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Original Article

Model of the Excitation Process of Chaotic Modes in Radio Electronic Devices Under the Influence of Fractalized Signals of Various Structures

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Abstract *- Usually, when describing the impact of signals, the informational approach and the thermal analysis approach are applied separately. This is due to the fact that the tasks of information suppression and thermal damage caused by high-power signals have traditionally been considered independently. However, in the case of using complex fractalized destructive signals, it is justified to apply an integrated approach. In particular, it is reasonable to employ not only a model describing the process of response formation of a radio-engineering system to low-amplitude electromagnetic information signals with a chaotic temporal structure, but also to assess, from the standpoint of the thermodynamic formalism, the possibility of the emergence of destructive chaotic regimes in the electrical circuits of radio-engineering systems. The object of the study is the process of propagation of fractalized destructive signals through the electrical circuits of radio-engineering systems. The article presents the results of the study on the processes leading to the emergence of chaotic regimes in the receiving path of radio-electronic devices under the influence of an electromagnetic pulse penetrating through the antenna-feeder system. The application of corresponding mathematical models and equations describing the dynamics of transition to chaotic states is considered. The analysis of fractal binary sequences revealed statistical patterns that correlate with the characteristics of complex natural and artificial systems. This opens up prospects for applying methods of generalized statistical thermodynamics in the study of complex information processes, which may be useful for a comprehensive understanding of their structure and dynamics. The obtained results support the hypothesis of a hierarchical organization of such sequences and their invariant nature.*

Keywords *- Symbolic Dynamics, Chaotic State, Attractor, Fractalized Signal, Thermodynamic, Attractor, Python.*

1. Introduction

In order to ensure national security, particularly in the field of information space protection under wartime conditions, it is necessary to employ advanced technologies in the field of Electronic Warfare (EW). Such technologies prevent or minimize disruptions in the operation regimes of Radio-Electronic Devices (REDs) within both civilian and military infrastructures. During the full-scale aggression against Ukraine, the enemy actively employs modern EW systems. In addition to traditional methods of jamming and interference, enemy EW systems utilize complex electromagnetic effects characterized by chaotic and fractal characteristics on (REDs). A comprehensive understanding of these complex electromagnetic signals characterized by chaotic and fractal features, actively applied by enemy EW systems, is particularly important for ensuring national security. The results of studying the statistical properties of such sequences may contribute to the development of more effective methods for their detection, analysis, and neutralization. This is critically important for the protection of the information space and the country's defense capabilities. For the purpose of analyzing systems aimed at protecting or disrupting the information space, the concept of informational characteristics of signals is introduced. The informational characteristics of signals acting on REDs are divided into two main classes. The first class - statistical - includes autocorrelation functions, correlation and fractal dimensions, entropy, Shannon information measures, and so on. The second class - linguistic (contextual) - encompasses the semantic and grammatical structures of signals, as well as the assessment of their informational value.

2. Analysis of recent research and achievements

It is evident that, when considering the information operating mode of REDs, a context-time structure of external influence is present. To address the tasks of controlling radio-physical devices at the information level, generators of unstable (non-stationary) pulse sequences are developed, which are characterized by a complex temporal-contextual structure [1]. Such generators, constructed on the basis of fractal and wavelet principles, allow the modeling and reproduction of signals that closely approximate the real complex influences observed under modern electronic warfare conditions. Simultaneously, new diagnostic methods for analyzing complex time series are being developed. Specifically, research focuses on new diagnostic approaches for complex time series that describe RED dynamics. Particular attention is given to studying the conditions under which chaotic regimes arise in receiving devices under the influence of fractalized signals. In other words, investigating the processes leading to the emergence of chaotic behavior is the key to the development of protective algorithms and systems aimed at enhancing the resilience of modern radio-electronic equipment against the effects of complex information signals.

Thus, the task arises to study and present some of the main results on the development of new diagnostic methods for complex trajectories that describe the evolution of radio-technical systems. These methods also allow the identification of ways to enhance their informational resilience.

Symbolic dynamics deals with global invariant properties of dynamics [2], without predefined phase trajectories.

A symbolic sequence may be considered within the framework of two conceptual approaches [3]. From the standpoint of information theory, it represents a specific message, while the dynamical system acts as a message generator. In this case, the analogue of metric entropy [3] is the specific entropy of the information source, i.e, the Shannon source entropy [4]. From a thermodynamic standpoint, a symbolic sequence may be treated as a classical one-dimensional lattice gas [5]. In this interpretation, a symbol specifies the particle type located at a lattice site. The ensemble for such a system is defined by a measure on the set of all possible configurations of the system within a finite zone Λ of length n. There exist m^n such configurations, specified by all possible word combinations $\omega_0 \ldots \omega_{n-1}$, ω each assigned a certain weight in the ensemble. The problem of determining distributions for the grand canonical ensemble of such thermodynamic systems, under appropriate conditions, can be solved within the framework of classical Gibbs-Boltzmann thermodynamics. This approach constitutes the basis of the thermodynamic formalism of symbolic dynamics developed by Ruelle, Bowen, and Sinai [6].

The informational and thermodynamic approaches to symbolic dynamics are mutually complementary and enable a constructive description of the global properties of chaotic systems by employing the notions of complexity, information, interaction, and equilibrium [7]. Therefore, in what follows, we consider the specific features of applying these approaches [8] to the description of destructive (fractalized) influences in a Radio-Engineering System (RES). The levels of these lie within the dynamic operating range of the RES.

However, the problem of describing the impact of complex fractalized destructive signals on elements of radio electronic equipment remains unresolved. This article proposes using not only models of the process of forming the response of a radio technical system to the influence of low-amplitude electromagnetic information signals with chaotic temporal structure, but also an assessment based on thermodynamics of the possibility of occurrence of destructive chaotic regimes in electrical circuits of radio technical systems.

The object of the study is the process of propagation of fractalized destructive signals through the electrical circuits of radio-engineering systems.

3. The purpose of the article

The problem of the research is the quantitative evaluation of correlation and the statistical structure of the destructive response signal arising in the electrical circuits of radio engineering systems during the transmission of a fractalized signal (symbolic sequence).

The problem consists of the following tasks, which are solved in the article:

1. Modeling state transitions in an information system.
2. Mathematical modeling of the Lorenz attractor using the formalism of Tsallis extended thermodynamics.

