SSRG International Journal of Electrical and Electronics Engineering
ISSN: 2348-8379/ https://doi.org/10.14445/23488379/IJEEE-V12I1P122

Volume 12 Issue 1, 222-229, January 2025

Original Article

Trajectory Control for Mobile Robot by Sliding Mode

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Received : 17 November 2024 Revised : 19 December 2024 Accepted : 19 January 2025 Published : 25 January 2025

Abstract - *This study presents a sliding mode control method for wheeled mobile robots designed to enhance trajectory tracking accuracy and robustness. The method begins with the formulation of constraint and dynamic equations that describe the robot's motion, accounting for the complex interactions between the wheels and the ground and the forces and moments that influence its movement. A comprehensive state model of the system is developed to enhance understanding of the robot's internal and external dynamics. A variable structure controller based on sliding mode principles is employed to achieve effective control, which is particularly advantageous for systems encountering uncertainties and disturbances, ensuring stability amid parameter variations. Specific modifications are applied to address the issue of chattering -rapid oscillations that can compromise precision and durability. Advanced techniques, such as saturation functions and higher-order sliding modes, are utilized to mitigate these oscillations while preserving the robustness of the control system. Finally, numerical simulations are performed to evaluate the performance of the proposed method under diverse conditions. The results illustrate that the controller facilitates precise trajectory tracking, even in the presence of external disturbances and dynamic uncertainties, confirming the effectiveness of sliding mode control for wheeled mobile robots in achieving stability, rapid convergence to target trajectories, and reduced chattering effects.*

Keywords - *Sliding mode, Lagrange formalism, Non-holonomic mobile robot, Trajectory pursuit.*

1. Introduction

Robotics has undergone significant development since the 1970s, from fixed manipulator systems mainly applied in production lines to autonomous mobile robots operating in diverse and unstructured environments. Though innovative for the manufacturing industry, the early robotic manipulators were incapable of mobility. These robots were designed mainly to replace humans in repetitive or physically demanding tasks but were confined to fixed, predefined environments [1]. With the advancement of needs related to logistics, exploration, and services, the research fraternity felt the requirement for mobile robots that could navigate autonomously in dynamic environments.

Mobile robots are autonomous machines that can move without anchoring their bodies to a fixed reference frame. Such robots have opened up new applications in fields far beyond manufacturing, including precision agriculture, logistics, hazardous area exploration, healthcare, military defense, and emergency response systems [2, 3]. In such robots, navigation through different environments with adaptability to various terrains makes them inseparable tools in most applications. Locomotion is at the core of mobile robots' functionality, and the design of their propulsion systems depends on the specific requirements of each application. They can be equipped with wheels, tracks, legs, or hybrid configurations depending on the terrain. Wheeled robots, for example, are particularly efficient on flat surfaces and are widely used for tasks requiring fast and precise movements [2]. However, in more complex environments, such as rough or unstable terrains, walking or tracked robots are preferred for their obstacle-conquering ability and better realization of stability [4].

Controlling a mobile robot, mainly regarding its stability and following a predetermined trajectory, is not easy. Real-world environments are usually uncertain, with uneven terrain and time-varying conditions that demand an efficient control strategy to handle these uncertainties.

Sliding Mode Control has appeared to be one of the robust ways to manage nonlinearities and external disturbances to which mobile robots are naturally exposed. This technique was developed in the 1950s and is effective, especially in systems that need increased robustness against uncertainties [5, 6]. SMC relies on using a sliding surface-a state to which the system converges to ensure stable behavior, even in the presence of disturbances. This control approach has been widely adopted for mobile robots, especially those that need to follow a trajectory over complex terrains where wheel-ground contact conditions may fluctuate [7]. However, one of the major drawbacks of this approach is the so-called chattering phenomenon, which is constituted by fast oscillations of the control signal that may cause premature mechanical wear and deteriorate performance. Modern solutions, such as introducing saturation functions and higher-order sliding mode control methods, have been developed to avoid this problem without losing system robustness [8].

The main objective of this research is to analyze and propose solutions to improve the stability of mobile robots during trajectory tracking, particularly through the use of sliding mode control. Integrating robust control techniques, such as SMC, with advanced dynamic models allows for better handling of real-world conditions and ensures the reliable performance of mobile robots in critical applications. This work shows readers that sliding mode control to mobile robots gives stability and better performance, illustrated by the position, speed and acceleration graphs; this control ensures flexibility for adding other control algorithms, such as neural network controls or fuzzy logic.

2. Methodology

2.1. Dynamic Modelling of the Mobile Robot

The choice of robot which was made has the following configuration:

Mobile robot with four wheels including two non-steering driving wheels placed at the rear and two idler wheels for better trajectory tracking and stability of the mobile robot. For the study of the system's movement, we consider an inertial frame of reference $R_0\ (o,\vec{\imath},\vec{\jmath},\vec{k})$ and another frame of reference linked to the robot $R_1\ (G,\vec{\imath_1},\vec{\jmath_1},\vec{k_1})$ where G is the center of gravity of the mobile robot.

