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Original Article

Capacitance in an Infinite Face-Centered Cubic Network-Accurate Calculations

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Abstract *- The infinite face-centered Cubic network in this study is composed of equal capacitors, each of which has a capacitance (C_o). Analytically and statistically, the equivalent capacitance $C(2m_1,0,0)$) between the lattice site ($2m_1$, 0, 0) and origin has been evaluated. The asymptotic behavior of the equivalent capacitance has also been studied. Finally, an asymptotic example is addressed, and the analogous capacitance is provided as a collection of numerical values.*

Keywords *- Lattice Green's function, Infinite network, Face centered cubic, Capacitors, Equivalent capacitance.*

1. Introduction

One of the well- known and fascinating challenges in electric circuit theory is finding out the equivalent resistance/capacitance in endless networks of the same resistors and capacitors. Numerous methods have been proposed to compute resistance/capacitance in infinite networks, including Net resistance between adjacent sites in infinite networks has been computed by superposing the current distribution [1-5]. Although analyzing infinite networks of capacitance is similar to networks of resistors, many efforts have been carried on them; for example, see the previous research and references therein. In these papers, the authors considered infinite square, simple cubic, and face-centred cubic lattices consisting of identical capacitors. They used the charge distribution and Lattice Green's Function methods to investigate these networks when they were perfect and disturbed. This work is unique since no prior research on analyzing infinite networks of capacitance has developed a precise and correct equation for the net capacitance between the lattice sites ($2m_1$, 0, 0) and ($2m_1$, 0, 0). Other works like [6-9] give an expression of the net capacitance between any lattice site and the origin in terms of the Lattice Green's Function values. The method presented here is valid only for infinite face centered networks and comprising of equal capacitors. Research on infinite networks of identical capacitors has important real-world applications, notably in the construction of new materials and energy storage devices. Understanding the effective capacitance of such networks can help optimize supercapacitors, metamaterials, and nanostructures for better performance in electronics, telecommunications, and power grid applications. This theoretical paradigm also promotes creativity in distributed sensor networks and large-scale circuit design. The remainder of this paper is written as follows: In Section 2, we offer some basic concepts, and in Part 3, we apply these principles to an endless network of equal capacitors. Finally, in Section 4, we conclude the study with our findings and discussions.

2. Literature Review

Analyzing infinite networks of identical resistors is a well-known approach in theoretical physics and electrical engineering. There are many methods used in literature, such as the current distribution method [6], the random walk method [7] and the mathematical principles of Green's functions [1, 2]. Many infinite networks (perfect or perturbed) have been investigated using these methods like 2-dimensional square, hexagonal, ….), 3 dimensional (simple cubic, body centered cubic, face-centered cubic, …) networks and references in. The application of Green's functions to infinite resistor networks has also received substantial attention. Notably, Cserti [1, 2, 7] was the first who use the lattice Green's function approach to determine the resistance between two random nodes in an infinite resistor network. This study offered precise formulae for resistance in a variety of lattice designs, confirming the usefulness of Green's function technique in resistor networks. Furthermore, studies have investigated the relationship between resistance in infinite networks and random walks, suggesting a link between electrical resistance and probabilistic processes. This multidisciplinary approach has improved our knowledge of electrical characteristics in infinite networks. The study of infinite networks composed of identical capacitors has been explored in various lattice structures, where authors utilized Green's function to compute the capacitance between arbitrary nodes in infinite linear chains and square lattices of identical capacitors. Their methodology demonstrated that the capacitance between two nodes depends on Green's function evaluated at specific lattice points, providing a systematic way to calculate capacitances in such networks. Further investigations extended this analysis to more complex lattice structures. For instance, studies have examined the capacitance in body-centered and face-centered cubic lattices, revealing that Green's function approach can be adapted to various geometries. These analyses highlight the versatility of Green's functions in handling different lattice configurations. The study of perturbations in infinite networks, such as the removal of a bond, has been addressed using Green's functions. Cserti et al. [2] investigated the resistance between arbitrary nodes in perturbed infinite resistor networks, establishing a relationship between the resistance and the lattice Green's function of the perturbed network. By solving Dyson's equation, they expressed Green's function and the resistance of the perturbed lattice in terms of those of the perfect lattice.

