SSRG International Journal of Electrical and Electronics Engineering
ISSN: 2348-8379/ https://doi.org/10.14445/23488379/IJEEE-V12I6P101

Volume 12 Issue 6, 1-13, June 2025
© 2025 Seventh Sense Research Group®

Original Article

Infinite Simple Cubic Network of Identical Capacitor: Analysis of Perfect and Perturbed Cases

Ataa Hikmat Almallak[1], Rabab Jarrar[1], Jihad Asad[1]*

[1]*Department of Physics, Faculty of Applied Science, Palestine Technical University- Kadoorie, Tulkarm P 305, Palestine.*

**Corresponding Author : j.asad@ptuk.edu.ps*

Received: 01 April 2025 Revised: 03 May 2025 Accepted: 02 June 2025 Published: 30 June 2025

Abstract *- Using the Lattice Green Function (LGF), the effective capacitance of an infinite SC network (made of identical capacitors) between the origin site and another site inside the network was computed in this study. Two situations were considered: the disturbed case of the infinite network, whereby one capacitance was eliminated between two sites, and the ideal instance of an infinite SC network. This study derives the general formula linking the capacitance of an endless network using the Lattice Green Function (LGF) for both circumstances (ideal and disturbed). Based on the (LGF) at the origin, the capacitance of the infinite SC network is stated in relative terms.*

Keywords *- Lattice Green Function, Infinite network, Capacitors, Perfect case, Perturbed case.*

1. Introduction

Physicists and engineers depend critically on studying electrical circuits inside infinite networks. Although much research has investigated effective resistance in infinite resistor networks, significantly less has been paid to capacitor networks, especially in Three-Dimensional (3D) designs such as the Simple Cubic (SC) lattice. This results in a major theoretical modelling gap in infinite capacitive networks, particularly in ideal and disturbed situations. Most current research has focused on resistor networks applying methods such as random walk approaches [1], Lattice Green's Function (LGF), and superposition distribution [2]. While these techniques have succeeded in resistance calculations [3-6], their adaptation to capacitive systems remains constrained. Cserti et al. [5-7] and Owaidat [8] achieved noteworthy development for resistor lattices. Author investigated infinite 2D and 3D networks of identical capacitors using LGF, therefore advancing some progress in the framework of capacitor lattices. Nevertheless, disturbed scenarios concerning their effect on network symmetry and effective capacitance are still understudied, in which one or more capacitors are eliminated.

This work uses the LGF methodology to compute the effective capacitance in an infinite 3D SC network under perfect and disturbed conditions, filling the research gap. We specifically enhance the work by analyzing many perturbation sites, measuring the change in effective capacitance, and comparing the results with the perfect case to underline symmetry breakdown and capacitive sensitivity to local defects.

For resistor-based systems, the literature on infinite electrical networks is plentiful; it is somewhat rare for capacitive networks, particularly in three-dimensional designs. Applying the Lattice Green's Function (LGF) to infinite resistor networks, Cserti [5] and Cserti et al. [6] investigated both ideal and disturbed situations. Further study of resistor tiling networks was conducted by Owaidat [8].

On the other hand, modelled infinite networks of identical capacitors, thus first investigated capacitive networks using LGF. Author expanded their study to perturbed configurations; nonetheless, thorough multi-site perturbations and numerical comparisons were not investigated.

Recurrence-based methods by Duffin & Shelly [9] and Horiguchi [10], as well as Watson's elliptic integral representations [11] and Economou's Green's Function formalism [12], establish mathematical underpinnings for these investigations.

This study addresses several perturbation sites in 3D SC lattices, validates results using simulations, and provides fresh numerical insights, building on and greatly expanding this body of work. The novelty of this work is in thoroughly treating several perturbations at different distances from the origin, using Dyson's equation and recurrence relations for efficient computation [10], and presenting new numerical examples and detailed tabulations not available in previous literature.

This study derives closed-form equations for the effective capacitance in both perfect and disturbed SC networks employing the LGF approach. To look at the local flaws' effect on the symmetry behaviour of the SC network, to offer a numerical structure for computing over several lattice configurations, and effective capacitance values. To investigate how defect proximity affects the effective capacitance and its convergence behaviour.

Beyond theoretical importance, this work has practical consequences in circuit design, electronic materials, and nano-scale engineering using infinite or periodic capacitor arrays. Knowing how perturbations impact capacitance can help one build fault-tolerant circuits, metamaterials, and brain processing arrays where uniformity and symmetry are crucial [9, 13].

However, the LGF method starts with an idealised infinite lattice of identical components. Hence, it cannot directly handle finite or non-uniform networks except by interpreting variations as perturbations to a perfect lattice [14]. Practically, closed-form solutions for lattice Green's functions are rare; one usually needs to calculate sophisticated Fourier integrals or apply recurrence relations to acquire values; many findings finally depend on numerical computing [15]. Multiple faults greatly increase the difficulty; for example, expanding the procedure to two simultaneous bond removals requires solving linked Dyson equations and usually uses numerical solutions for validation [15]. Even for ideal lattices, only asymptotic behaviours may be obtained in some circumstances (e.g., high spacing between nodes). The LGF integrals, such as van Hove singularities, typically contain nontrivial characteristics that prevent closed-form formulations [16].

The organisation of this work is as follows: Section II describes the general formality of LGF-based practical capacitance computation. Section III offers computing examples and formally treats a basic cubic lattice. Numerical results are presented in Section IV; Section V addresses the consequences of these conclusions; Section VI ends the study with important observations and future avenues of research.

2. General Formalism

2.1. Perfect Lattice

Imagine a perfect, infinite d-dimensional network in which every capacitor is denoted by C, and the following position vector determines all positions on the network. This implies symmetric capacitors.

$$\vec{u} = n_1\overrightarrow{a_1} + n_2\overrightarrow{a_2} + n_3\overrightarrow{a_3} + \ldots + n_d\overrightarrow{a_d} \tag{1}$$

Where $(n_1, n_2, n_3, \ldots, n_d)$ are integers (positive, negative, zero) and $(\overrightarrow{a_1}, \overrightarrow{a_2}, \overrightarrow{a_3}, \ldots, \overrightarrow{a_d})$ are independent primitive translation vectors.

While the charges are zero at all other lattice sites, and assuming the potential at the site $(\overrightarrow{u_l})$ will be designated as $\emptyset(\overrightarrow{u_l})$ and allow a charge (q) to enter the site $(\overrightarrow{u_l})$ and a charge (-q) escapes the site $(\overrightarrow{u_k})$. Then

$$q_m = q[\delta_{ml} - \delta_{mk}]. \tag{2}$$

For all m,

Now, based on Ohm's and Kirchhoff's laws,

$$\frac{q(\overrightarrow{u_l})}{C} = \Sigma[\emptyset(\overrightarrow{u_l}) - \emptyset(\overrightarrow{u_l} + \vec{x})]. \tag{3}$$

Where $\vec{x}$ are the vectors from the site $\vec{u}$ to its nearest neighbors ($\vec{x} = \pm\mathbf{a}_l$, l= 1,2, …, d),

Two state vectors can be formed, Ø and q, at a specific site $\overrightarrow{u_l}$ such that,

$$\emptyset = \textstyle\sum_l |l\rangle \emptyset_l ;$$

$$q = \textstyle\sum_l |l\rangle q_l . \tag{4}$$

Where $\emptyset_l = \emptyset(\overrightarrow{u_l}),\ q_l = q(\overrightarrow{u_l})$

Here, we can assume $\langle l|f\rangle = \delta_{lf}$ and $|l\rangle\langle l| = 1$. (i.e. $|l\rangle$ forms a complete orthonormal set).

Using Equation (4) and Equation (3). Then,

$$\textstyle\sum(z\delta_{lk} - \Delta_{lk})\langle k|\emptyset = \frac{\langle l|q}{C}. \tag{5}$$

Where z is the number of neighbors of each lattice site (e.g. z=3d for a d-dimensional hypercubic lattice).

Also,

$$\Delta_{ft} = \begin{cases} l, \overrightarrow{u_f}, \overrightarrow{u_t}, & \text{are nearest neighbors.} \\ \text{Zero,otherwise} \end{cases} \tag{6}$$

Multiplying both sides of (5) by ($|l\rangle$) and summing over l, getting,

$$\textstyle\sum_{l,k} |l\rangle(\Delta_{lk} - z\delta_{lk})\langle k|\emptyset = \frac{-q}{C} \tag{7}$$

Or

$$L_o\emptyset = -\frac{q}{C}. \tag{8}$$

Assuming $\sum_{l,k} |l\rangle(\Delta_{lk} - z\delta_{lk})\langle k| = L_o$

L_o is the so-called lattice Laplacian.