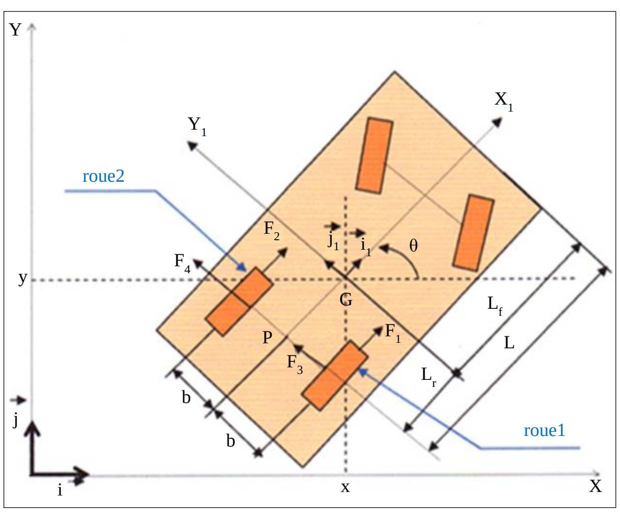

Fig. 1 Coordinated of the mobile robot

The state of the mobile robot is described by the following variables:

The position vector $\xi = (x, y, \theta)^T$ position and orientation of the robot. The speed vector $\dot{\xi} = (\dot{x}, \dot{y}, \dot{\theta})^T$ translation and rotation speed of the robot. The coordinates (φ_1, φ_2) and $(\dot{\varphi}_1, \dot{\varphi}_2)$ position and rotation speed of the two driving wheels (Figure 1).

(X,Y) : Coordinate system in the absolute reference frame R.
(X1,Y1) : Coordinate system in the reference frame attached to robot R1.
G : Center of gravity of the robot with its coordinates (X_G,Y_G).
b : Distance between the driving wheel and the symmetry axis of the robot.
Ir : Moment of inertia of the driving wheel around its axis.
Lr : Distance between the center of gravity and the rear axle.
Lf : Distance between the center of gravity and the front axle.
L : Mobile robot length.
θ : Orientation of the mobile robot (angle between R_0, R_1).
r : Radius of the driving wheel.
c : Constant equal to r/2b.
m_c : Mass of the mobile robot without wheels and without the motor rotor.
m_r : Mass of the driving wheel including the rotor of the engine.
m : total mass of the mobile robot.
φ_1 : Angular position of the left rear wheel.
φ_2 : Angular position of the right rear wheel.
$\dot{\varphi}_1$: Left rear wheel rotation speed.
$\dot{\varphi}_2$: Rotation speed of the right rear wheel.
Ic : Moment of inertia of the mobile robot.
Im : Moment of inertia of the driving wheel around its diameter.
F_1, F_2, F_3, F_4 : Tire-road interaction forces.

In order to establish the dynamic model of the robot, we choose the following generalized coordinate vector:

$$q = \begin{pmatrix} x_G \\ y_G \\ \phi_g \\ \phi_d \end{pmatrix} \tag{1}$$

q: Vector of joint positions.
ϕ_g: Angular position of the left rear wheel.
ϕ_d: Angular position of the right rear wheel.
x_G and y_G coordinates of the center of gravity.

The mobile robot is subject to constraints which are:
The inability to move in the lateral direction.

$$\dot{y}_G \cos\theta - \dot{x}_G \sin\theta - L_r.\dot{\theta} = 0 \quad (2)$$

The rolling stress without a slip for each wheel.

$$\dot{x}_G \cos\theta + \dot{y}_G \sin\theta - b.\dot{\theta} - r\dot{\varphi}_2 = 0 \quad (3)$$

$$\dot{x}_G \cos\theta + \dot{y}_G \sin\theta + b.\dot{\theta} - r\dot{\varphi}_1 = 0 \quad (4)$$

By subtracting (3) from (4), we will have:

$$\dot{\theta} = \frac{r}{2b}(\dot{\varphi}_1 - \dot{\varphi}_2) \quad (5)$$

By integrating this equation and considering the initial conditions to be zero, we obtain the following non-holonomic constraint:

$$\theta = c\,(\varphi_1 - \varphi_2) \quad (6)$$

By adding the equations of constraints (3) and (4) and dividing by 2 we will have the following:

$$\dot{x}_G \cos\theta + \dot{y}_G \sin\theta - \frac{r}{2}\dot{\varphi}_1 - \frac{r}{2}\dot{\varphi}_2 = 0 \quad (7)$$

On the other hand, we replace $\dot{\theta} = \frac{r}{2b}(\dot{\varphi}_1 - \dot{\varphi}_2)$in the Equation (2) we will have:

$$\dot{y}_G \cos\theta - \dot{x}_G \sin\theta - L_r \frac{r}{2b}(\dot{\varphi}_1 - \dot{\varphi}_2) = 0 \quad (8)$$

Equations (7) and (8) are two non-holonomic motion constraints, and they can be written in the following matrix form:

$A(q).\dot{q} = 0$ A(q) : Matrice des contraintes.