Similarly, Hijjawi et al. [8] studied the capacitance between arbitrary nodes in an infinite square lattice of identical capacitors when one bond is removed. They connected the capacitance to the Lattice Green's Function of the perturbed network, expressing it in terms of the perfect network's Green's function. Their numerical results provided insights into the effects of perturbations on capacitance in such networks. Finally, Tan Zhi Zhong has conducted significant research on the behavior and modeling of finite resistors in electrical systems. His work explores the impact of finite resistor sizes on circuit performance, including their influence on current distribution, thermal effects, and noise characteristics. By integrating both theoretical analyses and experimental data, Tan Zhi Zhong has developed refined models that account for real-world limitations, such as material properties and geometric constraints. These models enhance the accuracy of circuit simulations and have been instrumental in advancing the design of efficient and reliable electronic devices. His contributions are particularly valuable in high-precision applications, where even slight deviations in resistor behavior can lead to substantial performance discrepancies [9-13].

3. Elementary Definitions

We begin this part with a few key introductions and terminology pertaining to the Lattice Function of the infinite face lattice. There are several statistical scenarios when the face lattice's Green's Function is encountered. This is how it is defined [14]:

$$F(m_1, m_2, m_3; \varepsilon) = \frac{1}{\pi^3}\int_0^{\pi}\int_0^{\pi}\int_0^{\pi}\frac{\cos m_1\theta_1 \cos m_2\theta_2 \cos m_3\theta_3}{\varepsilon-(cos\theta_1 cos\theta_2+cos\theta_2 cos\theta_3+cos\theta_1 cos\theta_3)}\,d\theta_1 d\theta_2 d\theta_3. \quad (1)$$

Where $\varepsilon = \varepsilon_1 + i\varepsilon_2$ is a complex variable (n_1, n_2, n_3) is any lattice site where $m_1 + m_2 + m_3 =$ even. Watson [15] studied the Green's Function at the origin for $\varepsilon = 3$ $(i.e., F(0,0,0;\ 3) = f_o)$. Evaluation of the following result for the Green's Function of the infinite face lattice:

$$F(0,0,0;\ 3) = f_o = \frac{\sqrt{3}}{\pi^2}[K(k_3)]^2 = 0.44822039440. \quad (2)$$

Where $k_3 = sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}}$ (i.e., the elliptic integral's singular modulus), and $K(k_3)$ is known as the complete elliptic integral of the first kind. At the site $(2m_1, 0, 0)$ the Green's Function of the infinite face lattice may be expressed as follows, as demonstrated by Joyce and Delves [14] in their paper:

$$F(2m_1, 0, 0; 3) = (-1)^m \frac{\sqrt{3}}{3^m}\left\{\left[\frac{U_m^{(1)}K_3}{\pi}\right]^2 - \left[\frac{U_m^{(2)}}{K_3}\right]^2\right\}. \quad (3)$$

$U_m^{(j)}$(with $j = 1, 2$) are values that obey the following recurrence connection (i.e., rational):

$$(2m+1)U_{m+1}^{(j)} - 12mU_m^{(j)} - 3(2m-1)U_{m-1}^{(j)} = 0. \quad (4)$$

With the following initial conditions:

$U_0^{(1)} = 1, \quad U_1^{(1)} = 1, \quad U_0^{(2)} = 0, \quad U_1^{(2)} = 1, \quad K_3 = \frac{\pi}{2}\,{}_2F_1(\frac{1}{2}, \frac{1}{2}; 1; k_3)$, and with $m = 1, 2, 3, ...$

In the above ${}_2F_1(\frac{1}{2}, \frac{1}{2}; 1; k_3)$ is known as the entire first-kind elliptic integral.

