature is controlled by voltage *u(t)*. Therefore, the temperature distribution *q(x,t)* will depend on voltage *u(t)*.

The relationship between the provided voltage for the furnace *u(t)* and the temperature of the furnace *v(t)* is usually the first order inertia system with delayed-time as in [1,2,6,9,10,11,12]:

$$T.\dot{v}(t) + v(t) = k.u(t-\tau) \tag{5}$$

where T is the time constant, τ is the time delay; k is the static transfer coefficient; *v(t)* is the temperature of the furnace and *u(t)* is the provided voltage for the furnace (controlled function of the system).

B. The objective function and the constrained conditions

In this case, the problem is set out as follows: we have to determine a control function *u(t)* with $(0 \le t \le t_f)$ so as to minimize the temperature difference between the distribution of desired temperature $q^*(x)$ and real temperature of the object $q(x,t_f)$ at time $t = t_f$. It means at the end of the heating process to ensure temperature uniformity throughout the whole material:

$$J\left[u(t)\right] = \int_0^L \left[q^*(x) - q(x,t_f)\right]^2 dx \to \min \tag{6}$$

The constrained conditions of the control function is:

$$U_1 \le u(\mathrm{t}) \le U_2 \tag{7}$$

with U_1, U_2 are the lower and upper limit of the supply voltage respectively (V). This problem is called the most accurate burning problem.

III. THE SOLUTION OF PROBLEM

The process of finding the optimal solution includes 2 steps:

- S*tep 1*: Find the relationship between $q(x,t)$ and the control signal $u(t)$. Namely, we have to solve the equation of heat transfer (relationship between $v(t)$ and $q(x,t)$) with boundary condition type-3 combined with ordinary differential equation with time delay (relationship between $u(t)$ and $v(t)$)

- S*tep 2*: Find the optimal control signal $u^*(t)$ by substituting $q(x,t)$ found in the first step into the function (6), after that finding optimal solution $u^*(t)$.

A. Find the relationship between q(x,t) and the control signal u(t)

To solve the partial differential equation (1) with the initial and the boundary conditions (2), (3), (4), we apply the Laplace transformation method with the time parameter t. On applying the transform with respect to t, the partial differential equation is reduced to an ordinary differential equation of variable x. The general solution of the ordinary differential equation is fitted to the boundary conditions, and the final solution is obtained by the application of the inverse transformation.

Transforming Laplace (1), we obtained:

$$a\frac{\partial^2 Q(x,s)}{\partial x^2} = sQ(x,s) \tag{8}$$

where: $Q(x,s) = L\{q(x,t)\}$

After transforming the boundary conditions (3), (4), we have:

$$\lambda \left.\frac{\partial Q(x,s)}{\partial x}\right|_{x=0} = \alpha\left[Q(0,s) - V(s)\right] \tag{9}$$

$$\left.\frac{\partial Q(x,s)}{\partial x}\right|_{x=L} = 0 \tag{10}$$

From Eq. (5), assuming the delayed object satisfy the condition: $2 \le T/\tau < 6$ in [5], [9]. To solve this problem, the first order inertia system with delayed time is replaced by second-order Padé appximation (Padé-2) Transforming Laplace (5), we obtained:

$$(Ts+1)V(s) = k.U(s).\mathrm{e}^{-\tau s} \tag{11}$$

In which, the delayed object $\mathrm{e}^{-\tau s}$ is replaced by Padé-2.

with
$$e^{-\tau s} \approx \frac{1 - \dfrac{\tau s}{2} + \dfrac{\tau^2 s^2}{12}}{1 + \dfrac{\tau s}{2} + \dfrac{\tau^2 s^2}{12}}$$

or
$$e^{-\tau s} \approx \frac{12 - 6\tau s + \tau^2 s^2}{12 + 6\tau s + \tau^2 s^2} \tag{12}$$

(11) becomes:

$$(Ts+1)V(s) = k.U(s).\frac{12 - 6\tau s + \tau^2 s^2}{12 + 6\tau s + \tau^2 s^2} \tag{13}$$

$$\text{where: } V(s) = \mathbf{L}\{v(t)\}\ ;\ \ U(s) = \mathbf{L}\{u(t)\} \tag{14}$$