With :

$$A(q) = \begin{bmatrix} -\sin\theta & \cos\theta & c.L_r & -c.L_r \\ \cos\theta & \sin\theta & -\frac{r}{2} & -\frac{r}{2} \end{bmatrix} \quad (9)$$

The Lagrangian formalism is based on the calculation of the energies involved in the system [9]. We then calculate the first energy involved in the robot's movement, namely the kinetic energy; this energy decomposed into rotational energy and translational energy.

Rotational Energy:

$$E_{rot} = \frac{1}{2} I_G \dot{\theta}^2 + \frac{1}{2} I_{roue}\left(\dot{\varphi}_1^2 + \dot{\varphi}_2^2\right) + \frac{1}{2}\left(2 I_m \dot{\theta}^2\right) \quad (10)$$

Translational Energy:

$$E_{trans} = \frac{1}{2}\left(m_c + 2m_{roue}\right)\left(\dot{x}^2 + \dot{y}^2\right)$$
$$-2m_{roue} L_r \dot{\theta}\left(-\dot{x}\sin\theta + \dot{y}\cos\theta\right) + m_{roue}\left(b^2 + L_r^2\right)\dot{\theta}^2 \quad (11)$$

We then obtain the following expression:

$$L = E_{trans} + E_{rot} \quad (12)$$

$$L = \frac{1}{2}\left(\dot{x}_G{}^2 + \dot{y}_G{}^2\right) - 2m_{roue} L_r(\dot{\varphi}_1 - \dot{\varphi}_2)(-\dot{x}_G \sin\theta + \dot{y}_G \sin\theta)$$

$$+ + \frac{1}{2} I.c^2(\dot{\varphi}_1 - \dot{\varphi}_2) + \frac{1}{2} I_{roue}(\dot{\varphi}_1^2 - \dot{\varphi}_2^2) \quad (13)$$

With $\begin{cases} m = m_c + 2m_r \\ I = I_c + 2I_m + 2m_r(b^2 + L_r^2) \end{cases}$

The second energy involved is the potential energy U. Since the mobile robot moves only on the horizontal plane, the expression for U (q) = 0, [10]. We can now calculate the Lagrangian L (where L = T - U), corresponding simply to the expression for kinetic energy. According to the Lagrangian formalism, we write the following relationship:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_1}\right) - \frac{\partial L}{\partial q_1} = \tau'.\lambda \quad (14)$$

Where τ' is the vector of generalized external forces, this vector summarizes the torques applied by the actuators. λ is the vector of Lagrange multipliers due to the kinematic constraints of the system. We can then write the system in the form:

$$M(q)\ddot{q} + V(q,\dot{q}) = E(q).\tau - A^T(q).\lambda \quad (15)$$

M(q) : Inertia matrix.
V(q) : Coriolis matrix.
E(q) : Transformation matrix.
A(q) : Constraint matrix.

With $\tau' = E(q)\tau$

$$M(q) = \begin{bmatrix} m & 0 & 2m_r.c.L_r\sin\theta & -2m_r.c.L_r\sin\theta \\ 0 & m & -2m_r.c.L_r\cos\theta & 2m_r.c.L_r\cos\theta \\ 2m_r.c.L_r\sin\theta & -2m_r.c.L_r\cos\theta & I_r + I.c^2 & -I.c^2 \\ -2m_r.c.L_r\sin\theta & 2m_r.c.L_r\cos\theta & -I.c^2 & I_r + I.c^2 \end{bmatrix}$$

$$V(q) = \begin{bmatrix} 2m_r.L_r.\dot{\theta}^2\cos\theta \\ 2m_r.L_r.\dot{\theta}^2\sin\theta \\ 0 \\ 0 \end{bmatrix} \quad E(q) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (16)$$

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \qquad \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

With τ_1 and τ_2 right and left wheel torques.