The LGF for an infinite perfect can be defined as,

$$L_o G_o = -1. \tag{9}$$

Similar to the definition used in Economou [12].

The solution of Equation (9) can be given as,

$$\emptyset = -\frac{L_o^{-1} q}{C}, \text{ if } \; -L_o^{-1} = G_o$$

$$\emptyset = \frac{G_o q}{C}. \tag{10}$$

Inserting Equation (2) into Equation (10).

$$\emptyset = \langle f|\emptyset = \frac{\langle f|G_o q}{C} = \frac{1}{C}\sum_m \langle f|G_o|m\rangle q_m$$

$$\emptyset = \frac{q}{C}\,[G_o(f,l) - G_o(f,k)]\,. \tag{11}$$

Finally, the capacitance between sites $\overrightarrow{u_l}$ and $\overrightarrow{u_k}$ can be written as,

$$\frac{1}{C_o(l,k)} = \frac{\emptyset_l - \emptyset_k}{q} \text{ and using Equation (11), getting,}$$

$$\frac{1}{C_o(l,k)} = \frac{2}{C}[G_o(l,k) - G_o(l,k)]$$

The above formula can be rewritten as:

$$C_o(l,k) = \frac{C}{2[G_o(l,l) - G_o(l,k)]}\,. \tag{12}$$

2.1.1. Alternative Approach

Imagine a perfect network made of identical capacitors with capacitance C. First of all, we suppose that the following position vector $\vec{u}$ specifies all the lattice points,

$$\vec{u} = n_1\overrightarrow{a_1} + n_2\overrightarrow{a_2} + n_3\overrightarrow{a_3} +, \ldots, + n_d\overrightarrow{a_d}$$

Where $(n_1, n_2, n_3, \ldots, n_d)$ are integrals (+, -, zero), and $(\overrightarrow{a_1}, \overrightarrow{a_2}, \overrightarrow{a_3}, \ldots, \overrightarrow{a_d})$ are independent primitive translation vectors.

When all $\overrightarrow{a_l}$'s have the same magnitude, i.e., $|\overrightarrow{a_1}| = |\overrightarrow{a_2}| = |\overrightarrow{a_3}| = \ldots = |\overrightarrow{a_d}| = a$. Here $\vec{a}$ is the lattice constant of the d-dimensional, which is called a hypercubic lattice.

Under this condition of a capacitance network, we regard the hypercube to consist of similar capacitors-that is, the same capacitor C. This section presents the capacitance between the infinite hypercube's origin (0,0,0) and any other designated lattice point. Assuming a charge (+q) arrives at the origin, and a charge (-q) exits at a lattice point $(\overrightarrow{u_k})$, zero otherwise.

$$q_m = q[\delta_{ml} - \delta_{mk}]. \tag{13}$$

For all m,

At the lattice point $\vec{u}$ the potential is to be $\emptyset(\vec{u})$. According to Ohm's and kirchhoff's laws,

$$\frac{-q(\overrightarrow{u_l})}{C} = \sum_{\vec{x}}[\emptyset(\overrightarrow{u_l}) - \emptyset(\overrightarrow{u_l} + \vec{x}). \tag{14}$$

Where $\vec{x}$ are the vectors from the site $\vec{u}$ to its nearest neighbors ($\vec{x} = \pm\mathbf{a}_l$, l= 1,2, …, d).

Using the so-called lattice Laplacian [18] defined as;

$$\Delta_{(\vec{u})}F(\vec{u}) = \sum_{\vec{x}}[F(\vec{u} + \vec{x}) - F(\vec{u})]. \tag{15}$$

Then, Equation (14) can be rewritten as,

$$\Delta_{(\vec{u})}\emptyset_{(\vec{u})} = \frac{-q(\vec{u})}{C}. \tag{16}$$

The capacitance between the origin and the lattice site $\overrightarrow{u_k}$,

$$C_o(\vec{u}) = \frac{q(\vec{u})}{\emptyset(\overrightarrow{u_l}) - \emptyset(\overrightarrow{u_k})}. \tag{17}$$

We must solve Equation (16), a Poisson-like equation, using the LGF to determine capacitance.

$$\emptyset(\vec{u}) = \frac{1}{C}\sum_{\vec{m}} G_o(\vec{f} - \vec{m})q(\vec{m}). \tag{18}$$

The LGF is defined as:

$$\Delta_{\vec{m}} G_o(\vec{f} - \vec{m}) = -\delta_{f,m}. \tag{19}$$

Using Equation (13) and Equation (18), the potential at origin $(\emptyset(\overrightarrow{u_l}))$ and the potential at the lattice site $\overrightarrow{u_k}$ $(\emptyset(\overrightarrow{u_k}))$ can be written as:

$$\emptyset(\overrightarrow{u_l}) = \frac{q}{C}[G_o(\overrightarrow{u_l}) - G_o(\overrightarrow{u_k})]. \tag{20}$$

Also,

$$\emptyset(\overrightarrow{u_k}) = \frac{q}{C}[G_o(\overrightarrow{u_k}) - G_o(\overrightarrow{u_l})]. \tag{21}$$

Using Equation (21) then,

$$C_o(\vec{u}) = \frac{q}{\emptyset(\overrightarrow{u_l}) - \emptyset(\overrightarrow{u_k})} = \frac{C}{[G_o(\overrightarrow{u_l}) - G_o(\overrightarrow{u_k}) - G_o(\overrightarrow{u_k}) + G_o(\overrightarrow{u_l})]}$$

$$C_o(\vec{u}) = \frac{C}{2[G_o(\vec{u_l}) - G_o(\vec{u_k})]}. \quad (22)$$

The even nature of the LGFs has been exploited. Our primary outcome for the capacitance is Equation (22). Knowing LGF makes it simple to obtain the capacitance of a perfect network, therefore enabling one to compute (LGF) described in Equation (19). First, finding the periodic boundary conditions at the borders of a hypercube with L lattice points along each side allows one to get them.

Equation (19) may be written as [5],

$$G_o(\vec{u}) = s_o \int_{\vec{K}\in BZ} \frac{d^d\vec{K}}{2\pi^d} \frac{e^{(i\vec{K}\vec{u})}}{E(\vec{K})}. \quad (23)$$

Here $a^d = s_o$ is the volume of the unit cell of the d-dimensional hypercube and,

$$E(\vec{K}) = 2\sum_{i=1}^{d}(1 - \cos Ka_i) \quad (24)$$

From (22) and (23) in d-dimensional, the capacitor between the origin $\vec{u_l}$ and the lattice site $\vec{u_k}$ in an integral form as,

$$C_o(\vec{u_k}) = \frac{C}{2s_o \int_{\vec{K}\in BZ} \frac{d^d\vec{K}}{2\pi^d} \frac{1-e^{(i\vec{K}\vec{u_k})}}{E(\vec{K})}} \quad (25)$$

When the lattice site is specified by $\vec{u_k} = n_1\vec{a_1} + n_2\vec{a_2} + n_3\vec{a_3} + ,\dots, + n_d\vec{a_d}$, then the Equation (25) can be simplified as,

$$C_o(\vec{u_k}) = \frac{C}{\int_{-2\pi}^{2\pi}\frac{dx_1}{2\pi}\dots\int_{-2\pi}^{2\pi}\frac{dx_d}{2\pi}\frac{1-e^{\{i(\vec{n_1}x_1+,\dots,+\vec{n_d}x_d)\}}}{2\sum_{i=1}^{d}(1-\cos x_i)}} \quad (26)$$

Finally, we can write the LGF of a d-dimensional hypercube in the following formula,

$$G_o(\vec{n_1}, \vec{n_2}, \vec{n_3}, \dots, \vec{n_d}) =$$

$$\int_{-\pi}^{\pi}\frac{dx_1}{2\pi}\dots\int_{-\pi}^{\pi}\frac{dx_d}{2\pi}\frac{e^{(i\vec{n_1}x_1+,\dots,+i\vec{n_d}x_d)}}{2\sum_{i=1}^{d}(1-\cos x_i)} \quad (27)$$

2.2. Perturbed Lattice

In the perturbed case, a statistical approach is adopted by modeling the removal of a capacitor as a localized disturbance using Dyson’s equation. This perturbative method allows the calculation of the modified LGF and, hence, the new capacitance configuration. Multiple perturbation scenarios are considered, providing insight into how local symmetry breaking affects the overall network behavior.

