# Application of Taguchi-Based Response Surface Methodology to Optimize Process Parameters for a Small-Scale Wet Ball Mill for Use in Gold Mining 

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#### Abstract

A ball mill is a machine used in the mining industry to reduce the size of the ore to a desirable size in order to liberate the minerals. Effectively controlling the fineness of a ball mill's product necessitates an understanding of the impacts of various input parameters. In this study, Taguchi-based Response Surface Methodology (RSM) was employed to investigate the impact of feed size, slurry solid content and feed rate on the product's Particle Size Distribution (PSD) of a small-scale wet ball mill. Taguchi's L9 orthogonal array was utilized for experimental design to minimize the number of runs, while RSM was applied to optimize the input variables. Additionally, analysis of variance was employed to identify the most significant parameter influencing PSD within the range $-150+75 \mu \mathrm{~m}$. The findings revealed that, under the considered levels of the parameters, the average feed size exerted the most significant influence, followed by feed rate and solid content, respectively. The response optimizer indicated that a maximum PSD of $72.24 \%$ could be attained by maintaining an average feed size of 7.5 mm , a solid content of $60 \%$, and a feed rate of $0.025 \mathrm{~m}^{3} / \mathrm{h}$. The experimental PSD closely matches the predicted value with a $3.01 \%$ error, indicating a good fit.


Keywords - Ball mill, Feed rate, Feed size, Particle size distribution, Solid content, Taguchi-based Response Surface Methodology.

## 1. Introduction

In mineral processing, valuable ore is liberated from the valueless material in a process known as comminution [1]. This process involves various mining machines, and one key piece of equipment is the ball mill, which grinds crushed materials to a specific desired size depending on the separation technique to use in the next step [2]. A ball mill operates by the rotation of its cylindrical drum that generates a centrifugal force capable of lifting up the grinding media inside the drum to the shoulder of the cylinder and then dropping them down to the toe, causing an impact on the material, initiating comminution as presented in Figure 1 [3]. Due to forces acting parallel to and along the surfaces, a lot of abrasion occurs in the abrasion zone of the mill. The milling process ought to be tailored to grind material for effective separation downstream. It is, therefore, of essence to grind particles within a specified range to avoid producing very large or very fine particles.


Fig. 1 Operation of ball mill
For instance, in the conventional gravity concentration, facile separation is achievable within a mill's product particle size down to $75 \mu \mathrm{~m}$. In contrast, the effective size range for concentration by flotation lies within the spectrum range of 1 $\mu \mathrm{m}$ to $500 \mu \mathrm{~m}$ [3].

Artisanal and Small-Scale Gold Mining (ASGM) is present in over 80 countries, employing about 15 million miners and serving as the source of livelihood for millions more [4]. In the milling industry, two primary methods are used: dry and wet grinding. In dry grinding, the material is introduced into a mill and mechanical force is applied to break it down into smaller particles. In this method, the moisture content of the material to be ground should be below $2 \%$, which is difficult to achieve [5]. Dry grinding has a low efficiency, generates more heat, dust and produces non uniform particle size compared to wet grinding [5]. Prevalent use of batch dry ball mills in Artisanal and Small-Scale Mining (ASM) raises health concerns due to noise and dust emissions [6].

Additionally, machine downtime during material unloading and loading poses a further challenge in ASM, consuming additional energy and time [5]. On the other hand, wet grinding involves the use of a liquid, typically water, which helps reduce friction and heat build-up throughout the process. This results in a more efficient, uniform grinding and consumes less power compared to dry grinding [5]. However, in case it is needed for the downstream operation, dehydration and drying increase the cost of wet grinding.

In a wet ball mill, several factors play a crucial role in shaping the product size distribution. They include; slurry solid concentration, the feed rate, and the feed size [3, 7]. According to Ulusoy et al. [8], particle size is the predominant factor affecting the behaviour of materials. Finer feed particles result in increased slurry viscosity, impacting flowability and promoting agglomeration [9, 10]. This agglomeration behaviour often leads to the production of coarser particles. Dogacan et al. [9] reported that stability and agglomeration are directly influenced by the feed particle size distribution. Investigations by Tian et al. [11] confirmed that feed particle size significantly impacts product particle size distribution, peak viscosity, and final viscosity. However, there is no report on critical particle size to achieve stability and less agglomeration [9]. In wet milling, the effect of feed particle size can hardly be studied without considering the solid content. A study by Mangesana et al. [10] showed that viscosity increases with an increase in solid concentration. Excessive water content reduces the number of contact events between particles and media, resulting in decreased grinding.

Conversely, overly high solid content compromises the fluidization of the mill [3]. Another key parameter is the feed rate. While an increase in feed rate improves throughput, it may also lead to a $90 \%$ decline in plant efficiency [12].

In mineral processing, there is a great tendency to apply the classical One-Factor-at-A-Time (OFAT) approach to determine optimum process parameters. This involves varying one parameter at a time while keeping others constant
during the milling process [12]. This approach has been criticized for being time-consuming, resource-intensive, and lacking the ability to provide a comprehensive understanding of the interactions between multiple factors [13-15]. It has been noted that the OFAT approach does not allow for the estimation of interaction effects between factors, which is crucial in many studies [7]. The limitations of the OFAT led Makokha et al. [7] to assess the combined effects of ball loading and slurry solids concentration on the milling process using a statistically-based method of Design of Experiments (DOEs), namely Response Surface Methodology (RSM) and Central Composite Rotatable Design (CCRD). However, while RSM is a powerful tool for exploring and optimizing complex systems, it comes with the challenge of increased experimental complexity and resource requirements. This gap can be effectively addressed by integrating the Taguchi DOE with RSM. Employing Taguchi's DOE offers the advantage of having a reduced number of trials. RSM, on the complementary side, comes in handy for analyses and optimization of process parameters. This approach has demonstrated success in various engineering fields, like optimization of process parameters in plasma arc cutting [16].