To eliminate the Lagrange multipliers λ, we use the angular velocities of each wheel to determine the matrix S(q) such that:

$$\begin{cases} A(q)S(q) = [0] \\ \dot{q} = S(q)\eta \end{cases} \quad \text{With } \eta = \begin{bmatrix} \dot{\phi}_g \\ \dot{\phi}_d \end{bmatrix} \tag{17}$$

On the other hand, we have:

$$\ddot{q} = S(q).\dot{\eta} + \dot{S}(q).\eta \tag{18}$$

The resolution of this system allows us to explicitly express the matrix S(q):

$$S(q) = \begin{bmatrix} c.L_r \sin\theta + \frac{r}{2}\cos\theta & -c.L_r \sin\theta + \frac{r}{2}\cos\theta \\ -c.L_r \cos\theta + \frac{r}{2}\sin\theta & c.L_r \cos\theta + \frac{r}{2}\sin\theta \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{19}$$

By multiplying the equation by$S^T(q)$, and noting that $S^T(q)A^T(q) = [0]$ and $S^T(q)E(q) = I_{2\times2}$
and we also take $V(q,\dot{q}) = 0$

We will have:

$$S^T(q)M(q)\ddot{q} = S^T(q)E(q)\tau' = \tau \tag{20}$$

Substituting $\ddot{q}$ with its expression in Equation (2), we will have:

$$S^T(q)\,M(q)\,S(q).\dot{\eta} + S^T(q)\,M(q)\,\dot{S}(q).\eta = \tau \tag{21}$$

Let's choose the following state variable:

$$x = \begin{bmatrix} q \\ \eta \end{bmatrix} \tag{22}$$

The Equation (15) can be substituted into the equation as follows:

$$\dot{x} = f(x) + g(x).\tau \tag{23}$$

$$\dot{x} = \begin{bmatrix} \dot{q} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} S(q).\eta \\ f_1 \end{bmatrix} + \begin{bmatrix} [0] \\ (S^T(q)\,M(q)\,S(q))^{-1} \end{bmatrix}\tau \tag{24}$$

With

$$f_1 = \left(S^T(q)M(q)S(q)\right)^{-1}\left(-S^T(q)M(q)S(q)\right).\eta \tag{25}$$

The system is represented in state-space form, and for clarity, let's introduce a new input variable u such that:

$$\tau = \left(S^T(q)M(q)S(q)\right)(u - f_1) \tag{26}$$

We can then express the system in the form:

$$\dot{x} = \begin{bmatrix} S(q).\eta \\ [0] \end{bmatrix} + \begin{bmatrix} [0] \\ I \end{bmatrix} u \tag{27}$$

With $f(x) = \begin{bmatrix} S.\eta \\ [0] \end{bmatrix}$ and $g(x) = \begin{bmatrix} [0] \\ I \end{bmatrix}$

I is the 2x2 identity matrix.

2.2. Sliding Mode Control

To determine this sliding control, we use the equivalent control approach [11] to model robot behaviour using a simplified reference model. Our mobile robot is non-holonomous [12], which means it cannot move autonomously in all directions. Specifically, the robot's position is determined by its centre of gravity (G), subject to cinematic and dynamic constraints. However, we cannot directly control the center of gravity (G) position due to the robot's non-holonomic nature. To work around this problem, we have chosen another control point on the axis (X_1), which facilitates modeling and controlling the robot’s movement. This control point is chosen so that it is on the axis of symmetry of the robot and is at a distance from this control point, we can define the robot motion equations based on the control inputs and initial conditions [13, 14]. The motion equations are derived by applying mechanical laws while considering the kinematic and dynamic constraints imposed by the robot. By modeling the robot using this control point, we simplify motion equations and make implementing the control by sliding mode easier. This approach is particularly useful for non-holonomic mobile robots, as it bypasses the limitations imposed by non-holonomy and allows precise and controlled movements.

Therefore, we have chosen another control point located on the X_1 axis. [15] The coordinates of this point are given in the following form:

$$Z = \begin{bmatrix} z_x \\ z_y \end{bmatrix} = \begin{bmatrix} x_G + L\cos\theta \\ y_G + L\sin\theta \end{bmatrix} \text{ and } \dot{Z} = \begin{bmatrix} \dot{x}_G + L\dot{\theta}\cos\theta \\ \dot{y}_G + L\dot{\theta}\sin\theta \end{bmatrix}$$

On the other hand, we have the system with this expression:

$$\dot{x} = \begin{bmatrix} S(q).\eta \\ [0] \end{bmatrix} + \begin{bmatrix} [0] \\ I \end{bmatrix} u \tag{28}$$

This system has the particularity of possessing certain characteristics. We simplify its state representation by partitioning it into two sub-vectors x_1 and x_2, where the dimension of x_2 is equal to the dimension of u, which is m=2.