At the site $\vec{u_l}$ the charge contribution δq_l due to the bond (l_o, k_o) can be written as,

$$\frac{\delta ql}{C} = \delta_{ll_o}(\emptyset_{lo} - \emptyset_{ko}) + \delta_{lk_o}(V_{ko} - V_{lo});$$

$$= \langle l|l_o\rangle(\langle l_o| - \langle k_o|)\emptyset + \langle l|k_o\rangle(\langle k_o| - \langle l_o|)\emptyset;$$

$$\frac{\delta q_l}{C} = \langle l|. \quad (28)$$

Where the operator L_1 is of a so-called “dyadic” form,

$$L_1 = (|l_o\rangle - |k_o\rangle)(\langle l_o| - \langle k_o|) \quad (29)$$

Upon removing the bond (l_o, k_o) from the perfect lattice, the charge q_l at the site $\vec{u_l}$ is given by,

$$-(L_o\emptyset)_l - \frac{1}{C}\delta q_l = -\frac{q_l}{C} \quad (30)$$

For the perturbed lattice, Kirchhoff's and Ohm's equations may be expressed by substituting (28) into (29) as follows:

$$L_{o1}\emptyset = -\frac{q}{C} \quad (31)$$

Where,

$$L_{o1} = L_o + L_1 \quad (32)$$

The LGF G_{o1} for the disturbed lattice is defined similarly to that of the ideal lattice as:

$$L_{o1}G_{o1} = -1 \quad (33)$$

Therefore, Equation (31) becomes,

$$\emptyset = \frac{G_{o1}q}{C} \quad (34)$$

Equation (34) is similar to (10). Here the operator L_{o1} is now a sum of L_o related with the perfect lattice and a perturbation driven by L_{o1}. Assume that the charge to be supplied as in Equation (2) to determine the capacitance between the sites $\vec{u_l}$ and $\vec{u_k}$. Inserting it into (34) produces: To measure the capacitance between the sites. $\vec{u_l}$ and $\vec{u_k}$ We assume that the charge to be given as in Equation (2). So, inserting it into (34) gets:

$$\emptyset_f = \langle f|\emptyset = \frac{\langle K|G_{o1}q}{C};$$

$$= \frac{1}{c}\sum_m \langle f|G_{o1}|m\rangle q_m;$$

$$\emptyset_f = \frac{q}{C}[G_{o1}(f,l) - G_{o1}(f,k). \quad (35)$$

Thus, the capacitance between the sites $\vec{u_l}$ and $\vec{u_k}$ can be written as,

$$\frac{1}{C_{o1}(l,k)} = \frac{\emptyset_l - \emptyset_k}{Q};$$

$$= \frac{1}{C}\left[G_{o1}(l,l) - G_{o1}(l,k) + G_{o1}(k,k) - G_{o1}(k,l)\right] \tag{36}$$

Which can be rewritten as:

$$C_{o1}(l,k) = \frac{C}{\left[G_{o1}(l,l) - G_{o1}(l,k) + G_{o1}(k,k) - G_{o1}(k,l)\right]} \tag{37}$$

Here $G_{o1}(l,l) \neq G_{o1}(k,k)$ because the translational symmetry is broken, but $G_{o1}(l,k) = G_{o1}(k,l)$.

We shall discover that the effective capacitance lowers in the computation of the perturbed LGF. We build the perturbed capacitance of the networks for the ideal one. We gather from Equation (5), and Equation (33).

$$L_{o1}G_{o1} = -1, \text{But } L_{o1} = L_o + L_1$$

$$(L_o + L_1)G_{o1} = -1$$

But $L_o = -G_o^{-1}$ thus, the above relation becomes,

$$(-G_o^{-1} + L_1)G_{o1} = -1 \tag{38}$$

Multiplying both sides Equation (38) from left by G_o. Thus,

$$-G_oG_o^{-1}G_{o1} + G_oL_1G_{o1} = -G_o;$$

$$-G_{o1} + G_oL_1G_{o1} = -G_o. \tag{39}$$

Or,

$$G_o + G_oL_1G_{o1} = G_{o1} = G_o + G_oL_1G_o + G_oL_1G_oL_1G_o \tag{40}$$

Dyson's equation, Equation (40), is an equation for G_{o1} in terms of G_o (which is said to be known), and the perturbation L_1.

To solve the Equation (40), we can use the method presented by Economou [12]. Inserting Equation (29) into the Equation (40), we get,

$$G_{o1}(l,k) = \langle l|G_{o1}|k\rangle$$

$$G_{o1}(l,k) = G_o(l,k)$$

$$+ \frac{[G_o(l,l_o) - G_o(l,k_o)][G_o(l_o,k) - G_o(k_o,k)]}{1 - 2[G_o(l,l_o) - G_o(l_o,k_o)]} \tag{41}$$

The capacitance between $\vec{u_l}$ and $\vec{u_k}$ can be obtained by using Equations (39), (37), and (41). After some simply straight-forward algebra, getting,

$$\frac{C_{o1}(l,k)}{C} = \frac{1}{\frac{1}{C_o(l,k)} + \frac{\left[\frac{1}{C_o(l,k_o)} + \frac{1}{C_o(k,l_o)} - \frac{1}{C_o(l,l_o)} - \frac{1}{C_o(k,k_o)}\right]^2}{4\left[1 - \frac{1}{C_o(l_o,k_o)}\right]}} \tag{42}$$

When the bond (l_o, k_o) is eliminated, our last result for the perturbed effective capacitance between the $\vec{u_l}$ and the site $\vec{u_k}$ is Equation (42).

$$\frac{C_o(l_o,k_o)}{C} = d \quad \text{if } d > 1$$

And then from Equation (42), the capacitance between the two ends of the removed capacitor is,

$$\frac{C_{o1}(l_o,k_o)}{C} = d - 1$$

3. Application: Infinite SC Lattice

The effective capacitance in both ideal and disturbed infinite networks is computed in this work using the LGF approach as a key tool. Using Kirchhoff's and Ohm's laws to a lattice, LGF emerges as the solution to a discrete Poisson-like equation generated. It permits the estimation of potential distributions considering a particular charge configuration and shows the inverse of the lattice Laplacian operator. LGF is calculated in perfect lattices using Fourier transformations across the reciprocal space. Using Dyson's equation, LGF is altered in the perturbed condition, providing a strong instrument for investigating defect-induced fluctuations in capacitance.

An application of the formalism discussed in chapter two is found in this one. Here, we studied an infinite SC network; in section 3.1, we explored the perfect case, where the effective capacitance between the origin and specific sites in the perfect SC network has been computed, and here we expand computations carried out. Section 3.2 addresses the situation in which capacitance is lost (the network is disturbed), and we derive the effective capacitance of the perturbed SC network spanning several lattice sites from the origin. The eliminated capacitor has been collected at multiple locations for the first time.

3.1. Perfect Case

Applying the fundamental results in Chapter Two, we will determine the effective capacitance in this part's perfect case of a simple cubic network.

Now, one may define the energy-dependent LGF by specifying the Tight-Binding Hamiltonian of the SC lattice as follows [12].

$$G_o(E,\overrightarrow{n_1},\overrightarrow{n_2},\overrightarrow{n_3}) = \int_{-\pi}^{\pi}\frac{dx}{2\pi}\int_{-\pi}^{\pi}\frac{dy}{2\pi}\int_{-\pi}^{\pi}\frac{dz}{2\pi}\,\frac{1-\cos(\overrightarrow{n_1}x+\overrightarrow{n_2}y+\overrightarrow{n_3}z)}{E-\cos x-\cos y-\cos z} \tag{43}$$

This is the generalization of the LGF by using a new variable (E) instead of the value (3) in the Equation (44) for d=3.

Consider a Simple Cubic Network (SC) consisting of identical capacitors C. The capacitance between the origin $\overrightarrow{u_l}$ and the lattice site $\overrightarrow{u_k} = n_1\overrightarrow{a_1} + n_2\overrightarrow{a_2} + n_3\overrightarrow{a_3}$ can be obtained from the general formula given in Equation (43) by taking d = 3. Thus,

$$C_o(\overrightarrow{n_1},\overrightarrow{n_2},\overrightarrow{n_3}) = \frac{C}{\int_{-\pi}^{\pi}\frac{dx}{2\pi}\int_{-\pi}^{\pi}\frac{dy}{2\pi}\int_{-\pi}^{\pi}\frac{dz}{2\pi}\frac{1-\cos 0\overrightarrow{n_1}x+\overrightarrow{n_2}y+\overrightarrow{n_3}z)}{3-\cos x-\cos y-\cos z}} \tag{44}$$

We can easily calculate the effective capacitance between two lattice sites in the infinite SC network from Equation (45) (due to symmetry reason) as:

$$\frac{C}{C_o(1,0,0)} + \frac{C}{C_o(0,1,0)} + \frac{C}{C_o(0,0,1)} = \int_{-\pi}^{\pi}\frac{dx}{2\pi}\int_{-\pi}^{\pi}\frac{dy}{2\pi}\int_{-\pi}^{\pi}\frac{dz}{2\pi} = 1 \tag{45}$$

Here, the effective capacitance between two lattice sites is 3C; comparable results were achieved with the charge distribution approach.