The general solution of (1) is:

$$Q(x,s) = A(s).e^{\sqrt{\frac{s}{a}}.x} + B(s).e^{-\sqrt{\frac{s}{a}}.x} \tag{15}$$

where: $A(s); B(s)$ are the parameters need to be find.
After transforming, we have the function:

$$Q(x,s) = \frac{U(s).k.\left(12 - 6\tau s + \tau^2 s^2\right)\left[e^{-\sqrt{\frac{s}{a}}.(L-x)} + e^{\sqrt{\frac{s}{a}}.(L-x)}\right]}{\left(Ts+1\right)\left(12 + 6\tau s + \tau^2 s^2\right)\left\{\left[e^{-\sqrt{\frac{s}{a}}.L} + e^{\sqrt{\frac{s}{a}}.L}\right] - \lambda.\dfrac{\sqrt{\frac{s}{a}}}{\alpha}.\left[e^{-\sqrt{\frac{s}{a}}.L} - e^{\sqrt{\frac{s}{a}}.L}\right]\right\}} \tag{16}$$

Putting

$$G(x,s) = \frac{k.\left(12 - 6\tau s + \tau^2 s^2\right)\left[e^{-\sqrt{\frac{s}{a}}.(L-x)} + e^{\sqrt{\frac{s}{a}}.(L-x)}\right]}{\left(Ts+1\right)\left(12 + 6\tau s + \tau^2 s^2\right)\left\{\left[e^{-\sqrt{\frac{s}{a}}.L} + e^{\sqrt{\frac{s}{a}}.L}\right] - \lambda.\dfrac{\sqrt{\frac{s}{a}}}{\alpha}.\left[e^{-\sqrt{\frac{s}{a}}.L} - e^{\sqrt{\frac{s}{a}}.L}\right]\right\}} \tag{17}$$

We have: $Q(x,s) = G(x,s).U(s)$ (18)

From (18), according to the convolution theorem, the inverse transformation of (18) is given by

$$q(x,t) = g(x,t) * u(t) \tag{19}$$

We can write

$$q(x,t) = \int_0^t g(x,\tau).u(t-\tau)d\tau \tag{20}$$

or
$$q(x,t) = \int_0^t g(x,t-\tau).u(\tau)d\tau \tag{21}$$

where
$$g(x,t) = \mathbf{L}^{-1}\{G(x,s)\} \tag{22}$$

Therefore, if we know the function $g(x,t)$, we will be able to calculate the temperature distribution $q(x,t)$ from control function $u(t)$. To find $q(x,t)$ in (19), we need to find the function (22). Using the inverse Laplace transformation of function $G(x,s)$ we have the following result:

$$g(x,t)=\frac{k.k_0^2\left(12+6\tau k_0^2+\tau^2k_0^4\right).\cos\left(\frac{k_0}{\sqrt{a}}(L-x)\right)}{\left(12-6\tau k_0^2+\tau^2k_0^4\right).\left[\cos\left(\frac{k_0L}{\sqrt{a}}\right)-\frac{\lambda k_0}{\alpha\sqrt{a}}\sin\left(\frac{k_0L}{\sqrt{a}}\right)\right]}.e^{-k_0^2t}+$$
$$+\frac{k.\left(12+6\tau k_1^2+\tau^2k_1^4\right).\cos\left(\frac{k_1}{\sqrt{a}}(L-x)\right)}{2.\left(1-Tk_1^2\right)\left(3\tau-\tau^2k_1^4\right)\left[\cos\left(\frac{k_1L}{\sqrt{a}}\right)-\frac{\lambda k_1}{\alpha\sqrt{a}}\sin\left(\frac{k_1L}{\sqrt{a}}\right)\right]}.e^{-k_1^2t}+$$
$$+\frac{k.\left(12+6\tau k_2^2+\tau^2k_2^4\right).\cos\left(\frac{k_2}{\sqrt{a}}(L-x)\right)}{2.\left(1-Tk_2^2\right)\left(3\tau-\tau^2k_2^4\right)\left[\cos\left(\frac{k_2L}{\sqrt{a}}\right)-\frac{\lambda k_2}{\alpha\sqrt{a}}\sin\left(\frac{k_2L}{\sqrt{a}}\right)\right]}.e^{-k_2^2t}+$$
$$+\sum_{i=3}^{\infty}\frac{2\alpha.k\left(12+6\tau.\Psi_i^2+\tau^2\Psi_i^4\right)\cos\left(\frac{\Psi_i}{\sqrt{a}}(L-x)\right)}{\lambda\left(1-T\Psi_i^2\right)\left(12-6\tau.\Psi_i^2-\tau^2\Psi_i^4\right)\left[\frac{\lambda+\alpha L}{\lambda.\Psi_i\sqrt{a}}\sin\left(\frac{\Psi_i.L}{\sqrt{a}}\right)+\frac{L}{a}\cos\left(\frac{\Psi_i.L}{\sqrt{a}}\right)\right]}.e^{-\Psi_i^2t} \quad (23)$$