4. Materials and Methods

By partitioning the phase space (or state space) into a finite number of cells and associating each of them with a symbol (a letter) from a certain alphabet L, it is possible to move from the consideration of trajectories in phase space to the analysis of symbolic sequences [2-8]:

$$\omega = \{\omega_n\}_0^\infty; \quad \omega = \omega_0\omega_1\omega_2\omega_3\ldots, \tag{1}$$

where $\omega_n \in L$ is a letter from a certain alphabet. The symbolic sequence corresponds to the trajectories of the system under study. Meanwhile, the conversion $\sigma\{\omega_n\} = \{\omega_{n+1}\}$ reflects the evolution of the system along these trajectories. Metric characteristics of a chaotic system, such as correlation and statistical properties, are uniquely transferred to the properties of the corresponding symbolic sequences (under an appropriate partitioning).

Within the framework of symbolic dynamics, tha topological entropy is defined as [2]

$$n_T = sup \lim_{n\to\infty} \frac{log\, K(n)}{n}, \quad (2)$$

where $K(n)$ denotes the number of all admissible words of length n. If the alphabet consists of m symbols, that is, the number of corresponding elements of the partition is m, then the topological entropy attains its upper bound when $K(n) = m^n$

$$n_T = log\ m. \quad (3)$$

The metric entropy [2] is defined as

$$h_m = sup \lim_{n\to\infty} \frac{H(K(n))}{n}, \quad (4)$$

where $H(K(n)) = -\sum_n K(n)$.

Thus expression (4) provides an estimate of the overall level of complexity of the system (and of the corresponding symbolic systems) "as a whole".

A symbolic sequence may be considered within the framework of two conceptual approaches [3]. From the standpoint *of information theory*, it represents a specific message, while the dynamical system acts as a message generator. In this case, the analogue of metric entropy [3] is the specific entropy of the information source, i. e., the Shannon source entropy [4]

$$h_m = lim \frac{1}{n} - \sum_i P_i\,(n)\, log_2\, P_i\,(n), \quad (5)$$

Where $P_i(n)$ - denotes the probability of occurrence of the word of length n in an infinite message. When the alphabet consists of m distinct symbols, the total number of possible words is слів $N = m^n$, where n is the word length. In this case, the index of a word can be identified with a number in a base-m numeral system, whose representation corresponds exactly to the word itself. For chaotic sequences, $h > 0$. For a Markov process of order s, Claude Shannon demonstrated that the source attains its upper bound at $n = s$. If the sequence is periodic with period p, then $h = 0$.

Contemporary studies of dynamical systems increasingly rely on interdisciplinary approaches that combine concepts from information theory and mathematical linguistics. One such approach is the interpretation of a symbolic sequence as a “text”. Its characteristics can be evaluated through the analysis of its structural and statistical properties, in particular using Zipf’s distribution [8]. From the perspective of *information theory*, a symbolic sequence generated by a dynamical system can be considered as a message [2-8]. From the standpoint of mathematical linguistics, it is treated as a text. In both cases, it becomes necessary to introduce new characteristics for dynamical systems. Specifically, *the complexity of a symbolic sequence* is understood as a measure of its structural organization and informational richness. The theoretical question of the complexity of a given sequence is, in principal, semi-infinite, and, in practice, is addressed using the theory of algorithmic complexity [9], which is substantially computationally intensive.

Let A denote an abstract computational machine (for example, a Turing machine) into which “programs”-finite sequences of 0s and 1s-are input, producing output words x over some alphabet Z. The length of a program is defined as the number of symbols it contains. The $K_A(x)$ of a word x with respect to the machine A is defined as the minimal length among all programs that output x (if no such program exists, then то $K_A(x) = +\infty$). A central issue is A. N. Kolmogorov’s theorem [9] on the existence of a machine B (the “minimal machine”) such that for all finite words x

$$K_A(x) £ K_B(x) + C_B, \quad (6)$$

where C_B depends only on B and not on x, the algorithmic complexity of the word x is defined as the quantity $K_B(x)$, which will hereafter be denoted simply as $K(x)$. Consider a symbolic sequence generated by a dynamical system f, namely: $\omega = \omega_0, \omega_1, \dots \omega_{n-1}, \omega_n,$ which corresponds to an initial point x. We now define the complexity of the trajectory of the point x in the dynamical system f [10]: $K_f(x) = sup K_f(x/\varepsilon)$. The value $K_f(x)$ always lies between 0 and 1. If $K_f(x)$ is equal to 1, this implies that a random sequence can be described only by explicitly specifying all of its elements.

The complexity of the trajectory of the point x is closely related to the rate at which the system “forgets” its initial conditions, i. e., to the problem of the precision required in specifying the initial point. In this context, the dynamical system itself is regarded as a “machine”, while the initial conditions are treated as “programs”. As an illustrative example, consider the “tent map”, defined by the transformation.

$$T(x) = 2x\ mod 1, X_{n+1} = T(x_n). \quad (7)$$

The transformation defined in (7) is closely related to the binary (base-2) expansion of points within the unit interval. The formula $T(x) = \begin{cases} 0, x < 1/2, \\ 1, x \geq 1/2, \end{cases}$ represents an n-dimensional symbolic partition. Hence, for an initial condition x, the symbolic trajectory unfolds as the binary representation of this number, where each iteration corresponds to a shift operation $\sigma(\omega_n) = \omega_{n+1}$. Accordingly, the transformation can be written as $T(\omega_0, \omega_1, \omega_2,\ldots) = (\omega_1, \omega_2, \omega_3,\ldots)$. If it is required to obtain the first N symbols of the dynamical trajectory $\omega^N = \omega_0, \omega_1,\ldots, \omega_{N-1}$, then the initial value x must be specified with an accuracy of $(N - 1)$ digits in its binary expansion. In other words, it is necessary to construct a sequence for which $K_f(x) = 1$. This implies that the value of x possesses maximal entropy and generates a fully determined yet unpredictable symbolic trajectory of length N.

For an uncountable set of initial conditions x that are rational numbers**,** the corresponding binary expansions are either finite or periodic in nature, which leads to $K_f(x) = 0$. This indicates that such initial conditions carry a limited amount of information. In these cases, the symbolic trajectory exhibits a repetitive or trivial structure, rendering it non-informative from the perspective of chaotic dynamics.

Thus, complexity enables us to distinguish individual trajectories, whereas topological and metric entropy characterize the overall "complexity" of the dynamical system as a whole [9]. Kolmogorov complexity, which evaluates the regularities lying "beyond" statistical properties, constitutes a new invariant that can not re be reduced to the entropic characteristics of a sequence.