The LGF of the SC at the origin ($G_o\ (3; 0,0,0) = g_o$) was estimated, as it was expressed by a closed form within the elliptic integrals as follows [10],

$$G\ (3; 0,0,0) = g_o = (\tfrac{2}{\pi})^2(18 + 12\sqrt{2} - 10\sqrt{3} - 7\sqrt{6}\,)[K(k_o)]^2 = 0.5054620197 \tag{46}$$

Where,

$K(k)$ is the complete elliptic integral of the first kind ($K(k) = \int_0^{\frac{\pi}{2}} d\theta\ \frac{1}{\sqrt{1-k^2(\sin\theta)^2}}$).

and,

k_o is its modulus ($k_o = (2-\sqrt{3})(\sqrt{3}-\sqrt{2})$).

Then, we can express the LGF of a SC by known values. g_o and π as;

$$G_o(3; n_1, n_2, n_3) = p_1 g_o(3; 0,0,0) + \frac{p_2}{\pi^2 g_o(3;0,0,0)} + p_3 \tag{47}$$

We can rewrite the above equation as follows;

$$G_o(3; n_1, n_2, n_3) = p_1 g_o + \frac{p_2}{\pi^2 g_o} + p_3 \tag{48}$$

Here, $p_1 = \beta_1 + \frac{5}{12}\beta_2$, $p_2 = -\frac{1}{2}\beta_2$, $p_3 = -\frac{1}{3}\beta_3$,

While p_1, p_2, p_3, related to Duffin and Shelly's parameter [19], and $\beta_1, \beta_2, \beta_3$ are rational numbers.

Finally, the effective capacitance of an infinite SC network of identical capacitors between the origin (0,0,0) and any other site (n_1, n_2, n_3) can be expressed as;

$$C_o(n_1, n_2, n_3) = \frac{C}{[G_o(3;0,0,0) - G_o(3;n_1,n_2,n_3)]} \tag{49}$$

We can rewrite the Equation (49) as follows,

$$C_o(n_1, n_2, n_3) = \frac{C}{\gamma_1 g_o + \frac{\gamma_2}{\pi^2 g_o} + \gamma_3} \tag{50}$$

Where γ_1, γ_2 and γ_3 are rational numbers related to Duffin and Shelly's [9] parameters p_1, p_2 and p_3 as;

$$\gamma_1 = 1 - p_1$$

$$\gamma_2 = -p_2$$

$$\gamma_3 = -p_3$$

Various values for γ_1, γ_2 and γ_3 can be obtained from (Table 1) [17] for (n_1, n_2, n_3)ranging from (0,0,0) - (5,5,5) and Various values for γ_1, γ_2 and γ_3 can be obtained from (Table 1) for (n_1, n_2, n_3)ranging from (6,0,0) - (6,5,5).

We can calculate different values of γ_1, γ_2 and γ_3 for other sites using the following recurrence relation [10];

$$G_o(n_1+1, n_2, n_3) + G_o(n_1-1, n_2, n_3) + G_o(n_1, n_2+1, n_3) + G_o(n_1, n_2-1, n_3) + G_o(n_1, n_2, n_3+1) + G_o(n_1, n_2, n_3-1) = -2\delta_{n_{10}}\delta_{n_{20}}\delta_{n_{30}} + 2EG_o(n_1, n_2, n_3). \tag{51}$$

In some cases, one may use Equation (51) two or three times to calculate different values of γ_1, γ_2 and γ_3 for (n_1, n_2, n_3) beyond (6,5,5).

We aim to find the effective capacitance between the origin and another lattice site (n_1, n_2, n_3). First, we are calculating the LGF for the lattice site (n_1, n_2, n_3) by substituting into Equation (51) and then substituting into Equation (48).

Below, we present some examples showing the application mechanism for calculating the effective capacitance between the origin and other sites. Our results are shown in Table 1. Consider the following examples:

To study the asymptotic behavior of the effective capacitance as the separation between the origin and the site (n_1, n_2, n_3) goes to an infinity or significant value in this case from Equation (49), the effective capacitance goes to a finite value, we insert that $G_o(n_1, n_2, n_3) \to 0$ into Equation (49) where we got,

$$C_o(n_1, n_2, n_3) = \frac{C}{G_o(0,0,0)} = \frac{C}{g_o} = \frac{C}{0.5054620197} = 1.9783880$$

$C =$ finite value when any of $(n_1, n_2, n_3) \to \infty$.

3.2. Perturbed Case

In this section, we will calculate the effective capacitance in the perturbed case of the SC network by applying the basic results in section two.

We consider a perturbed case where we removed a capacitor from the perfect, infinite SC network and let the removed capacitor be between the sites. $l_o = (l_{ox}, l_{oy}, l_{oz})$ and $k_o = (k_{ox}, k_{oy}, k_{oz})$. We aim to find the effective capacitance between the sites $l = (l_x, l_y, l_z)$ and $k = (k_x, k_y, k_z)$ for the perturbed case.

Firstly, let us consider the removed capacitance to be between the site $l_o = (0,0,0)$ and the site $k_o = (1,0,0)$, we need to find the capacitance between any sites $l = (l_x, l_y, l_z)$ and $k = (k_x, k_y, k_z)$. To do this, one should use Equation (42). Our results are shown in Table 2. Below, we show an illustrative example.

Example: Find the effective capacitance between $l = (0,0,0)$ and $k = (7,0,0)$

$[l = (0,0,0)$ and $k = (7,0,0)]$,$[l_o = (0,0,0)$,$k_o = (1,0,0)]$

$$\frac{C_{o1}(l,k)}{C} = \frac{1}{\frac{1}{C_o(l,k)} + \frac{\left[\frac{1}{C_o(l,k_o)} + \frac{1}{C_o(k,l_o)} - \frac{1}{C_o(l,l_o)} - \frac{1}{C_o(k,k_o)}\right]^2}{4\left[1 - \frac{1}{C_o(l_o,k_o)}\right]}}$$

$(C\_o1\ (l,k))/C = 1/(1/(C\_o\ (l,k)\ )\ +\ ⟦[1/(C\_o\ ((l\_x - k\_ox), (l\_y - k\_oy), (l\_z - k\_oz))\ ) + 1/(C\_o\ ((k\_x - l\_ox), (k\_y - l\_oy), (k\_z - l\_oz))\ ) - 1/(C\_o\ ((l\_x - l\_ox), (l\_y - l\_oy), (l\_z - l\_oz))\ ) - 1/(C\_o\ ((k\_x - k\_ox), (k\_y - k\_oy), (k\_z - k\_oz))\ )]/4[1 -\ 1/(C\_o\ ((l\_ox - k\_ox), (l\_oy - k\_oy), (l\_oz - k\_oz))\ )]\ ⟧\text{^}2\ )$

$(C\_o1\ (l,k))/C\ = 1/(1/(C\_o\ (l,k)\ )\ +\ ⟦[1/(C\_o\ ((0 - 1), (0 - 0), (0 - 0))\ ) + 1/(C\_o\ ((7 - 0), (0 - 0), (0 - 0))\ ) - 1/(C\_o\ ((0 - 0), (0 - 0), (0 - 0))\ ) - 1/(C\_o\ ((7 - 1), (0 - 0), (0 - 0))\ )]/4[1 -\ 1/(C\_o\ ((0 - 1), (0 - 0), (0 - 0))\ )]\ ⟧\text{^}2\ )$

$$\frac{C_{o1}(7,0,0)}{C} = \frac{1}{\frac{1}{C_o(7,0,0)} + \frac{\left[\frac{1}{C_o(-1,0,0)} + \frac{1}{C_o(7,0,0)} - \frac{1}{C_o(0,0,0)} - \frac{1}{C_o(6,0,0)}\right]^2}{4\left[1 - \frac{1}{C_o(-1,0,0)}\right]}}$$

But, $C_o(n_1, n_2, n_3) = C_o(-n_1, -n_2, -n_3)$

Then

$$\frac{C_{o1}(7,0,0)}{C} = \frac{1}{\frac{1}{C_o(7,0,0)} + \frac{\left[\frac{1}{C_o(1,0,0)} + \frac{1}{C_o(7,0,0)} - \frac{1}{C_o(0,0,0)} - \frac{1}{C_o(6,0,0)}\right]^2}{4\left[1 - \frac{1}{C_o(1,0,0)}\right]}}$$

$$\frac{C_{o1}(7,0,0)}{C} = \frac{1}{\frac{1}{2.07210} + \frac{\left[\frac{1}{3} + \frac{1}{2.07210} - \frac{1}{\infty} - \frac{1}{2.08885}\right]^2}{4\left[1 - \frac{1}{3}\right]}} = 1.903884819$$

$C_{o1}(7,0,0) = 1.903884819.$

Secondly, the removed capacitance is shifted and becomes between the site $l_o = (1,0,0)$ and the site $k_o = (2,0,0)$, then we need to find the capacitance between any site $l = (l_x, l_y, l_z)$ and $k = (k_x, k_y, k_z)$. To do this, one should use Equation (41). Our results are arranged in Table 2. Below, we show an illustrative example.