$$k_0=\frac{1}{\sqrt{T}};\; k_1^2=\frac{3-\sqrt{3j^2}}{\tau}$$
$$k_2^2=\frac{3+\sqrt{3j^2}}{\tau};$$

where Ψ_i is calculated from the formula:

$$\Psi_i=\phi_i\sqrt{a}\,/\,L \quad (24)$$

- ϕ_i is the solution of the equation:

$$\phi.tg\phi=\alpha L/\lambda=B_i \quad (25)$$

- *Bi* is the coefficient BIO of the material.
- α is the heat-transfer factor (W/m^2.^{0}C)
- λ is the heat-conducting factor of object (W/m.^{0}C)
- *L* is the thickness of object (m),
- *a* is the temperature-conducting factor (m^2/s)
- τ is the delayed time of the furnace (s)
- *T* is the time constant of the furnace (s)
- *k* is the static transfer coefficients of the furnace

Conclusions:

We have also solved a system of parabolic-type partial differential equation with boundary conditions of type-3 (the relationship between *v(t)* and *q(x,t)*) combined with the ordinary differential equation with time delay (the relationship between *u(t)* and *v(t)*).

The relationship between the supplied voltage for the furnace u(t) and the temperature field distribution in the object q(x,t):

$$q(x,t)=g(x,t)*u(t)=\int_0^t g(x,t-\tau).u(\tau)d\tau \quad (26)$$

with t_f is the allowed burning time (s).

B. Find the optimal control signal $u^*(t)$ by using numerical method

To find the $u^*(t)$, we have to minimize the objective function (6).
It means:

$$\mathrm{J}\left[u(t)\right]=\int_0^L\left[q^*(x)-q(x,t_f)\right]^2dx\rightarrow\min \quad (27)$$

where

$$q(x,t_f)=\int_0^{t_f}g(x,t_f-\tau).u(\tau)d\tau \quad (28)$$

and $q^*(x)$ is the desired temperature distribution; $q(x,t_f)$ is the real temperature distribution of the object at time $t = t_f$.
As calculated in [2,6,9,10,11,12], in order to solve this problem, we also use the integral numerial method.
Thus, the optimal control problem is here to find u^*_j in order to minimize the objective function:

$$\mathrm{J}[u^*]=L\sum_{i=0}^{n}\xi_i\left[q^*(x_i)-q(x_i,t_f)\right]^2 \quad (29)$$

where ξ_i are the weights assigned to the values of integrand at the points x_i. The values of x_i and the weights ξ_i are known for each integration formula.

If the Simpson's composite formula is used, the values of x_i and ξ_i are given by in [2], [6],[10].

$$\left.\begin{array}{l}x_i=L_i/n;(i=0,1,...,n)\\ \xi_0=\xi_n=1/3n\\ \xi_1=\xi_3=\xi_{n-1}=4/3n\\ \xi_2=\xi_4=\xi_{n-2}=2/3n\end{array}\right\}\; n \text{ is an even number}$$

Therefore, $q(x_i,t_f)$ is calculated:

$$q(x_i,t_f)\cong t_f\sum_{j=0}^{m}\xi_jg(x_i,t-\tau_j).u(\tau_j) \quad (30)$$

the values of τ_j and ξ_j are given by in [2],[10],[11].

$$\left.\begin{aligned}&\tau_j = jt_f / m\\&\xi_0 = \xi_m = 1/3m\\&\xi_1 = \xi_3 = \xi_{m-1} = 4/3m\\&\xi_2 = \xi_4 = \xi_{m-2} = 2/3m\end{aligned}\right\} \quad (j = 0,1,2,...,m)$$

Putting

$$c_{ij} = t_f.\xi_j.g(x_i, t-\tau_j);\ u(\tau_j) = u_j;\ q^*(x_i) = q_i^* \quad (31)$$

Substituting (30) and (31) into (29), we obtained:

$$J[u^*] = L\sum_{i=0}^{n}\xi_i\left[q_i^* - \sum_{j=0}^{m}c_{ij}.u_j\right]^2 \quad (32)$$

The constrained conditions of the control function (Limit of the supplied voltage for the furnace) are described as follows:

$$U_1 \le u_j \le U_2 \ (j = 0 \div m) \quad (33)$$

The performance index (32) is a quadratic function of the variables u_j with constraints (33) are linear, the problem becomes a quadratic programming problem. This problem can be obtained by using numerical method after a finite number of iterations of computation.