The author [11] proved that if a Polynomial $P(x)$ takes integer values for all natural arguments x, then for any integer $q \geq 2$, the number a, represented as $0,q_1,q_2,q_3\ldots q_n\ldots,$ where q_n is the base-q digit representation of the value $P(n)$, is transcendental. According to the definition introduced by Borel [12], a number a is said to be normal in base q if every digit block $\omega_0,\ldots,\omega_{N-1}$ of length N occurs in the base-q expansion of a with asymptotic frequency (probability) $p = 1/q^N$ where q denotes the size of the alphabet (i. e., the base of the numeral system). In information theory, where $h_M(\omega)$ denotes the topological entropy of a sequence and $h_M(\omega)$ its metric (information) entropy**,** when all possible combinations occur with equal probability, the entropy attains its maximum, and the sequence is regarded as **"**maximally chaotic" or random. That is, for a system with base q, the maximal entropy is given by $h_T(\omega = q_1q_2\ldots) = h_M(\omega) = log q$. In particular, let $P(x) = x$ i $q = 2$. Then the corresponding symbolic sequence is: $\omega = 01\ 10\ 11\ 100\ 101\ 110\ 111\ldots$, while its representation in a decimal system corresponds to the sequence *123456789101112*....

Observing the sequence $\omega = 011011100101110111$ one may conclude that it has zero algorithmic complexity, since the generating set $X = \{P(x) = x;\ q = 2;\ x_{n+1} = x_n + 1\}$ is fully specified and the sequence itself is of finite length.

The language of chaotic systems constitutes a powerful tool for investigating their internal structure and underlying regularities. In [13], it is hypothesized that the complexity of a dynamical system is directly related to the number of grammatical rules that determine admissible or forbidden combinations of symbols appearing in its symbolic sequence.

The latter may be interpreted as a text written in a certain language with a finite alphabet. This approach exhibits strong analogies with the theory of grammatical complexity developed by Stephen Wolfram for the study of cellular automata [14]. The approach has proven to be highly productive in the analysis of discrete mappings and has been further developed in a number of subsequent works [15]. In [16], the following quantity was proposed as a measure of complexity

$$C = \lim_{n\to\infty} \frac{\ln N_+(n)}{n}. \qquad (8)$$

Existing approaches to modeling the state evolution of information systems have direct analogues in theoretical and mathematical linguistics:- the "probabilistic" theory of Markov chains [17-19], used to describe nonlinear systems possessing the property of hyperbolicity;- the alternative theory of Chomsky [17-19], which treats the process of text generation as strictly deterministic and governed by the "deep structure" of language, i. e., a fixed set of grammatical rules.

Both approaches were critically examined by George A. Miller. According to his viewpoint, the process of text generation occupies an intermediate position between complete determinism and complete randomness. Complete determinism, in Chomsky's sense, is interpreted as a specific realization of a Turing machine structure. Complete randomness, in the probabilistic interpretation, treats text as a message generated by a Markov source. The reductionism inherent in both concepts is one-sided. The dependencies between successive symbols impose certain constraints on the sequence. In the case of natural languages, these symbols should be interpreted as words rather than individual letters, even though the latter may follow a Markov distribution. However, these dependencies do not uniquely determine the evolution of the sequence.

The system of rules establishes boundaries for admissible symbol combinations in a given language, thereby defining a "vector" for the subsequent "text formation". However, within this "vector", there theoretically exists a multitude of "correct" continuation variants of the initial text. The decisive role, therefore, belongs to the dynamics of the text "generation" process itself. Consequently, complexity differs fundamentally from entropy, which characterizes only the degree of "randomness" within a sequence. Let us consider a case in which a genetic connection with a linguistic situation exists, analogous to the example of strange attractors in continuous dynamical systems. As an illustrative example, we examine the Lorenz dynamical system [20]

$$\left.\begin{aligned}\dot{x} &= -\sigma x + \sigma y \\ \dot{y} &= rx - y - xz \\ \dot{z} &= -bz + xy\end{aligned}\right\}. \qquad (9)$$

The system characterized by a specific set of (σ, b, r) generates strange attractors with an embedding dimension of three.

The three-dimensional phase space is partitioned according to the two "lobes" of the attractor, each assigned a symbolic label "0" or "1" (Figure 1, *a*, *b*). The simulation of the strange attractor of the Edward Lorenz system was simulated using the PyCharm software environment with the Python programming language.

In this context, each trajectory can be uniquely encoded as a symbolic sequence $\omega = \omega_0, \omega_1, \omega_2\ldots$, where the symbol "0" or "1" indicates the rotation of the trajectory around an unstable fixed point $x_1 < 0$ (or $x_2 > 0$) respectively (Figure 2).

Specifically, if the local maximum of the component satisfies, $x_m < 0$, then $\omega_i = 0$, otherwise, $\omega_i = 1$.

The symbolic dynamical system is thus defined as follows.

$$\omega \to \hat{\delta}(\omega) = \omega', \qquad (10)$$

consequently $\omega' = \{\omega_{i+1}\}$.

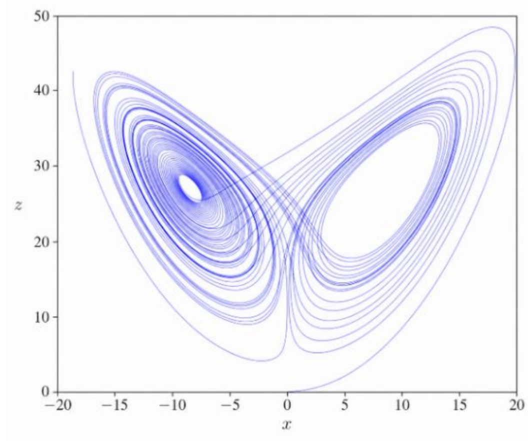
Of particular interest is the set of all admissible binary sequences that arise in the Edward Lorenz system [20] for specific values of the parameters (σ, r, b) and for all initial states x0 belonging to the attractor.

Numerical experiments indicate that the statistical structure of admissible sequences ∑F is consistent with the Zipf-Mandelbrot law (Figure 3).