Therefore, the dimension of $n - m = 4$. We can then express the system (15) in the following form:

$$\dot{x} = f(x) + g(x)\tau$$

$$\begin{cases} \dot{x}_1 = f_1(x) + g_1(x) \\ \dot{x}_2 = f_2(x) + g_2(x) \end{cases} \quad \text{Where} \begin{cases} f_1(x) = S(q).\eta \\ g_1(x) = [0] \\ f_2(x) = [0] \\ g_2(x) = I \end{cases}$$

This allows us to express the system in its reduced form:

$$\begin{cases} \dot{x}_1 = f_1(x) \\ \dot{x}_2 = g_2(x)u \end{cases}$$

Now, we consider s(z) as the vector defining the switching functions that determine the sliding surfaces of the m-dimensional system. By definition, these functions are expressed as:

$$s(z) = \begin{bmatrix} s_1(z_x) \\ s_2(z_y) \end{bmatrix} \quad \text{Where} \quad \begin{cases} S_1 = \dot{\tilde{z}}_x + \lambda_x \tilde{z}_x \\ S_2 = \dot{\tilde{z}}_y + \lambda_y \tilde{z}_y \end{cases}$$

$$\text{With} \quad \begin{cases} \dot{\tilde{z}}_i = \dot{z}_i - \dot{z}_{id} \\ \tilde{z}_i = z_i - z_{id} \end{cases} \qquad i = x, y$$

On the other hand, we have:

$$Z = \begin{bmatrix} z_x \\ z_y \end{bmatrix} = \begin{bmatrix} x_G + L\cos\theta \\ y_G + L\sin\theta \end{bmatrix} \quad \text{and} \quad \dot{Z} = \begin{bmatrix} \dot{x}_G + L\dot{\theta}\cos\theta \\ \dot{y}_G + L\dot{\theta}\sin\theta \end{bmatrix}$$

Z_{id} and $\dot{Z}_{id}$ are the respective coordinates of the desired position and velocity vectors. λ_x and λ_y are the coefficients of the sliding surfaces. These coefficients determine the system's response time. To determine the system's dynamics, we use the reaching law approach [2], which allows us to directly specify the approach dynamics of the system such that:

With Q and K being two diagonal matrices with positive elements, and:

$$\dot{s} = -Q signe(s) - Kh(s) \tag{29}$$

With Q and K two diagonal matrix with positive elements

$$signe(s) = \left[signe(s_x), signe(s_y)\right]^T$$

$$h(s) = \left[h(s_x), h(s_y)\right]^T$$

The function h(s) is a function depending on the sliding surface. We now determine the control law.

$$\dot{s} = \frac{ds}{dt} = \frac{\partial s}{\partial x}\dot{x} = \frac{\partial s}{\partial x}\left(f(x) + g(x)u\right)$$
$$= -Q signe(s) - Kh(s) \tag{30}$$

Finally, we will have: $u =$
$$-\left(\frac{\partial s}{\partial x_2} g_2(x)\right)^{-1}\left(Q signe(s) + Kh(s) + \frac{\partial s}{\partial x_1} f_1(x)\right) \tag{31}$$

We used MATLAB/SIMULINK (v.2024b) in this study.

3. Results and Discussion

To validate and enhance the performance of the proposed control strategy, the following Figure 2 illustrates the dynamic model for trajectory tracking in wheeled mobile robots implemented in the MATLAB/Simulink environment.

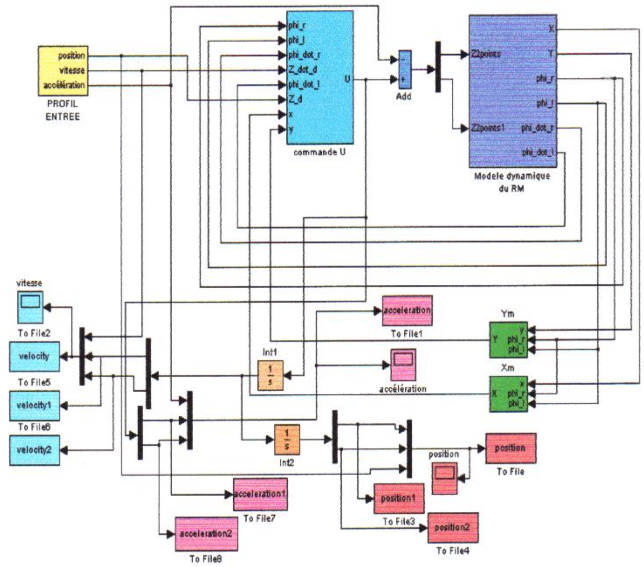

Fig. 2 Block diagram of the dynamic model for trajectory tracking in wheeled mobile robots