Although a solution of the quadratic programming problem is obtained after a finite number of iterations of computation, but its algorithm is more complicated than that of the simplex method for linear programming. If the performance index is taken as

$$J = \int_0^L \left|q^*(x) - q(x, t_f)\right| dx \quad (34)$$

instead of (32), the linear programming technique can be used directly. On applying the same procedure as mentioned above, the approximate performance index corresponding to (32) is written as

$$J \cong \overline{J} = L\sum_{i=0}^{n}\xi_i\left|q_i^* - \sum_{j=0}^{m}c_{ij}.u_j\right| \quad (35)$$

The problem of minimizing (35) under the constraints (33) can be put into a linear programming form by using known techniques [2],[9],[10],[11].

Thus, we can replace the solution of (32) with the constraint (33) by minimizing the problem (35) with the constraint (33).

By using the simplex method in [2],[4],[6],[9]… the optimal solution of (33), (35) can be obtained by using numerical method after a finite number of iterations.

C. Calculate the temperature of the furnace v(t) and the temperature distribution in the object q(x,t)

a) Calculate the temperature of the furnace v(t)

We know that *v(t)* and *u(t)* have the relation:

$$T.\dot{v}(t) + v(t) = k.u(t-\tau) \quad (36)$$

or

$$\dot{v}(t) = \frac{k.u(t-\tau) - v(t)}{T} = f(v, u) \quad (37)$$

Based on improved Euler formula, we have:

$$v(j+1) = v(j) + l.f(u, v(j))$$

$$v(j+1) = v(j) + \frac{t_f}{m}.\left[\frac{k.u(j) - v(j)}{T}\right] \quad (38)$$

where $l = t_f / m$; t_f is the allowed burning time (s); with m is the number of time intervals, T is the time constant of the furnace.

After transforming, we get

$$v(j+1) = \frac{k.l.u(j) + v(j)[T-l]}{T} \quad (39)$$

or

$$v(j) = \frac{k.l.u(j-1) + v(j-1).[T-l]}{T} \quad (40)$$

with $j = 0 \div m$

So, when knowing $u^*(t)$ we can calculate *v(t)* from Eq. (40).

b) Calculate the temperature distribution in the object q(x,t)

To calculate *q(x,t)* when knowing $u^*(t)$, we use the privious calculated results. Here also use the numerical method [2], [3], [4]...

From Eq. (30), we have

$$q(x_i, t_j) = \int_0^t g(x_i, t_j - \Gamma).u(\Gamma)d\Gamma \quad (41)$$

$i = 0 \div n$; $j = 0 \div m$; $t = 0 \div t_f$; n is the number of layers of space, m is the number of time intervals, t_f is the allowed burning time (s);

According to trapezoidal formula [3], [4]. After calculating, we obtained

$$q(x_i, t) \approx jl\sum_{\varepsilon=0}^{j\delta}\xi_\varepsilon.g(x_i, jl - \Gamma_\varepsilon).u(\Gamma_\varepsilon) \quad (42)$$

IV. CALCULATING PROGRAMS

To calculate the optimal control signals as well as build real-time control programs, the author used a real control program thanks to Matlab software.

To make calculating programs, assuming the system's parameters are given, include:

- Object's physical parameters: α ; λ ; a ; L
- Parameters of the heating furnace: T; τ ; k
- Required temperature distribution: q^*
- Allowed burning time: t_f
- Limited conditions: $U_1 \div U_2$

To solve the above optimal control problem, we set up the following calculating programs:

✓ Make a program to solve the optimization problem (35), to do so, we must calculate the parameters c_{ij}. The parameters c_{ij} are determined from Eq. (31).

✓ Make a program to calculate the function $g(x,t)$ from Eq. (23), to calculate this function, we must calculate the values ϕ_i and Ψ_i .