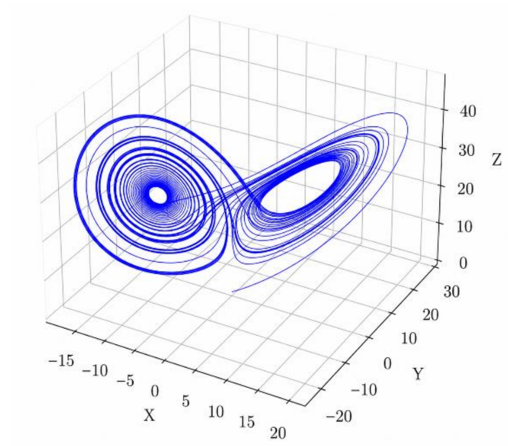
This result implies that mathematical modeling of the Lorenz attractor must be carried out within the framework of the extended thermodynamic formalism proposed by Constantino Tsallis [21, 22].

The statistical structure of these sequences fundamentally differs from that of Markov processes. In particular, the correlation function exhibits static "tails," and the correlations are non-integrable.

The resulting sequence should therefore be regarded as an indivisible whole, since the interaction energy between its "subsystems" within this "system" exceeds the energy of the individual "subsystems". Let us consider this issue in greater detail.



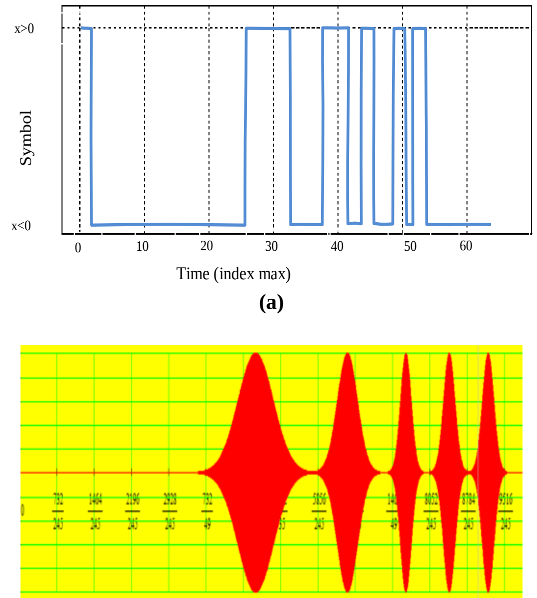
(a)



(b)

Fig. 1 The Lorenz attractor: a - two-dimensional phase space; b - three-dimensional phase space representation

The thermodynamics of fractal sequences, including Tsallis entropy and the Zipf law, provides an effective framework for analyzing statistical structure. This is because any symbolic sequence can be interpreted as a specific "text" characterized by well-defined statistical regularities. The statistical properties of such a "text" reflect stable invariant features of the underlying dynamics, in particular the nature and structure of correlations. Without loss of generality, we restrict the analysis to binary sequences, i. e., sequences defined over the finite alphabet {0, 1}. Previously [8], an *n*-level Zipf analysis was applied to investigate statistical properties of binary sequences exhibiting either short-range (exponentially decaying) Markov correlations or long-range correlations. The essence of this method consists in determining the normalized frequency $\omega(R)$ of occurrence of a given "word", i. e., a binary combination of length n, as a function of its rank R [8]. The rank R is defined as the ordinal position of the word within the set of all possible words of length n (whose total number equals $N = 2^n$), after ordering them in descending order of their observed frequencies. Thus, $R = 1$ corresponds to the most frequent word, $R = 2$ to the next most frequent, and so on.



(a)

(b)

Fig. 2 Symbolic sequence derived from the maxima of the x-component of the Lorenz system attractor: a - LF circuits; b - HF circuits.

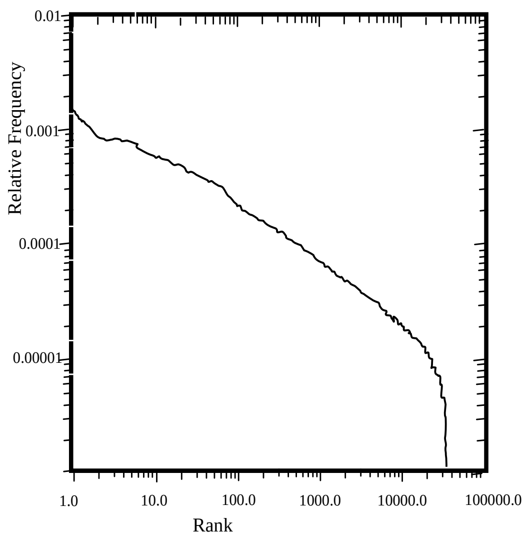


Fig. 3 Zipf plot for the symbolic sequence generated by the Lorenz system attractor at parameter values σ = 10, r = 25, b = 8/3

It has been shown that for the correlated sequences, over a wide range of ranks R (except for the limiting cases $R = 1$ and $R = 2^n$), the frequency histogram of $\omega(R)$ decreases with increasing rank R approximately according to a power law.

$$\omega(R) \sim R^{-\zeta}, \qquad (11)$$

where ζ is the Zipf exponent, which can be determined experimentally as the slope of the Zipf plot (i. e., the graph of $\omega=\omega\ (R)$ in logarithmic scale).

It has also been found that there exists a simple, approximately linear relationship between the parameter α [8], which characterizes long-range correlations, and the Zipf exponent ζ. Markov's sequences, in contrast to the above, do not exhibit the power-law dependence (11). Numerical data indicate an exponential decay of $\omega=\omega\ (R)$ with increasing rank [8].

From a thermodynamic standpoint, a binary sequence may be represented as a gas on a one-dimensional crystalline lattice, where the symbol "0" corresponds to an empty site and the symbol "1" to a site occupied by an atom.

Under certain conditions, the study of correlations and statistical properties of various n-level combinations can be carried out within the framework of classical Gibbs-Boltzmann thermodynamics using the notions of "energy" and "interaction." This approach forms the basis of the Sinai-Ruelle-Bowen (SRB formalism) thermodynamic formalism [6], which is widely employed in modeling chaotic dynamical processes in radio-engineering systems. The energy of a state X of an infinite binary sequence is defined as

$$X = \ldots x_{-2}\ x_{-1}\ x_0\ x_1\ x_2 \ldots = \{x_i\}_{i\in Z};\ x_i \in \{0,1\}. \quad (12)$$

That is, it is considered the total contribution of local "interactions" between symbols. The contribution to the total energy of the "system" associated with the occurrence of a symbol x_0 is determined by the corresponding interaction potential.

$$\Phi(x_0) = \sum_{n=1}^{N} \phi_n(x_{-n}, \ldots,\ x_{-1},\ x_0,\ x_1 \ldots x_n), \quad (13)$$

where ϕ_n is a symmetric function describing the interaction of a symbol x_0 with its textual surrounding context of length $2n + 1$. Thus, ϕ_n correlates directly with the structural properties of the text sequence.