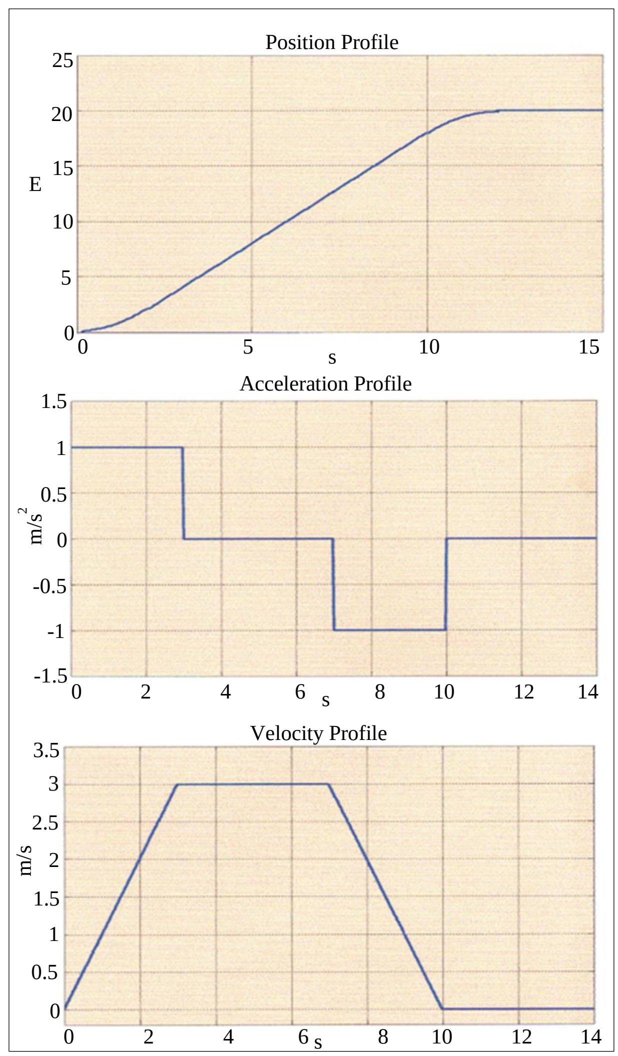

Fig. 3 Selected position, velocity, and acceleration profiles

Figure 3 depicts the selected position, velocity, and acceleration profiles utilized for testing the proposed control system.

3.1. Robot Parameters and Experiments during the Simulation, the Robot is Required to Follow a Straight Reference Trajectory

The initial position of the robot in the absolute frame is

$$(x, y, \theta) = (0,0,0)$$

The final position to reach is

$$\left(x_f, y_f, \theta_f\right) = \left(2{,}0, \frac{\pi}{6}\right)$$

The gains used in this simulation are

$Q = 20m/s^2$; K=5Hz; λ =0.5Hz

The robot parameters are

$m_c = 99Kg$; m_r =0,5Kg; $b = 0{,}5148m$; $r = 0{,}0228m$; $L = 1{,}05m$; $L_r = 0{,}3m$; $I_r = 8{,}26 \times 10^{-3} Kg.m^2$; $I_c = 14{,}4 \times 10^{-3} Kg.m^2$

3.2. Analysis of Position Tracking, Velocity and Acceleration

The analysis of position tracking results indicates that the robot's trajectory closely aligns with the desired path, as illustrated in Figure 4. The tracking performance is both accurate and remains within acceptable thresholds, highlighting the effectiveness of the control method in guiding the robot to its target positions. However, some nuances in the robot's behavior, particularly during specific movement phases, warrant further investigation to comprehensively understand its dynamics. A key observation involves the velocity errors of one of the wheels during the deceleration phase, as illustrated in Figure 5(a). During this phase, a notable dip in velocity error is evident. This phenomenon arises from the robot's decreased sensitivity to rapid velocity changes while decelerating, leading to a slower response to speed adjustments. Although this might initially appear concerning, it indicates that the system is stabilizing effectively; however, it does highlight a slight delay in response during sharp deceleration. Importantly, despite this initial lag in velocity adjustment, position errors are corrected after a short period, becoming nearly negligible within seconds. This suggests that, while temporary discrepancies in velocity occur, the robot successfully compensates for them, ensuring accurate position tracking over time.

Another notable observation relates to the acceleration oscillations shown in Figure 5(b), which are linked to the specific motion profile being followed, in this case, a straight trajectory. These oscillations are unsurprising, as they are inherent to the control dynamics of systems operating along such paths. A reassuring aspect is that, while these oscillations are present, they do not cause any damage to the actuators. This indicates that the control system is robust enough to accommodate these variations without resulting in mechanical stress or degradation.