✓ Make a program to calculate the values ϕ_i from Eq. (25), after calculating the series of solutions ϕ_i , next, we will calculate the values Ψ_i from Eq. (24).

V. SOME SIMULATION RESULTS

In order to test the calculating results, the author also have proceeded to run the simulation programs on two object samples as a flat-slab of Carbon steel at a set temperature of 1000^0C and a flat-slab of Diatomite at a set temperature of 400^0C.

A. The simulation for a flat-slab of Carbon steel

- The physical parameters of the object

$\alpha = 335\ (w/m^2.{}^0C);\ \lambda = 56\ \ (w/m.{}^0C)$

$a=1.03*e^{-5}(m^2/s);\ L= 0.2\ (m)$

- The parameters of the furnace

$T = 1200\ (s);\ \tau = 210\ (s);\ k\ = 5$

- The desired temperature distribution $q^* = 1000^0C$
- The period of heating time $t_f = 6800$ (s)
- Limit the temperature of furnace $u(t) \leq 3000^0$C
- Limit the temperature of flat-slab surface:

$q(0,t) \leq 1050\ {}^0$C

- Limit under voltage: $U_1=150$ (V)
- Limit upper voltage: $U_2=65$ (V)

With these parameters, the coefficient Bi is calculated

$$Bi = \alpha.\mathrm{L}/\ \lambda = 335.0,2/56 \approx 1,2$$

Thus, the flat-slab of Carbon steel is a thick object because the coefficient Bi is greater than 0.5.

To calculate the optimal heating process, we choose $n = 4$ and $m=16$.

We have: $2 \leq \frac{T}{\tau} = \frac{1200}{210} \approx 5,7 < 6$

After the simulation, we have result like in Figure 1.

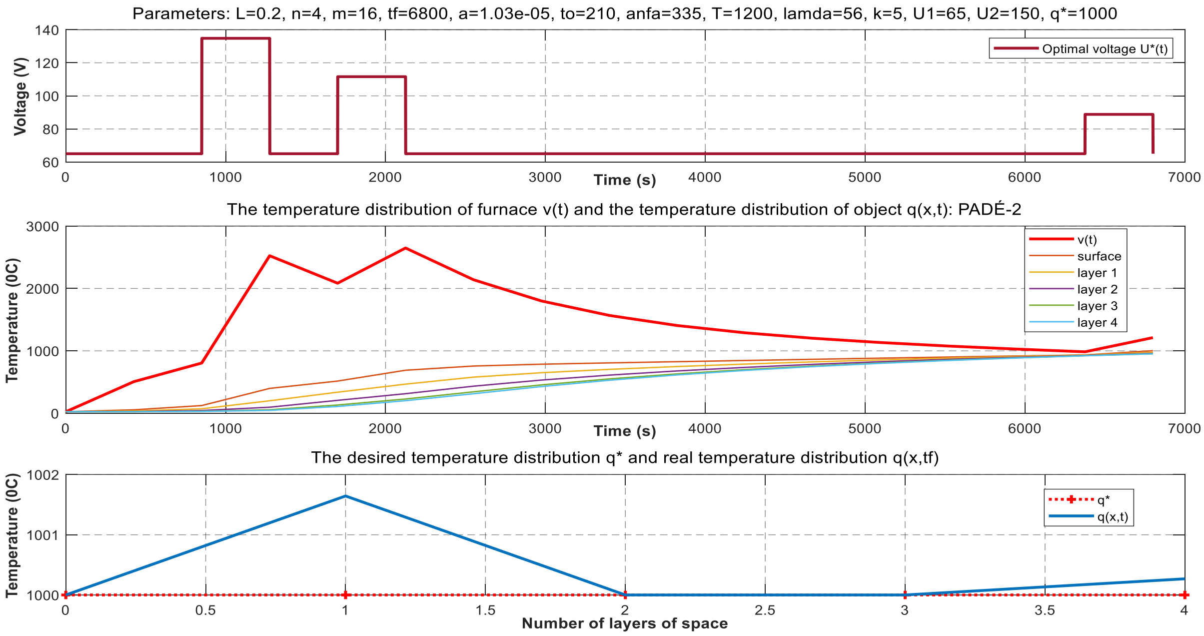

Fig. 1 The optimal heating process for a flat-slab of Carbon steel with $q^*=1000^0$C ($e \approx 0.010154$)

Remark 1

In Figure. 1, simulation result shows that at the time $t=t_f$ $=6800(s)$, the temperature distributions of the layers in a flat-slab of Carbon steel $q(x,t_f)$ is approximately equal at a set temperature $q^*=1000^0C$ with the error of objective function J as $e \approx 0.010154$. Therefore, the optimal solution has been testified.