For more information about the derivation of Equation (4), interested readers can follow the derivation presented in [15].

4. Calculation of the Capacitance C(2m₁,0,0; 0,0,0)

The goal of this section is to give the net capacitance in an infinite face-centered cubic network of equal capacitors between the lattice site ($2m_1$,0,0) and the origin (i.e., $C(2m_1, 0,0)$) in terms of $F(2m_1, 0,0;\ 3)$. Firstly, it has been demonstrated that the equivalent capacitance between any lattice site and origin in a three-dimensional infinite network made up of equal capacitors with capacitance C_o is,

$$C(\vec{0}, \vec{r}) = \frac{C_o}{2[G(\vec{0}) - G(\vec{r})]}\,. \quad (5)$$

Here $\vec{r}$ denotes the position vector of the lattice's points. For a 3- D lattice, it takes the form:

$$\vec{r} = m_1\overrightarrow{b_1} + m_2\overrightarrow{b_2} + m_3\overrightarrow{b_3}. \quad (6)$$

With m_1, m_2, m_3 are integers, and $\overrightarrow{b_1}, \overrightarrow{b_2}, and\, \overrightarrow{b_3}$ are independent primitive translation vectors. Also, the equal capacitance between any lattice site and origin in a three-dimensional lattice may be stated in integral form as:

$$C(m_1, m_2, .., m_d) = \frac{C_o}{\int_{-\pi}^{\pi} \frac{dx_1}{2\pi} \cdots \int_{-\pi}^{\pi} \frac{dx_d}{2\pi} \frac{1-\exp(im_1x_1+\cdots+im_dx_d)}{\sum_{i=1}^{d}(1-cosx_i)}}. \quad (7)$$

Furthermore, the Green's Function for a 3D hypercube reads:

$$G(m_1, m_2, .., m_d) = \int_{-\pi}^{\pi} \frac{dx_1}{2\pi} \cdots \int_{-\pi}^{\pi} \frac{dx_d}{2\pi} \frac{1-\exp(im_1x_1+\cdots+im_dx_d)}{2\sum_{i=1}^{d}(1-cosx_i)}. \quad (8)$$

For the case of an infinite face lattice (i.e., $d = 3$). Then using (7), (8) and comparing with (5), we got:

$$C(m_1, m_2, m_3) = \frac{C_o}{[f_o - F(m_1, m_2, m_3)]}. \quad (9)$$

Finally, plugging (3) into (9) results in our final required relation

$$C(2m_1, 0,0) = \frac{C_o}{\left(\frac{\sqrt{3}}{\pi^2}[K(k_3)]^2 - (-1)^n \frac{\sqrt{3}}{3^n}\left\{\left[\frac{U_n^{(1)}K_3}{\pi}\right]^2 - \left[\frac{U_n^{(2)}}{K_3}\right]^2\right\}\right)}. \quad (10)$$

This is our basic relation, and using the initial conditions given after (4) in addition to (4) and (10), one can evaluate the required effective capacitance. Below, in Table 1 we give some numerical results. It is important to mention two points: firstly, $C(2m_1, 0,0) = C(-2m_1, 0,0)$ which is known as symmetric, and this is due to the fact that FCC is pure (it has no impurities). Second, consider the capacitance's asymptotic behavior (i.e., when the separation between the site ($2m_1$, 0, 0) and origin grows larger or reaches infinity). In this case, it has been shown in the previous research, that as $m_1 \longrightarrow \infty$, then $F(2m_1, 0,0) \longrightarrow 0$. Thus from (9),

$$C(m_1, m_2, m_3) \longrightarrow \frac{C_o}{[f_o - 0]} = \frac{C_o}{0.44822039440} = 2.23104529043\ (for\ C_o = 1)$$