There exists a sequence $\alpha=\{\alpha_n\}_{n\to Z}$, $\alpha \to 0$ for which the following condition holds:

$$sup(\phi_n(x_{-n-1}, \ldots, x_{-1}, x_0, x_1 \ldots x_{n+1}) - \phi_n(x_{-n}, \ldots, x_{-1}, x_0, x_1 \ldots x_n)) \le C\alpha_m. \quad (14)$$

It means that the interaction potential can vary by at most $2C\alpha$ under arbitrary variations of the variables $2C\alpha$ ($m \le n + 1$). In the case of short-range correlations, this corresponds to the limit $\alpha_n=p^n$ $0 \le p < 1$, whereas in the case of long-range correlations $\alpha_s=p^{-s}$, $s > 0$.

The presence of a stational statistical structure in the sequence indicates invariance under shift homeomorphism, thereby highlighting hidden order even within complex dynamical systems. This, in turn, implies that condition (14) holds for each x_i. This property is analogous to translational invariance in the structure of equilibrium states. Within the admissible assumptions of the Sinai-Ruelle-Bowen (SRB) formalism, two key requirements are imposed on the sequences:

$$\sum_{n=1}^{\infty} \alpha_n < \infty \rightarrow requirement \quad a,$$
$$\sum_{n=1}^{\infty} n\alpha_n < \infty \rightarrow requirement \quad b. \qquad (15)$$

Sequences that satisfy both requirements can be interpreted, following Ruelle [6], as systems in which interactions are organized in such a way that they exhibit thermodynamic behavior. Equilibrium states for such systems are defined constructively, and their structure in the thermodynamic limit is described by the Gibbs distribution [23].

$$p_j = \frac{e^{-\beta E_j}}{\sum_{j=1}^{2^n} e^{-\beta E_j}}, \qquad (16)$$

where β is a parameter analogous to the inverse temperature in statistical physics, this approach makes it possible to describe not only the statistics of individual symbols or *n*-tuples, but also the correlation structure and the occurrence of "phase transitions" in the space of symbolic dynamics. The sequences satisfying conditions (15) can therefore be regarded as canonical ensembles of microsystems at the *n*-th hierarchical level.

In the case of a first-order Markov chain, the energy spectrum is completely determined by the values of probabilities *p*, and the frequency of occurrence of a given *n*-level configuration depends solely on the number *k* of consecutive pairs with differing binary values.

$\omega(k) = 2^{-1}\ \{x^k,\ p^{n-1-k}\}$. The proposed thermodynamic approach allows one to derive the statistical structure (15) under the assumption that the energy spectrum reflects nearest-neighbor interactions with a symmetric potential, namely $U(x_i, x_{i+1}) = -ln\ p(x_i, x_{i+1})$.

Under these conditions, the energy of the system possesses the property of additivity, which turns the Markov sequence into an unambiguous example of a so-called "thermodynamic" sequence. Figure 4 illustrates the standard Zipf plot for a Markov sequence.

The step-like structure observed in the plot (Figure 4) arises due to a strong degeneracy of the energy spectrum. In the case of long-range correlated sequences, three distinct scenarios are possible. These scenarios depend on whether conditions (*a*) and (*b*) in expression (15) are satisfied. When *s* > 2, both assumptions are fulfilled; consequently, the sequence is "thermodynamic" and admissible within the framework of SRB formalism. If 1 < *s* < 2, condition (*b*) in (15) *s* violated. This case was investigated by Freeman Dyson [24]. It was demonstrated that, in contrast to thermodynamic sequences, a phase transition occurs in this regime, associated with the emergence of long-range correlations. Specifically, the correlation between distant positions x_i and x_j does not tend to zero as $|i - j| \rightarrow \infty$, so that for the conditional probability $p(i/j) = p(\{x = 1\}/\{x_j = 1\})$, we get

$$lim_{|i-j| \rightarrow \infty} p(i/j) > 2^{-1}. \qquad (17)$$

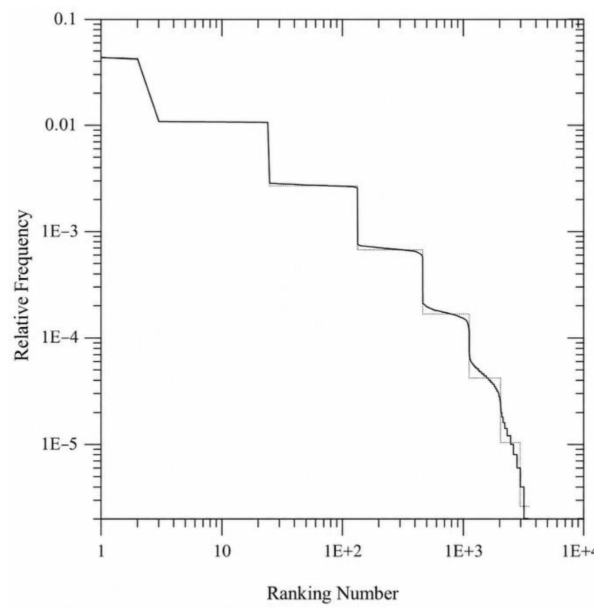


Fig. 4 Zipf distribution for a Markov sequence with a given transition probability: the solid line represents numerical simulation results; the dashed curve corresponds to the analytical solution of the energy spectrum equation

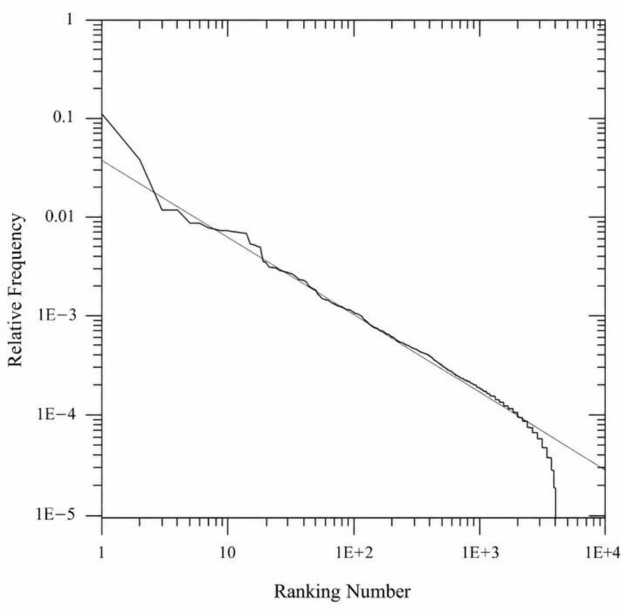
However, it should be noted that the question of the structure of the set of limiting states remains unresolved. Thus, for modeling the response of a radio-engineering system to low-amplitude electromagnetic signals with a chaotic temporal structure, the most universal approach is the use of long-range correlated fractal sequences.