Example: Find the effective capacitance between $l = (0,0,0)$ and $k = (-7,0,0)$

$[l = (0,0,0)$ and $k = (-7,0,0)]$, $[l_o = (1,0,0)$ and $k_o = (2,0,0)]$

$(C\_o2\ (l,k))/C\ \ = 1/(1/(C\_o\ (l,k)\ )\ +\ ⟦[1/(C\_o\ ((0 - 2), (0 - 0), (0 - 0))\ ) + 1/(C\_o\ ((-7 - 1), (0 - 0), (0 - 0))\ ) - 1/(C\_o\ ((1 - 0), (0 - 0), (0 - 0))\ ) - 1/(C\_o\ ((-7 - 2), (0 - 0), (0 - 0))\ )]/4[1 -\ 1/(C\_o\ ((1 - 2), (0 - 0), (0 - 0))\ )]\ ⟧\text{^}2\ )$

$$\frac{C_{o2}(-7,0,0)}{C} = \frac{1}{\frac{1}{C_o(-7,0,0)} + \frac{\left[\frac{1}{C_o(-2,0,0)} + \frac{1}{C_o(-8,0,0)} - \frac{1}{C_o(1,0,0)} - \frac{1}{C_o(-9,0,0)}\right]^2}{4\left[1 - \frac{1}{C_o(-1,0,0)}\right]}} = \frac{1}{\frac{1}{C_o(7,0,0)} + \frac{\left[\frac{1}{C_o(2,0,0)} + \frac{1}{C_o(8,0,0)} - \frac{1}{C_o(1,0,0)} - \frac{1}{C_o(9,0,0)}\right]^2}{4\left[1 - \frac{1}{C_o(1,0,0)}\right]}}$$

$$\frac{C_{o2}(-7,0,0)}{C} = \frac{1}{\frac{1}{2.0721} + \frac{\left[\frac{1}{2.38275} + \frac{1}{2.05979} - \frac{1}{3} - \frac{1}{2.05034}\right]^2}{4\left[1 - \frac{1}{3}\right]}} = 2.060770959$$

$C_{o2}(-7,0,0) = 2.060770959$

Thirdly, the removed capacitance is shifted and becomes between the site $l_o = (0,0,0)$ and the site $k_o = (-1,0,0)$, Our results are arranged in the Table 3. Below, we show an illustrative example.

Example: Find the effective capacitance between $l = (0,0,0)$ and $k = (-7,0,0)$

$[l = (0,0,0)$ and $k = (-7,0,0)]$, $[l_o = (1,0,0)$ and $k_o = (2,0,0)]$

$$(C\_o2\ (l,k))/C \ = 1/(1/(C\_o\ (l,k)\ )\ +\ 〖[1/(C\_o\ ((0-2),(0-0),(0-0))\ ) + 1/(C\_o\ ((-7-1),(0-0),(0-0))\ ) - 1/(C\_o\ ((1-0),(0-0),(0-0))\ ) - 1/(C\_o\ ((-7-2),(0-0),(0-0))\ )]/4[1 - 1/(C\_o\ ((1-2),(0-0),(0-0))\ )]〗\text{^}2\ )\ \frac{C_{o2}(-7,0,0)}{C} =$$

$$\frac{1}{\frac{1}{C_o(-7,0,0)}+\frac{\left[\frac{1}{C_o(-2,0,0)}+\frac{1}{C_o(-8,0,0)}-\frac{1}{C_o(1,0,0)}-\frac{1}{C_o(-9,0,0)}\right]^2}{4\left[1-\frac{1}{C_o(-1,0,0)}\right]}} = \frac{1}{\frac{1}{C_o(7,0,0)}+\frac{\left[\frac{1}{C_o(2,0,0)}+\frac{1}{C_o(8,0,0)}-\frac{1}{C_o(1,0,0)}-\frac{1}{C_o(9,0,0)}\right]^2}{4\left[1-\frac{1}{C_o(1,0,0)}\right]}}$$

$$\frac{C_{o2}(-7,0,0)}{C} = \frac{1}{\frac{1}{2.0721}+\frac{\left[\frac{1}{2.38275}+\frac{1}{2.05979}-\frac{1}{3}-\frac{1}{2.05034}\right]^2}{4\left[1-\frac{1}{3}\right]}} = 2.060770959$$

$$C_{o2}(-7,0,0) = 2.060770959$$

Fourthly, the removed capacitance is shifted and becomes between the site $l_o = (-1,0,0)$ and the site $k_o = (-2,0,0)$. Our results are arranged in Table 3. Below, we show an illustrative example.

Example: Find the effective capacitance between $l = (0,0,0)$ and $k = (-7,0,0)$

$[l = (0,0,0)$ and $k = (-7,0,0)]$, $[l_o = (1,0,0)$ and $k_o = (2,0,0)]$

$$(C\_o2\ (l,k))/C \ = 1/(1/(C\_o\ (l,k)\ )\ +\ 〖[1/(C\_o\ ((0-2),(0-0),(0-0))\ ) + 1/(C\_o\ ((-7-1),(0-0),(0-0))\ ) - 1/(C\_o\ ((1-0),(0-0),(0-0))\ ) - 1/(C\_o\ ((-7-2),(0-0),(0-0))\ )]/4[1 - 1/(C\_o\ ((1-2),(0-0),(0-0))\ )]〗\text{^}2\ )\ \frac{C_{o2}(-7,0,0)}{C} =$$

$$\frac{1}{\frac{1}{C_o(-7,0,0)}+\frac{\left[\frac{1}{C_o(-2,0,0)}+\frac{1}{C_o(-8,0,0)}-\frac{1}{C_o(1,0,0)}-\frac{1}{C_o(-9,0,0)}\right]^2}{4\left[1-\frac{1}{C_o(-1,0,0)}\right]}} = \frac{1}{\frac{1}{C_o(7,0,0)}+\frac{\left[\frac{1}{C_o(2,0,0)}+\frac{1}{C_o(8,0,0)}-\frac{1}{C_o(1,0,0)}-\frac{1}{C_o(9,0,0)}\right]^2}{4\left[1-\frac{1}{C_o(1,0,0)}\right]}}$$

$$\frac{C_{o2}(-7,0,0)}{C} = \frac{1}{\frac{1}{2.0721}+\frac{\left[\frac{1}{2.38275}+\frac{1}{2.05979}-\frac{1}{3}-\frac{1}{2.05034}\right]^2}{4\left[1-\frac{1}{3}\right]}} = 2.060770959$$

$$C_{o2}(-7,0,0) = 2.060770959$$

4. Potential Practical Application

Interconnect resistance in large-scale integrated circuits is greatly modelled using LGF techniques. LGFs used in infinite resistor networks allow engineers to analytically simulate complicated current paths inside semiconductor layouts analytically, hence optimising connection designs to lower power loss and improve device dependability [18].

Widely used in touch-sensitive user interfaces and proximity detection systems in industrial automation and robotics, lattice models of capacitive networks are vital in analysing capacitive sensor arrays. The strong design of high-resolution sensor grids is supported by their consistent behaviour under lattice perturbations [19].

Materials science investigates how structural flaws like microvoids or dislocations impact mechanical or electrical performance using lattice network models-including those utilising LGFs. These techniques improve material dependability by predicting stress concentration zones or changes in conductivity [20].

Lattice-based design ideas inspire self-healing circuit technologies especially those incorporating soft or wearable electronics. Designed utilising lattice networks, liquid-metal composites replicate the transfer of current to other routes when damage occurs, hence allowing self-repair mechanisms in next-generation electronics [21]. Crucially in contexts like aerospace or defence, where system robustness is vital, lattice-based frameworks help to create adaptable electronic systems with customisable paths. By dynamically rerouting signal routes, these networks can preserve operation even with localised failure [22].