5. Results and Discussion

The net capacitance in an infinite face network composed of identical capacitors with a capacitance of C_o between the lattice site ($2m_1$,0,0) and the origin (0,0,0) (i.e.($2m_1$,0,0)) has been represented in the precise form provided by Eq. (10). The entire first-kind elliptic integral and rational integers, π, are utilized to get the equivalent capacitance ($C(2m_1,0,0)$). We were able to get numerical values for these predicted effective capacitances using Mathematica, as shown in Table 1. Figure 1 displays the estimated capacitances plotted along the [100] direction against the site ($2m_1$,0,0) in the infinite fcc lattice. The capacitance ($C(2m_1,0,0)$) is symmetric and approaches the finite value 2.23104529043 (for C_o=1). This is due to the fact that $f_o(3,0,0,0)$ = 0.4482203944 as m→∞.

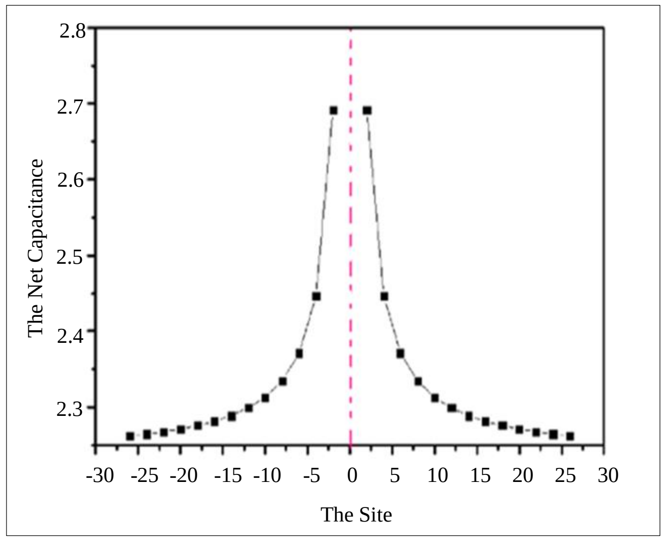

Fig. 1 The effective capacitance of the site ($2m_1$,0,0) and origin in an infinite face lattice

Table 1. Numerical values of effective capacitance $C(2n_1,0,0)$ in an infinite face lattice

Here we consider C_o=1

The site ($2m_1$,0,0)	$U_m^{(1)}$	$U_m^{(2)}$	$C(2m_1,0,0)$
(0,0,0)	1	0	∞
(2,0,0)	1	1	2.69124672004
(4,0,0)	5	4	2.44633355758
(6,0,0)	129/5	21	2.37083681056
(8,0,0)	717/5	816/7	2.33445231414
(10,0,0)	825	4695/7	2.31307465448
(12,0,0)	266859/55	27612/7	2.29901441252
(14,0,0)	1593171/55	2142999/91	2.28906285767
(16,0,0)	9615591/55	12934080/91	2.28165684794
(18,0,0)	994789431/935	78712155/91	2.27592391131
(20,0,0)	1218673431/187	9160550820/1729	2.27135929497
(22,0,0)	3410853057/85	56405302965/1729	2.26763540051
(24,0,0)	5336440769529/21505	348809334480/1729	2.26450813751
(26,0,0)	1948213488537/1265	10824102013941/9645	2.2629810249

6. Conclusion

The effective capacitance $C(2m_1,0,0)$ in an infinite face-centred cubic lattice has been precisely calculated and described in terms of elliptic integrals and rational integers, as shown in Eq. (10). Numerical simulations with Mathematica verify the theoretical predictions, and the findings are given in Table 1. Figure 1 shows how $C(2m_1,0,0)$ converges in the [100] direction and reaches a finite value of 2.23104529043 as m approaches infinity (with Co=1).

This behavior is due to the limiting value for f_o = 0.4482203944. These findings contribute to a better knowledge of capacitance characteristics in infinite lattice networks and can be used as a basis for future research in related systems.

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