These sequences are generated by the procedure of successive random additions, as described in detail by Voss [25]. This method produces a sequence of real-valued numbers y(t), separated by the time interval Δt, from which the corresponding binary sequence is constructed using the inverse RW algorithm:

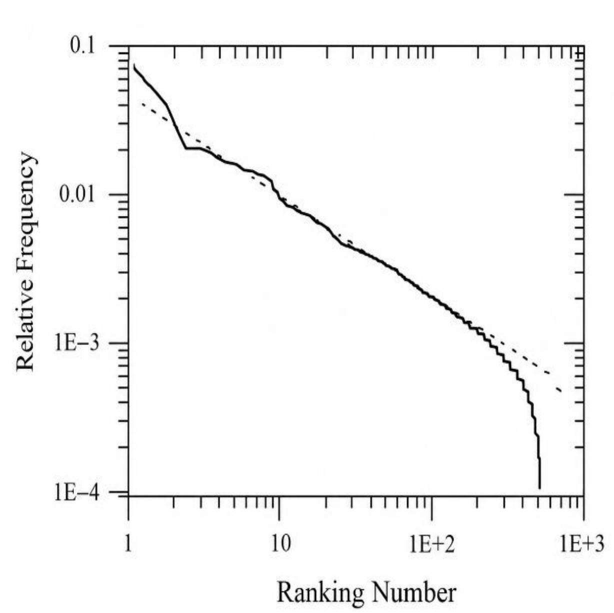
$$x_i = \begin{cases} 1,\ y\big((i+1)\Delta t\big) \geq y(i\Delta t), \\ 0,\ y\big((i+1)\Delta t\big) < y(i\Delta t). \end{cases} \qquad (18)$$

Sequences of length $L = 10^7$ bits were studied, with the number of steps in the Foss procedure set to $m = 18$. It should be emphasized that linearity of the Zipf plot is already observed for sequence lengths typical of practical applications.

Figure 5, *a*, *b* present the Zipf plots of the fractal binary sequence for *n* = 12.



(a)



(b)

Fig. 5 Zipf plot for a fractal binary sequence: *a* - at *n* = 12, α=0,9 (*D* = 1.1); *b* - at *n* = 12, α=0,7 (*D* = 1.3). The straight dashed line corresponds to the power-law function (*R*) = ∝ $R^{-\xi}$, ξ = 2α - 1

Thus, for $L > 10^5$ and $m > 13$, the relative error ζ was found to be less than 3% (Figure 5).

The obtained results indicate that, over a wide range of parameter values, the relationship between α and ζ is accurately described by the linear relation: $\zeta = 2\alpha - 1 = 3 - 2D$. Long-range correlated sequences (17) exhibit scale invariance resulting from the self-affinity of the corresponding fractal curves. The statistical properties of such curves remain invariant with respect to "coarse-graining" procedures:

$$\{ 111,110,101,011 \} \rightarrow 1, \{ 000,001,010,100 \} \rightarrow 0;$$
$$\{ 11111,11110,\ldots,11100 \} \rightarrow 1, \{ 00000,00001,\ldots,00011 \} \rightarrow 0. \qquad (19)$$

which corresponds to the transformation $\Delta t \rightarrow b\Delta t$, b = 3, 5, …, for fractal curves.

From a thermodynamic perspective, this behavior reflects the homogeneity of thermodynamic functions, in particular entropy and free energy. Consequently, the equilibrium distribution (18) remains invariant under the application of a renormalization procedure.

The formalism of generalized thermodynamics is closely related to the multifractal approach [21, 25], where the Constantino Tsallis entropy is associated with the spectrum of fractal dimensions D_q. In this context, a fractal binary sequence can be regarded as a "conductor" that contains all admissible states at the *n*-level. It should be emphasized that the parameter *q* is not arbitrary, but plays a determining role.

The obtained results support the hypothesis proposed by Benoit Mandelbrot, which explains the Zipf law through the hierarchical structure of language [8, 31]. Markov sequences are capable of "reproducing" Zipf-like behavior with low values ζ that slowly vary with increasing rank *R*. The principal distinction between sequences with long-range correlations and those with short-range correlations lies in their non-extensivity.

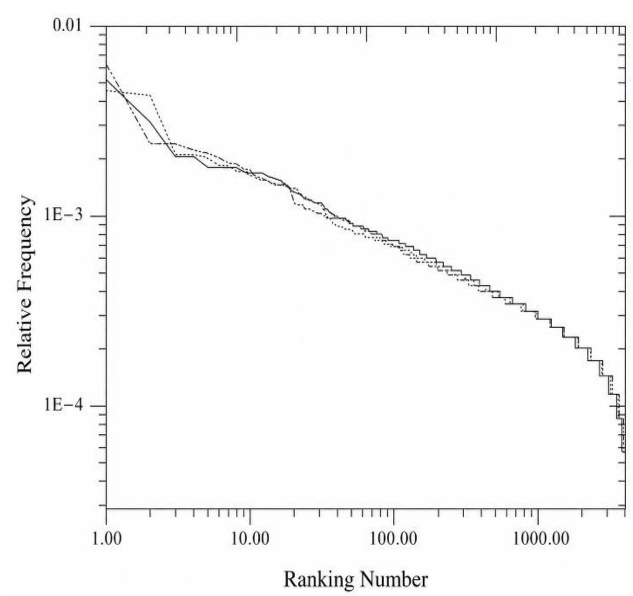


Fig. 6 Zipf plot of a fractal binary D = 1.3 after two successive renormalization procedures

Interactions among elements in such sequences (Figure 6) give rise to an integral structure, bringing their properties closer to those observed in real textual materials, such as DNA molecules and texts of natural or artificial languages.

5. Results of Experimental Investigations

The block diagram of the experimental test bench is presented in Figure 7. The setup is designed to investigate the effects of various types of fractal signals on the Microwave (MW) transmission path. The signals are generated using a programmable arbitrary waveform generator (AWG 2041) (1, Figure 7).