Modern robotics benefit from lattice analysis techniques guiding the construction of dense, sensitive capacitive networks, hence enabling high-resolution touch sensors. These sensors provide responsive feedback systems in robotic hands by emulating human-like touch sensitivity, facilitating dexterous handling [23]. Especially when grounded in LGF theory, lattice network analysis offers a basis for estimating failure locations in materials exposed to environmental conditions, stress, or fatigue. The development of durable materials in mechanical, aeronautical, and infrastructural systems depends on these predictive instruments [24].

Pressure sensors designed with LGF-based capacitive networks help dependably operate in demanding environments. These are essential in aerospace fields, where materials must be stable at high temperatures, pressure, or radiation exposure [25]. The idea of lattice networks underpins the construction of flexible electronics. LGF techniques help to simulate stretchable interconnects and bendable sensors such that they retain conductivity and function under mechanical deformation [26].

Finally, strain distribution and defect interactions in semiconductors are modelled extensively using LGF approaches. Better strain engineering and defect mitigating are possible by enabling carrier mobility and device stability in modern semiconductor devices [27].

5. Results and Discussion

Tables 1–4 present detailed numerical results for perfect and perturbed cases to aid clarity. These include specific lattice points, associated LGF values, and computed capacitances. This step-by-step tabulation demonstrates the application of theoretical results to practical computation.

We conducted numerical simulations by truncating the infinite lattice and solving the network equations with boundary conditions. These results were compared with the analytical LGF-based solutions, and close agreement was found. For instance, the capacitance at (7,0,0) matched within a 0.1% margin between analytical and numerical approaches, validating the robustness of our formulation. In this section, we will show numerical calculations of the infinite SC network in the perfect and perturbed cases.

Table 1. Computed values of the effective capacitance between the origin and the other site $(\mathbf{n_1}, \mathbf{n_2}, \mathbf{n_3})$ along [100] direction in the perfect, infinite SC network

The site (n_1, n_2, n_3)	$C_o\ (n_1, n_2, n_3)$	The site (n_1, n_2, n_3)	$C_o\ (n_1, n_2, n_3)$
(7,0,0)	2.072103394	(8,1,1)	2.058435514
(7,1,0)	2.071047851	(8,2,0)	2.057148846
(7,1,1)	2.070031587	(8,2,1)	2.056531667
(7,2,0)	2.068115189	(8,2,2)	2.054762611
(7,2,1)	2.067199287	(8,3,0)	2.054208354
(7,2,2)	2.064628469	(8,3,1)	2.053662616
(7,3,0)	2.063842462	(8,3,2)	2.052082385
(7,3,1)	2.063056997	(8,3,3)	2.04967388
(7,3,2)	2.060832431	(9,0,0)	2.05034269
(7,3,3)	2.057507711	(9,1,0)	2.049875386
(7,4,0)	2.058816897	(9,1,1)	2.049419901
(7,4,1)	2.058166208	(9,2,0)	2.048503844
(7,4,2)	2.056303077	(9,2,1)	2.048078545
(7,4,3)	2.053484593	(9,2,2)	2.046807203
(7,4,4)	2.050020002	(10,0,0)	2.042835586
(8,0,0)	2.059790358	(10,1,0)	2.042531548
(8,1,0)	2.05910342	(11,0,0)	2.036591443

Table 2. Computed values for the effective capacitance of an infinite SC lattice between $\boldsymbol{l} = (\mathbf{0}, \mathbf{0}, \mathbf{0})$ *and* $\mathbf{k} = (\mathbf{k_x}, \mathbf{k_y}, \mathbf{k_z})$ along [100] direction for a perfect lattice $\boldsymbol{C_o}(\boldsymbol{l}, \boldsymbol{k})$; perturbed lattice due to removing a capacitance between (0,0,0), (1,0,0) and (1,0,0), (2,0,0): $\boldsymbol{C_{o1}}(\boldsymbol{l}, \boldsymbol{k})$ and $\boldsymbol{C_{o2}}(\boldsymbol{l}, \boldsymbol{k})$

The site (n_1, n_2, n_3)	C_o (l, k)	C_{o1} (l, k)	C_{o2} (l, k)
(7,0,0)	2.072103394	1.903884819	2.058613251
(8,0,0)	2.059790358	1.894380216	2.046920523
(9,0,0)	2.05034269	1.886963888	2.037863403
(-7,0,0)	2.072103394	1.910034159	2.060770959
(-8,0,0)	2.059790358	1.898990981	2.048476157
(-9,0,0)	2.05034269	1.890558872	2.039053279

Table 3. Computed values for the effective capacitance of an infinite SC lattice between $\boldsymbol{l} = (\mathbf{0}, \mathbf{0}, \mathbf{0})$ $\boldsymbol{and}$ $\mathbf{k} = (\mathbf{k_x}, \mathbf{k_y}, \mathbf{k_z})$ along [100] direction for a perfect lattice $\boldsymbol{C_o(l, k)}$; perturbed lattice due to removing a capacitance between (0,0,0), (-1,0,0) and (-1,0,0), (-2,0,0): $\boldsymbol{C_{o3}(l, k)}$ and $\boldsymbol{C_{o4}(l, k)}$

The site (n_1, n_2, n_3)	C_o (l, k)	C_{o3} (l, k)	C_{o4} (l, k)
(0,0,0)	∞	∞	∞
(1,0,0)	3	2.807346939	2.989582698
(2,0,0)	2.382748573	2.202493543	2.371813429
(3,0,0)	2.220387888	2.047131045	2.209205938
(4,0,0)	2.151072924	1.982224017	2.139779276
(5,0,0)	2.113003994	1.947151619	2.101693611
(6,0,0)	2.088848597	1.925161104	2.077632697
(7,0,0)	2.072103394	1.910034159	2.060770959
(8,0,0)	2.059790358	1.898990981	2.048476157
(9,0,0)	2.05034269	1.890558872	2.039053279
(-1,0,0)	3	2.000002667	2.807346939
(-2,0,0)	2.382748573	2.058742658	2.058742658
(-3,0,0)	2.220387888	1.999747566	2.166585037
(-4,0,0)	2.151072924	1.95978949	2.127561543
(-5,0,0)	2.113003994	1.934057844	2.096102857
(-6,0,0)	2.088848597	1.916523842	2.074270518
(-7,0,0)	2.072103394	1.903884819	2.058613251
(-8,0,0)	2.059790358	1.894380216	2.046920523
(-9,0,0)	2.05034269	1.886963888	2.037863403

Table 4. Calculated values for the effective capacitance of an infinite SC lattice between $\boldsymbol{l} = (\mathbf{0}, \mathbf{0}, \mathbf{0})$ $\boldsymbol{and}$ $\mathbf{k} = (\mathbf{k_x}, \mathbf{k_y}, \mathbf{k_z})$ along [100] direction for a perfect lattice $\boldsymbol{C_o(l, k)}$; perturbed lattice due to removing a capacitance between (3,0,0), (4,0,0): $\boldsymbol{C_{o5}(l, k)}$

The site (n_1, n_2, n_3)	C_o (l, k)	C_{o5} (l, k)
(0,0,0)	∞	∞
(1,0,0)	3	2.999120298
(2,0,0)	2.382748573	2.371813429
(3,0,0)	2.220387888	2.047131045
(4,0,0)	2.151072924	1.959789491
(5,0,0)	2.113003994	2.096102857
(6,0,0)	2.088848597	2.087736212
(7,0,0)	2.072103394	2.070744255
(8,0,0)	2.059790358	2.058956759
(9,0,0)	2.05034269	2.049710546
(-1,0,0)	3	2.999875863
(-2,0,0)	2.382748573	2.382575945
(-3,0,0)	2.220387888	2.220180573
(-4,0,0)	2.151072924	2.150618359
(-5,0,0)	2.113003994	2.112747718
(-6,0,0)	2.088848597	2.088585173
(-7,0,0)	2.072103394	2.071827454

In this part, section one is dedicated to the perfect case results, and section two is concerned with the disturbed case findings. The results of the capacitance for an infinite SC network are discussed.

5.1. Perfect Case Result

Perfect case results of the effective capacitance for a 3D lattice (SC) between the origin site and the other site (n_1, n_2, n_3). are shown in Figure 1. This quantity shows the effective capacitance against the site (n_1, n_2, n_3) along [100] direction for a perfect, infinite SC network. Figure unequivocally indicates that the capacitance is symmetric along [100] direction (i.e., $(\mathrm{n}_1, \mathrm{n}_2, \mathrm{n}_3) = (-\mathrm{n}_1, -\mathrm{n}_2, -\mathrm{n}_3)$). Due to the inversion symmetry of the lattice, this is simple and obvious for the ideal condition.