To calculate the optimal heating process, the author choose $n = 4$ and $m=16$

B. The simulation for a flat-slab of Diatomite

- The physical parameters of the object

$\alpha = 60\ (w/m^2.{}^0C);\ \lambda = 0.2\ \ (w/m.{}^0C)$

$a = 3.6*e^{-7}\ (m^2/s);\ L= 0.04\ (m)$

- The parameters of the furnace

  $T = 1200$ (s); $\tau = 210$ (s); $k = 0.3$
- The desired temperature distribution $q^* = 400\ ^0C$
- The period of heating time $t_f = 5400$ (s)
- Limit the temperature of furnace $u(t) \leq 650^0$C
- Limit the temperature of flat-slab surface

  $q(0,t) \leq 500\ ^0$C
- Limit under voltage: $U_1 = 220$ (V)
- Limit upper voltage: $U_2 = 150$ (V)

With these parameters, the coefficient Bi is calculated as follows:

$$Bi = \alpha.L / \lambda = 60.0{,}04 / 0.2 \approx 12$$

Hence, the flat-slab of Diatomite is also a thick object because the coefficient Bi is greater than 0.5.

To calculate the optimal heating process, the author chooses $n = 6$ and $m = 36$. After the simulation, we have result like in Figure 2.

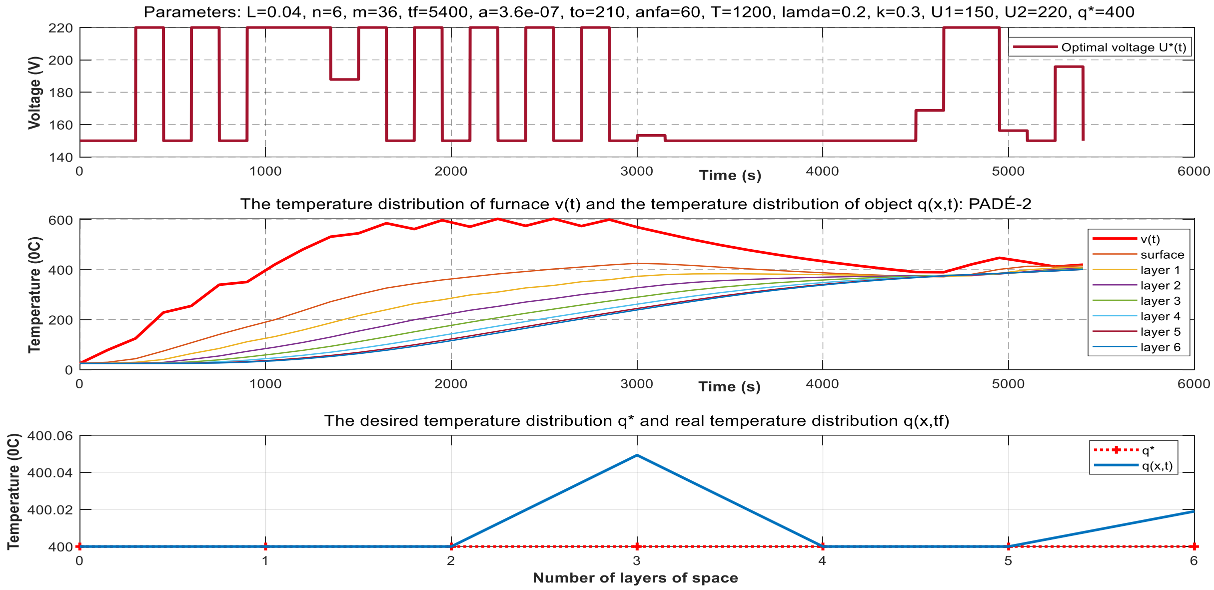

Fig. 2 The optimal heating process for a flat-slab of Diatomite with $q^*= 400^0$C (in case Padé-2 with e ≈ 0.00194)