5.1. Description of the Test Bench Operation.

The time-domain signal realization is defined either by a digital data array uploaded from a computer or by directly inputting an analytical expression describing the generated signal into the waveform generator. The resulting analog fractal signal is then fed to the specially designed transmitter module (2) and subsequently to the radiating antenna (3). As radiating elements, the test bench employs either custom-designed fractal antennas [30]. The carrier frequency is supplied to the fractal signal transmitter module from the generator (4). The developed generator operates at eight selectable frequencies, each tunable such that the carrier frequency can be adjusted within the range of 1.5-8.8 GHz

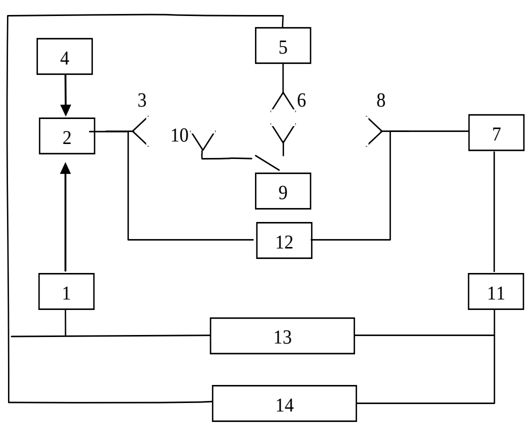


Fig. 7 Block diagram of the experimental test bench for generating and studying fractal signals

1-programmable signal generator; 2-fractal signal transmitter; 3 - fractal/broadband transmitting antenna; 4 - programmable carrier frequency generator; 5 - stochastic signal generator; 6 - transmitting antenna of the stochastic generator; 7 - fractal signal receiver; 8 - receiving antenna; 9 - digital spectrum analyzer; 10 - spectrum analyzer antenna; 11 - wideband dual-channel ADC (2 GHz sampling rate); 12 - four-channel digital oscilloscope; 13 - computer-based control unit; 14 - data acquisition and processing system.

To investigate fractal signal detection in complex electromagnetic environments, the setup includes a stochastic signal generator (1.8-9 GHz)(5) with a radiating antenna(6). For fractal signal reception, a specially designed receiver module (7) with antenna (8) is used. As receiving antennas, custom-developed fractal antennas are also employed [30]. The output signal from the receiver is fed to a high-speed dual-channel ADC (GageScope 3.5) (11) (sampling rate 2 GHz) and subsequently transmitted to the data acquisition and processing system (14). System operation is controlled via unit (13), which comprises two computers with specialized software enabling full control of the test bench, high-resolution analysis of the amplitude-time signal characteristics, measurement of spectral parameters, as well as correlation and wavelet analysis of the investigated signals.

For monitoring the characteristics of transmitted and received signals, the setup also includes a four-channel wideband digital oscilloscope Infiniium 54585A with an 8 GHz sampling rate and 1.5 GHz bandwidth, as well as a digital spectrum analyzer HP 8592A, covering the frequency range from 100 MHz to 22 GHz. The system further incorporates a software-based computational complex that enables mathematical processing of the signal database and calculation of the main characteristics of the studied signals.

It includes the wavelet spectrum, Fourier spectrum, phase portrait, autocorrelation function, cross-correlation function, Poincaré map, and correlation dimension of the time realizations. Using the numerical representation of the previously described FW, a fractal signal (Figure 2) was synthesized on the test bench both with and without a carrier frequency. Subsequently, its main characteristics were measured. The spectra of the investigated signals were obtained using the spectrum analyzer.

5.2. Comparison of Simulation and Experimental Results

Figure 8 presents the measured power spectral density (PSD) of the FW signal with a sinusoidal carrier at 2 GHz, $\xi = 1/3$, and $f_0 = 0.125$ GHz. The carrier frequency position is clearly observed at the center of the spectrum. The experimental results are in good agreement with the theoretical predictions (see the previous subsection). However, in the theoretical FW spectrum, the amplitude at 2 GHz is equal to zero. From a theoretical standpoint, the spectrum of the FW signal with a sinusoidal carrier is shifted by the carrier frequency and has the same form as the baseband FW spectrum. Figure 8 also presents results obtained in different frequency ranges. In this case, the self-similarity coefficient is $\xi^{-1} = 3$.

Experimental data show that the ratio of the frequency band width of 610.3 MHz to that of 204.1 MHz is approximately 3, which corresponds to the ratio between the 204.1 MHz and 73.72 MHz bandwidths. At the receiver output, the time-domain realization and the autocorrelation function of the FW-type fractal signal with the same sinusoidal carrier were measured in real time using the ADC. Figure 8 additionally shows the results of modeling the fractal signal generation scheme in MicroCap and Mathcad, which confirm the theoretical and experimental findings.