5.2. Perturbed Case Result

Figures 1-5 demonstrate the outcomes of the effective capacitance for SC lattice between the origin and any other site (n_1, n_2, n_3) under the perturbed condition.

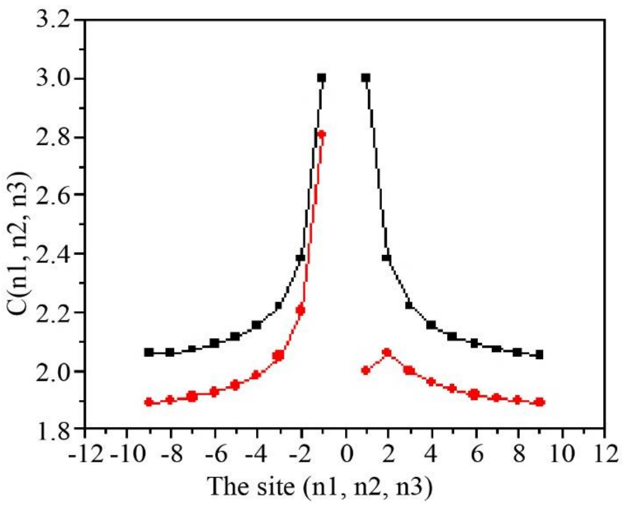

Fig. 1 Along [100], the effective capacitance between the origin and the site (n1, n2, n3) is computed. While red (circles) show a disturbed case $\mathbf{C_{o1}(l,k)}$, black (squares) denotes perfect case.

The net capacitance between the origin and any other lattice site against the site (n_1, n_2, n_3) along [100] direction for a perfect SC network (squares) and a disturbed SC network (circles) where the link between $l_o = (0,0,0)\ and\ k_o = (1,0,0)$ is broken in Figure 1.

Figure 2 shows the net capacitance between the origin and any other lattice site against the site along [100] direction for a perfect SC network (squares) and a perturbed SC network (circles) where the bond between $l_o = (1,0,0)\ and\ k_o = (2,0,0)$ is broken.

The equivalent capacitance between the origin and any other lattice site against the site along [100] direction is displaced for a perfect SC network (squares) and a disturbed SC network (circles) when the bond between $l_o = (0,0,0)\ and\ k_o = (-1,0,0)$ is broken in Figure 3.

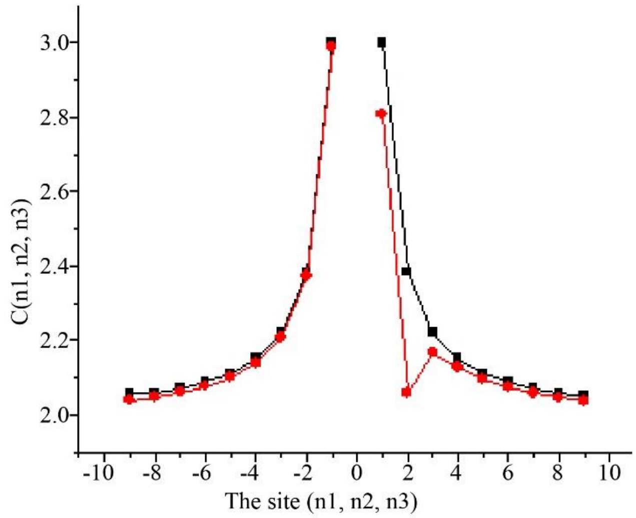

Fig. 2 Along [100], the effective capacitance between the origin and the site (n1, n2, n3) is computed. While red (circles) show a disturbed case $\mathbf{C_{o2}(l,k)}$, black (squares) denotes perfect case.

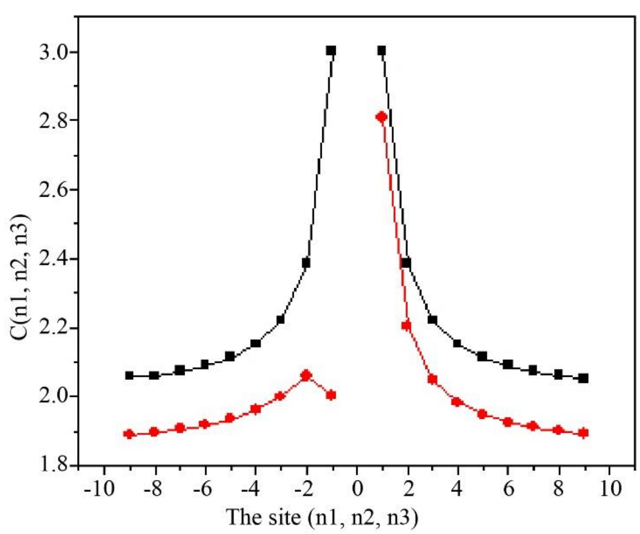

Fig. 3 Along [100], the effective capacitance between the origin and the site (n1, n2, n3) is computed. While red (circles) show a disturbed case $\mathbf{C_{o3}(l,k)}$, black (squares) denotes perfect case.

Figure 4 displays the effective capacitance between the origin and any other lattice site against the site along [100] direction for a perfect SC network (squares) and a disturbed SC network (circles) where the link between l_o= (-1,0,0) and k_o= (-2,0,0) is broken.

Figure 5 shows the equivalent capacitance between the origin and any other lattice site against the site along [100] direction for a perfect SC network (squares) and a perturbed SC network (circles) where the bond between l_o= (3,0,0) and k_o= (4,0,0) is broken.

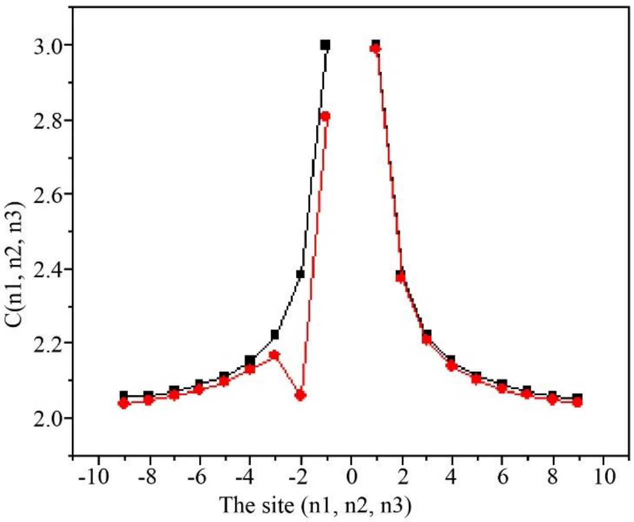

Fig. 4 Along [100], the effective capacitance between the origin and the site (n1, n2, n3) is computed. While red (circles) show a disturbed case $C_{o4}(l, k)$, black (squares) denotes perfect case.

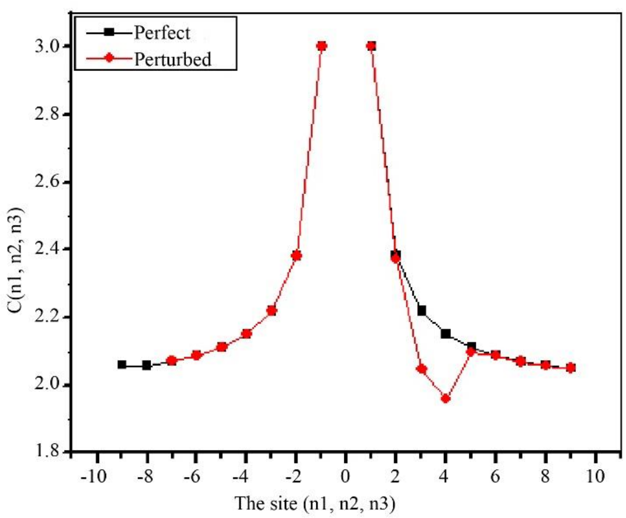

Fig. 5 Along [100], the effective capacitance between the origin and the site (n1, n2, n3) is computed. While red (circles) shows disturbed case $C_{o5}(l, k)$, black (squares) denotes perfect case.

One can see that removing a capacitor from the perfect SC network decreases the effective capacitance, especially when the removed capacitor is close to the origin, as shown in Figure 1. Also, it affects the symmetry around the removed capacitor $(n_1,n_2,n_3) \neq (-n_1,-n_2,-n_3)$.

However, comparing Figure 1 with Figure 3, and Figure 2 with Figure 4, we see a mirror reflection of results such that the measured values in both directions (positive and negative) were nearly affected by the same amount after removing the capacitor.

On the other hand, if the capacitor is removed from the origin, the effective capacitance will slightly change in far lattice points, as shown in Figures 4-5.