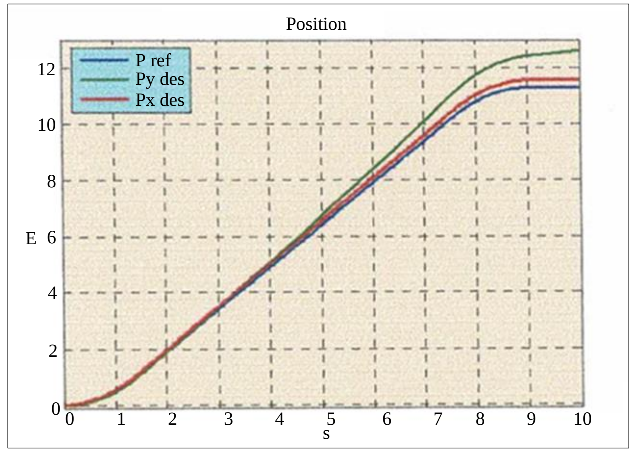

Fig. 4 Right tracking simulation

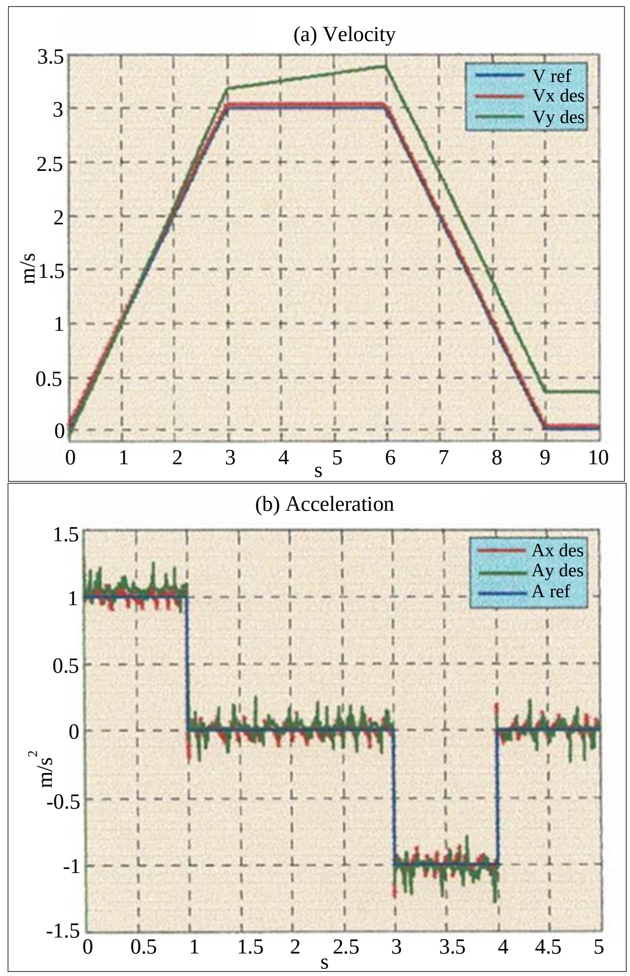

Fig. 5 Right tracking simulation: (a) Velocity, and (b) Acceleration.

3.3. Analysis of Control Commands and Slip Surfaces

The analysis also reveals the presence of the chattering phenomenon, an issue commonly associated with sliding mode control. Chattering is manifested by spikes in the control commands, as seen in Figure 6. Chattering occurs when the control signal oscillates rapidly near the sliding surface, resulting in high-frequency fluctuations in the system's inputs. While chattering can lead to mechanical wear or inefficiency in control, the observed spikes here do not seem to significantly affect the robot's ability to track its trajectory. This can be attributed to the fact that the sliding surfaces Sx and Sy tend towards zero, as shown in Figure 7. When the sliding surfaces approach zero, the system is converging towards the desired state, ensuring that any deviations are minimal and corrected promptly. This convergence suggests that the chattering, although present, is sufficiently controlled to prevent any major negative impact on the robot's performance. In summary, although the velocity errors during deceleration and the chattering phenomenon are noteworthy, they do not substantially hinder the robot's overall trajectory-tracking performance. The control system effectively minimizes position errors, and the acceleration oscillations remain within acceptable limits without inflicting damage on the actuators. While the impact of chattering could be further mitigated through more refined control techniques, its current effects are limited to minor fluctuations that do not compromise the system's overall effectiveness in maintaining precise trajectory tracking.

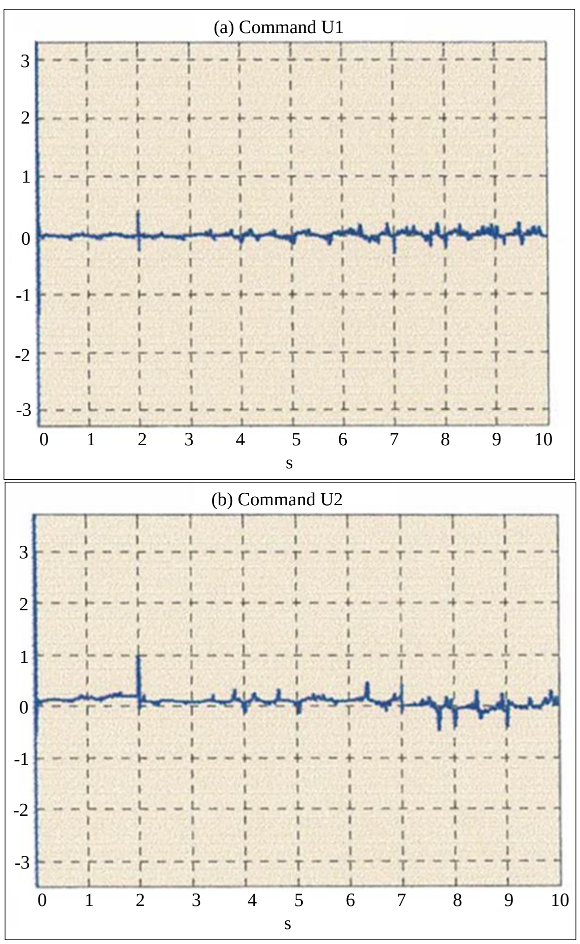

Fig. 6 Graphs of commands: (a) Command U1, and (b) Command U2.