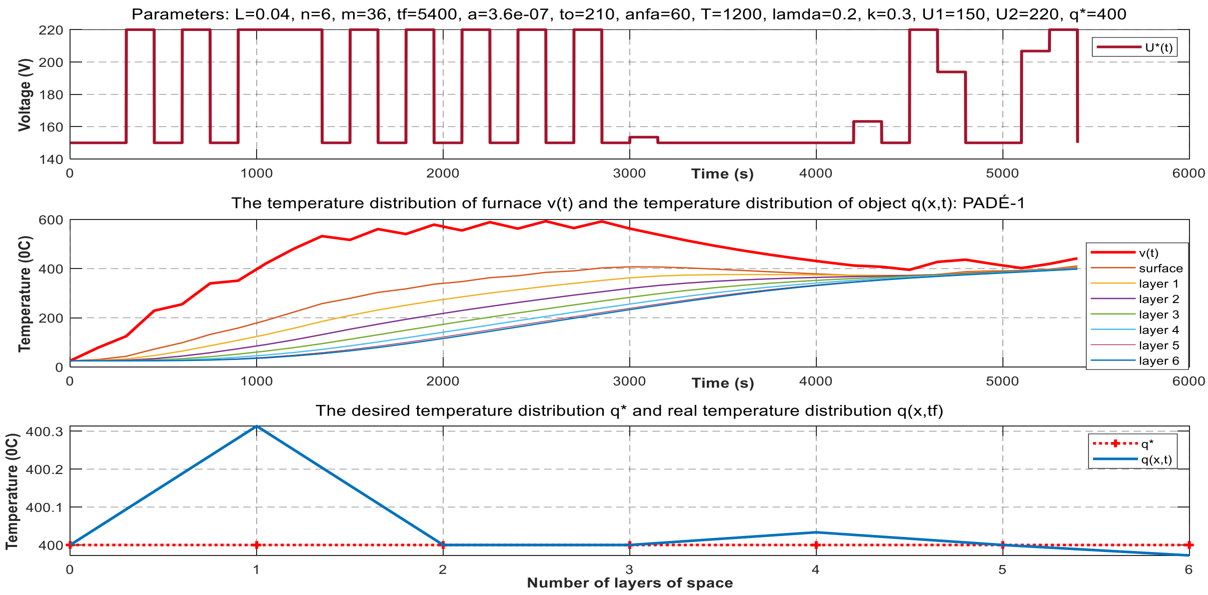

Fig. 3 The optimal heating process for a flat-slab of Diatomite with $q^*=400^0$C (in case Padé-1 with e ≈ 0.1571)

Remark 2

In Figure 2, simulation result shows that at the time $t=t_f =5400(s)$, the temperature distributions of the layers in a flat-slab of Diatomite $q(x,t_f)$ is also approximately equal at a set temperature $q^* = 400^0C$ with the error of objective function J as $e \approx 0.00194$. So, the optimal solution has been testified.

In Figure 3, in order to verify the conclusions in [5], [9], the author have also proceeded to run more the simulation program on sample of Diatomite with parameters are the same as in Figure 2, but in case Padé-1.

From Figure 3, we see that, the optimal solution has been testified, too. However, the error of objective function J in case Padé-2 ($e \approx 0.00194$) is smaller than in case Padé-1 ($e \approx 0.1571$).

Although this deviation is very small, it does not mean much in actual heat transfer, but it is also mathematically correct.

In Figures from Figure1 to Figure 3

where $U^*(t)$ is the optimal control signal (optimal voltage) of the furnace; $v(t)$ is the temperature of the furnace; $q(x,t)$ is temperature distribution of the flat-slab, including the temperature of the two surfaces and the temperature of the inner layers of the flat-slab. q^* is the desired temperature distribution

VI. CONCLUSION

The paper has continued to develope in some previous studies as in [2,5,6,9…]. In this paper, the author has proceeded to replace a delayed object $e^{-\tau s}$ by using Padé-2 approximation model in order to solve the problem of optimal control for a DPSDT, typically a controlled object is described by heat transfer equation, which is one of the physical processes with distributed parameters.

Through calculating and simulating programs, the optimal solution of the problem has been verified

Furthermore, through researches [5], [9], [12], it can be seen that depending on the ratio T/τ , we can replace a delayed object $e^{-\tau s}$ by a suitable approximation model, then the accuracy of the problem will be higher.

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