6. Conclusion

The approach yields improved accuracy and coverage compared to earlier studies due to three factors:

1) The use of Dyson's equation for treating perturbations rigorously,
2) The use of elliptic integrals and recurrence relations for precise LGF evaluation, and
3) Extensive validation through simulations. These enable calculation for longer-range interactions and multiple perturbations, which were not previously reported.

The results show improvement over previously reported findings due to several key methodological advancements.

First, we extend the use of Dyson's equation to analyze multiple perturbations systematically, while prior studies focused primarily on single perturbation effects.

Second, we utilize more accurate computation of the Lattice Green's Function (LGF) using closed-form expressions involving elliptic integrals [11, 14] and recurrence relations [10], allowing us to calculate capacitances at larger lattice separations with greater precision. Third, we validate our analytical expressions against numerical simulations and observe excellent agreement, confirming the reliability of our model. These refinements enable us to compute the effective capacitance not just for limited cases but across a broader range of perturbed configurations and spatial separations-something not fully addressed in earlier capacitor network literature.

This extended capability makes our approach more versatile and comprehensive compared to existing models in the literature. Theoretically, this study aimed to calculate, in an infinite SC network of equal capacitance, the effective capacitance between the origin (0,0,0) and every other lattice site (n1, n2, n3) under both the perfect and disturbed conditions. In an infinite SC network, the effective capacitance between the origin (0,0,0) and every other lattice site (n1, n2, n3) is rational regarding LGF using a recurrence formula. This scientific study has various benefits.

1) It may be applied in intricate constructions like Body Centre Cubic (BCC) and Face Centre Cubic (FCC).
2) Their outcomes show the symmetry of infinite lattice constructions.
3) Eventually, our research on capacitance networks might be a suitable illustration of the LGF introduction at the undergraduate level.

Acknowledgments

The authors thank Palestine Technical University-Kadoorie for supporting them financially.

References

[1] Peter G. Doyle, and J. Laurie Snell, *Random Walk and Electrical Networks*, The Carus Mathematical Monograph, The Mathematical Association of America, vol. 22, 1984. [Google Scholar] [Publisher Link]

[2] D. Atkinson, and F.J. Van Steenwijk, "Infinite Resistance Lattices," *American Journal of Physics*, vol. 67, pp. 486-492, 1999. [CrossRef] [Google Scholar] [Publisher Link]

[3] G. Rickayzen, *Green's Functions and Condensed Matter*, Dover Publications, Academic Press, London, 1980. [Google Scholar] [Publisher Link]

[4] Giulio Venezian, "On the Resistance Between Two Points on a Grid," *American Journal of Physics*, vol. 62, no. 11, pp. 1000-1004, 1994. [CrossRef] [Google Scholar] [Publisher Link]

[5] József Cserti, "Application of the Lattice Green's Function: Resistance of an Infinite Network of Resistors," *American Journal of Physics*, vol. 68, no. 10, pp. 896-906, 2000. [CrossRef] [Google Scholar] [Publisher Link]

[6] József Cserti, Gyula Dávid, and Attila Piróth, "Perturbation of Infinite Networks of Resistance," *American Journal of Physics*, vol. 70, no. 2, pp. 153-159, 2002. [CrossRef] [Google Scholar] [Publisher Link]

[7] J. Cserti, G. Széchenyi, and G. Dávid, "Uniform Tiling with Electrical Resistors," *Journal of Physics A: Mathematical and Theoretical*, vol. 44, no. 21, 2011. [CrossRef] [Google Scholar] [Publisher Link]

[8] M. Owaidat, "On the Uniform Tiling with Electrical Resistors," *arXiv preprint*, 2013. [CrossRef] [Google Scholar] [Publisher Link]

[9] R.J. Duffin, and E.B. Shelly, "Difference Equation of Poly-Harmonic Type," *Duke Mathematical Journal*, vol. 25, no. 2, pp. 209-238, 1958. [CrossRef] [Google Scholar] [Publisher Link]

[10] Tsuyoshi Horiguchi, "Lattice Green's Function for the Simple Cubic Lattice," *Journal of the Physical Society of Japan*, vol. 30, no. 5, pp. 1261-1271, 1971. [CrossRef] [Google Scholar] [Publisher Link]

[11] George N. Watson, "Three Triple Integrals," *The Quarterly Journal of Mathematics*, vol. os-10, no. 1, pp. 266-276, 1939. [CrossRef] [Google Scholar] [Publisher Link]

[12] Eleftherios N. Economou, *Green's Functions in Quantum Physics*, 2nd ed., Springer-Verlag, Berlin, 1983. [CrossRef] [Google Scholar] [Publisher Link]

[13] Robert B. Laughlin, "Anomalous Quantum Hall Effect: An Incompressible Quantum Fluid with Fractionally Charged Excitations," *Physical Review Letters*, vol. 44, pp. 225-236, 2005. [CrossRef] [Google Scholar] [Publisher Link]

[14] G.S. Joyce, "Lattice Green Function for the Simple Cubic Lattice," *Journal of Physics A: General Physics*, vol. 5, no. 8, 1972. [CrossRef] [Google Scholar] [Publisher Link]

[15] Sayak Bhattacharjee, and Kabir Ramola, "Green's Functions for Random Resistor Networks," *Physical Review E*, vol. 108, no. 4, 2023. [CrossRef] [Google Scholar] [Publisher Link]

[16] Joseph A. Yasi, and Dallas R. Trinkle, "Direct Calculation of the Lattice Green Function with Arbitrary Interactions for General Crystals," *Physical Review E-Statistical, Nonlinear, and Soft Matter Physics*, vol. 85, no. 6, 2012. [CrossRef] [Google Scholar] [Publisher Link]

[17] M.L. Glasser, and J. Boersma, "Exact Values for Cubic Lattice Green Functions," *Journal of Physics A: Mathematical and General*, vol. 33, no. 28, pp. 5017-5023, 2000. [CrossRef] [Google Scholar] [Publisher Link]

[18] Xinsheng Wang et al., "On Model Order Reduction of Interconnect Circuit Network: A Fast and Accurate Method," *Mathematics*, vol. 9, no. 11, pp. 1-13, 2021. [CrossRef] [Google Scholar] [Publisher Link]

[19] Bingnan Wang, Jiang Long, Koon Hoo Teo, "Multi-Channel Capacitive Sensor Arrays," *Sensors*, vol. 16, no. 2, pp. 1-12, 2016. [CrossRef] [Google Scholar] [Publisher Link]

[20] Scott Kirkpatrick, "Percolation and Conduction," *Reviews of Modern Physics*, vol. 45, no. 4, pp. 574-588, 1973. [CrossRef] [Google Scholar] [Publisher Link]

[21] Michael D. Bartlett, Michael D. Dickey, Carmel Majidi, "Self-Healing Materials for Soft-Matter Machines and Electronics," *NPG Asia Materials*, vol. 11, no. 1, pp. 1-4, 2019. [CrossRef] [Google Scholar] [Publisher Link]

[22] Jiong Pan et al., "Principles and Applications of Two-Dimensional Semiconductor Material Devices for Reconfigurable Electronics," *Nanomaterials*, vol. 15, no. 3, pp. 1-21, 2025. [CrossRef] [Google Scholar] [Publisher Link]

[23] Jaemin Kim et al., "Stretchable Silicon Nanoribbon Electronics for Skin Prosthesis," *Nature Communications*, vol. 5, no. 1, pp. 1-11, 2014. [CrossRef] [Google Scholar] [Publisher Link]

[24] Cun Zhao et al., "Stress Wave Propagation and Energy Absorption Properties of Heterogeneous Lattice Materials Under Impact Load," *Advances in Materials Science and Engineering*, vol. 2021, no. 1, pp. 1-14, 2021. [CrossRef] [Google Scholar] [Publisher Link]

[25] Xuelong Wang et al., "Microstructured Flexible Capacitive Sensor with High Sensitivity based on Carbon Fiber-Filled Conductive Silicon Rubber," *Sensors and Actuators A: Physical*, vol. 312, 2020. [CrossRef] [Google Scholar] [Publisher Link]

[26] Sheng Xu et al., "Soft Microfluidic Assemblies of Sensors, Circuits, and Radios for the Skin," *Science*, vol. 344, no. 6179, pp. 70-74, 2014. [CrossRef] [Google Scholar] [Publisher Link]

[27] Martin Hÿtch et al., "Nano-Scale Holographic Interferometry for Strain Measurements in Electronic Devices," *Nature*, vol. 453, no. 7198, pp. 1086-1089, 2008. [CrossRef] [Google Scholar] [Publisher Link]