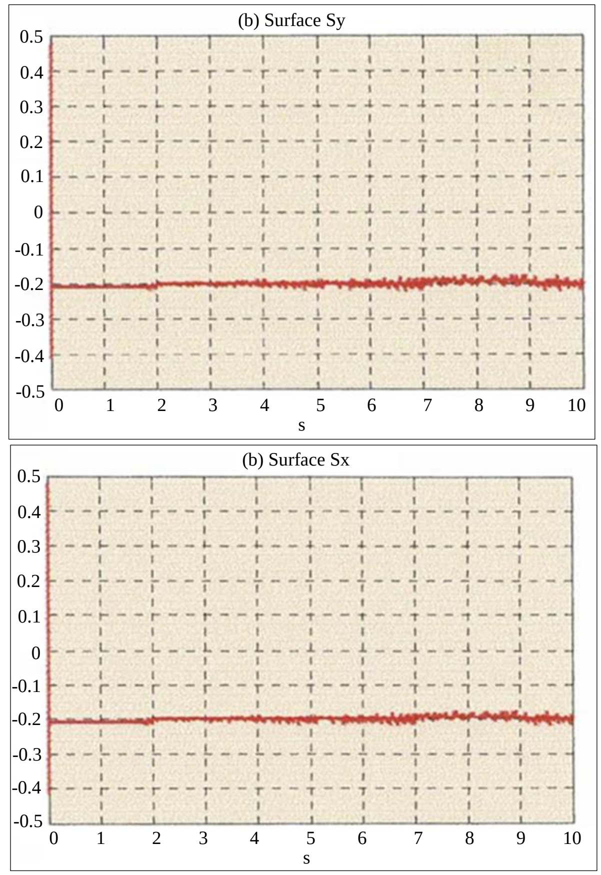

Fig. 7 Graphs of slip surfaces: (a) Surface S_x, and (b) Surface S_y.

4. Conclusion

This study has presented a trajectory tracking control method for a nonholonomic wheeled mobile robot based on sliding mode control techniques. Effective trajectory tracking is vital for enabling mobile robots' autonomous and efficient operation in varied and potentially disturbed environments. Our methodology highlights the advantages of sliding mode control, particularly its robustness to dynamic uncertainties and external disturbances, which are common challenges mobile robots face in practical applications. The simulation results confirmed the relevance and effectiveness of sliding mode control for this type of robot. A significant finding of this study was the critical role of the λ parameter, which influences the system's dynamics. This parameter is essential for optimizing the control system's performance by accounting for the robot's specific physical constraints and the surrounding environmental conditions. Our findings demonstrated that carefully selecting this parameter enables the robot to achieve effective stabilization while providing a rapid and precise dynamic response. Sliding mode control provides several advantages over alternative control techniques, notably its capacity to manage nonlinear systems and withstand unexpected disturbances. The simulation results demonstrated that this approach can maintain the robot's stability despite surface variations, external disturbances, and uncertainties in the dynamic model parameters. This robustness is a key strength of sliding mode control, distinguishing it from conventional control methods, which may be more susceptible to parameter variations or rely on incomplete models. Another important aspect to highlight is the management of chattering. While sliding mode control is known for its robustness, it is also known to produce rapid oscillations in the control signal, a phenomenon called chattering, which can affect system performance and cause premature wear of mechanical components. In our study, we applied techniques to reduce chattering, such as using saturation functions or higher-order sliding modes. This allowed us to minimize this undesirable effect while maintaining a precise and stable dynamic response. In conclusion, this research has demonstrated that sliding mode control is a particularly well-suited method for the trajectory tracking of nonholonomic wheeled mobile robots, thanks to its robustness and flexibility in the face of uncertainties. The simulation results validate this approach and show that it offers an efficient solution for applications requiring a high degree of reliability, such as mobile robotic systems operating in varied and potentially unpredictable environments. However, while this method shows promising results, there are still opportunities for improvement. For example, optimizing control parameters could further enhance system performance, especially in reducing chattering without compromising stability. Further studies could be conducted to test this approach in real-world environments with more complex conditions, such as highly irregular terrains or unpredictable interaction scenarios. Combining this method with artificial intelligence techniques, such as machine learning, could also open new perspectives for real-time dynamic adaptation, thereby enhancing the autonomy of mobile robots in critical applications.

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