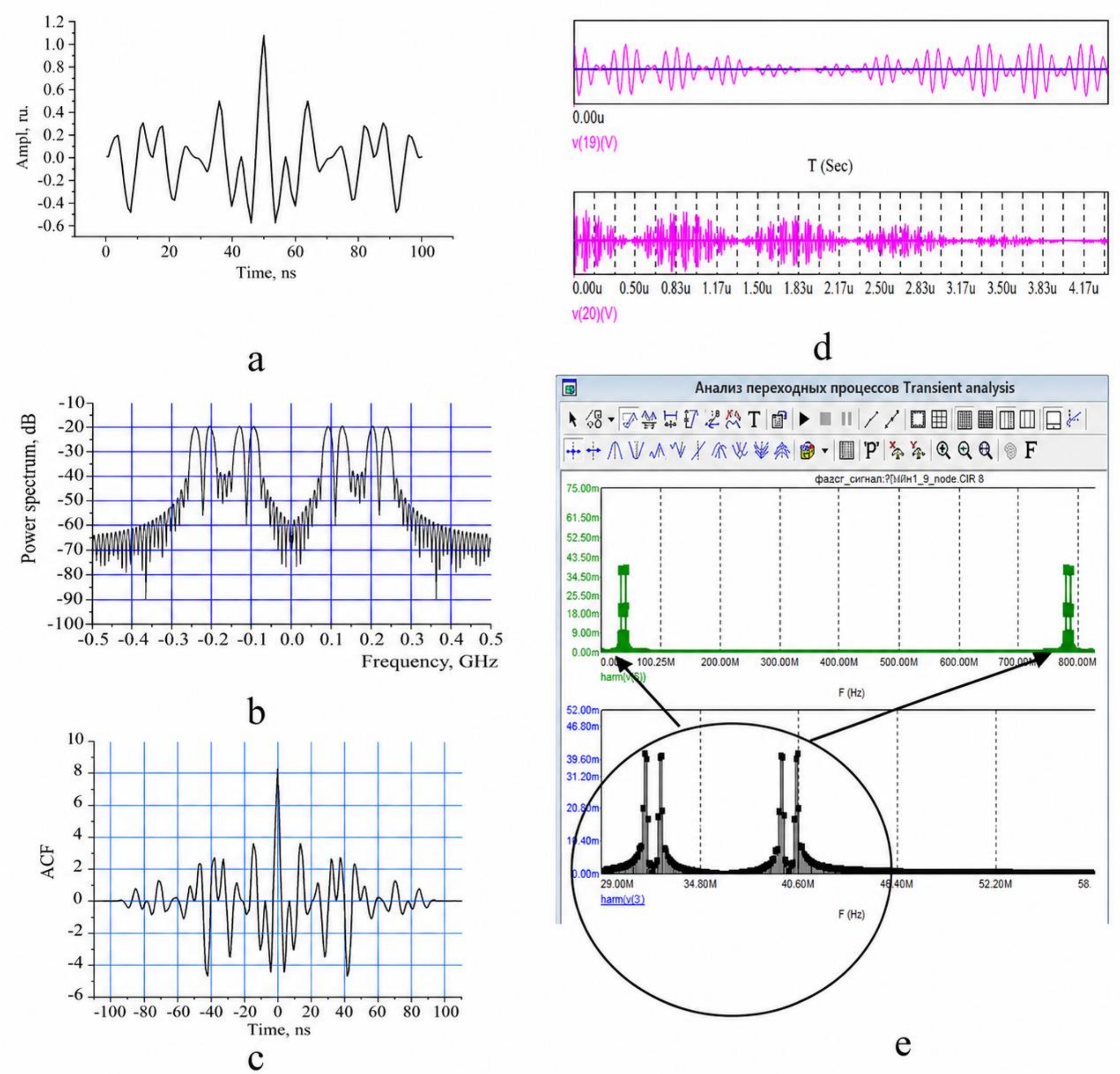


Fig. 8 Structure of the Fractal Wavelet (FW) signal:(a) FW signal in the time domain; b, c - FW signal spectrum and its Autocorrelation Function (ACF) (Ts = 100 ns, f_0 = 2 GHz, $\xi = 1/3$); d, e FW signal and its spectrum in the MicroCap environment.

6. Results and Discussion

1. Model the state evolution of information systems
   From a theoretical perspective, the novelty lies in the application of the thermodynamics of fractal sequences, including Tsallis entropy and the Zipf law. The proposed modeling approach makes it possible to take into account the statistical structure of signals, since any symbolic sequence can be interpreted as a specific "text" characterized by certain statistical regularities. From a practical point of view, a simple, approximately linear relationship has been identified between the parameter α, which characterizes long-range correlations, and the Zipf exponent ζ. In contrast, Markov sequences do not demonstrate such a power-law dependence. From a thermodynamic perspective, a binary sequence can be represented as a gas on a one-dimensional crystal lattice. In this representation, the symbol "0" corresponds to a free lattice site, whereas the symbol "1" represents a lattice site occupied by an atom.
2. Mathematical model of the Lorenz (formalism proposed by constantino Tsallis)
   The application of mathematical models and equations describing the dynamics of the system's transition to

chaotic states is considered. In particular, it is shown that the most accurate description of the onset of chaotic regimes in Radio-Engineering Systems (RES) is achieved using thermodynamic modeling according to Foss, depending on the structural properties of the signal penetrating through the AFT.

The analysis of fractal binary sequences has demonstrated the existence of statistical dependencies that correlate with the properties of complex natural and artificial systems.

This opens up possibilities for applying generalized statistical thermodynamics to the study of complex information processes, enabling a deeper understanding of their structure and dynamics. The obtained data confirm the hypothesis of hierarchical organization and scale invariance of the investigated sequences.

The results may be integrated into the development of advanced fractalized Electronic Warfare (EW) systems for the Armed Forces of Ukraine.

Their implementation can enhance the effectiveness of both suppressing adversary radio-engineering systems and countering emerging threats in the modern electromagnetic environment, which is critically important for ensuring national security.

A direction for future research is the further development and application of generalized statistical thermodynamics to model destructive processes caused by noise-driven information flows affecting the operation of information-analytical systems in the field of cybersecurity.

7. Conclusions

Thus, the application of the extended symbolic dynamics formalism appears promising for modeling the emergence of "strange" kinetics in radio-engineering systems, where electromagnetic anomalies of dynamic chaos arise.

These anomalies are associated with the complex, hierarchical, fractal structure of the chaos boundary in phase space, resembling Cantor-type sets. Despite the exponential local instability of motion, this approach leads to a power-law decay of correlations.

$C(\tau)\propto\tau^{-p}$, $p < 1$ as well as an increased probability of conditions leading to destructive chaotic electromagnetic effects may damage sensitive elements of the receiving chain of the radio-engineering system.

Given the observed similarity between the statistical properties of the studied fractal binary sequences and those of complex systems such as natural languages and DNA molecules, the generalized statistical thermodynamics can be regarded as a promising tool for the analysis of complex information flows. Reliable mathematical modeling of the impact of such signals on Radio-Electronic Devices (RED) facilitates the detection of latent (hidden) fractalized signal threats and helps outline pathways for improving the effectiveness of radiotechnical protection of military command-and-control systems.

Conflicts of Interest

The authors declare that they have no conflict of interest related to this study, including financial, personal, authorship-related, or any other interests that could have influenced the research process or the results presented in this article.

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Data Availability

The data supporting the findings of this study are available from the authors upon reasonable request.

Use of Artificial Intelligence

The authors confirm that no artificial intelligence technologies were used in the preparation of this work.

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Author's Contributions

Oleksandr Fyk: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing - original draft, Writing - review and editing; Stanislav Nikul: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Software, Supervision, Validation, Writing - original draft; Oleksandr Florin: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Software, Supervision, Validation, Visualization, Writing - original draft; Oleh Nazarenko: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Supervision, Validation, Writing - original draft; Olena Novykova: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Supervision, Validation, Writing - original draft; Dmytro Kosov: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Software, Supervision, Validation, Writing - original draft, Writing - review and editing; Pavlo Bordiian: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Resources, Software, Supervision, Validation, Visualization, Writing - original draft.

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