This study investigated the influence of average particle size, solid content, and feed rate on the product particle size distribution. Taguchi-based response surface methodology was adopted to explore the interactions amongst these factors and their impact on achieving a product particle size within the range of $-150+75 \mu \mathrm{~m}$ (material passing through a 150 $\mu \mathrm{m}$ sieve and getting retained by a $75 \mu \mathrm{~m}$ sieve). The study additionally seeks to optimize these parameters to attain the maximum product particle size within the aforementioned size range.

## 2. Materials and Methods

The optimization process flow chart, depicted in Figure 1, summarizes the sequential steps involved in optimizing key parameters of a wet ball mill for improved performance using the Taguchi-based Response Surface Methodology.

### 2.1. Design of Experiment

In this Design of Experiments (DOE), the Taguchi method proves valuable by significantly reducing the number of required experiments while preserving reliability and robustness. For instance, compared to a full factorial design involving 27 experimental runs for studying 3 parameters at 3 levels, the Taguchi Design only necessitates 9 sets of experiments.

In this study, statistical analysis was conducted using Minitab 20 software. The Taguchi method was employed to systematically vary different levels of factors and generate sets of experiments. Taguchi's L9 (33, a standard three-level orthogonal array) with 8 degrees of freedom (DOF) was selected, and each chosen parameter was analysed at three
levels, as detailed in Table 1. Following the experimental phase, Response Surface Methodology (RSM) was applied for the analysis and optimization of responses.


Fig. 2 Optimization process flow chart
Table 1. Taguchi's L9 for studying the feed size, the feed rate and the solid content

| S. <br> No. | Average <br> Feed Size <br> $(\mathbf{m m})$ | Solid <br> Content <br> $(\%)$ | Feed Rate <br> $\left(\mathbf{m}^{3} / \mathbf{h}\right)$ |
| :---: | :---: | :---: | :---: |
| 1 | 7.500 | 0.6 | 0.0073 |
| 2 | 7.500 | 0.7 | 0.0163 |
| 3 | 7.500 | 0.8 | 0.0250 |
| 4 | 3.680 | 0.6 | 0.0163 |
| 5 | 3.680 | 0.7 | 0.0250 |
| 6 | 3.680 | 0.8 | 0.0073 |
| 7 | 1.805 | 0.6 | 0.0250 |
| 8 | 1.805 | 0.7 | 0.0073 |
| 9 | 1.805 | 0.8 | 0.0163 |

### 2.2. Experimental Setup

A continuous feed small-scall ball mill (Figure 3) was employed throughout the experimental analysis. The grinding conditions of the experimental analysis are presented in Table 2. The ball mill discharge was of overflow type and was run in an open circuit. The mill had a drum with an internal diameter of 365 mm and a length of 630 mm ,
which was fitted with 4 rectangular lifters of $630 \times 30 \times 25$ mm . The grinding media, which accounted for $40 \%$ of the total volume of the drum, was a blend of steel balls of different diameters. Specifically, the ball diameters were 50 $\mathrm{mm}, 35 \mathrm{~mm}$, and 25 mm , representing $60 \%, 20 \%$, and $20 \%$ of the total volume of balls, respectively. The total weight of the grinding media was 123 kg , and the same weight of balls was used consistently across all experiments. A constant speed of 52 rpm , which represents $75 \%$ of the critical speed of the mill, was maintained across all experiments. In this investigation, the input variables were the average feed size, the feed rate and the solid content while the output variable was the specified range of PSD. The objective function aimed to maximize material within the range of $-150+75 \mu \mathrm{~m}$.


Fig. 3 Experimental wet small-scale ball mill (a) Ball mill, and (b) Inside the drum.

The feed material utilized throughout the investigations was gold ore collected from Masara mines in Migori County, Kenya. The extracted ore underwent initial processing through a jaw crusher to reduce coarse particles to the desired feed sizes. Subsequently, sieving was conducted using a series of mesh sieves, categorizing the feed particle size into three ranges: $-10 \mathrm{~mm}+5 \mathrm{~mm},-5 \mathrm{~mm}+2.36 \mathrm{~mm}$, and -2.36 $\mathrm{mm}+1.25 \mathrm{~mm}$. The feed rate was systematically varied within the range of 0.0073 to $0.025 \mathrm{~m}^{3} / \mathrm{h}$, while the slurry solid concentration was adjusted between $60 \%$ and $80 \%$ by weight. Each experiment was conducted for a duration of 1 hour, and the resulting product was collected and subsequently dried up in an oven at $120^{\circ} \mathrm{C}$. After drying, a representative sample of 50 g was taken for sieve analysis using Jones riffle splitter. The sample was dry sieved for 15 minutes using a vibrator sieving machine composed of a series of sieves including $3.35 \mathrm{~mm}, 1.7 \mathrm{~mm}, 850 \mu \mathrm{~m}, 600 \mu \mathrm{~m}$, $425 \mu \mathrm{~m}, 300 \mu \mathrm{~m}, 212 \mu \mathrm{~m}, 150 \mu \mathrm{~m}, 106 \mu \mathrm{~m}, 75 \mu \mathrm{~m}$ and 53 $\mu \mathrm{m}$. The amount of material retained on each sieve was then recorded, and the weight percentage was calculated relative to the total weight. The weight percentage of material passing through the $150 \mu \mathrm{~m}$ sieve and retained by the $75 \mu \mathrm{~m}$ sieve ($150+75 \mu \mathrm{~m}$ ) was then recorded for each set of experiments.

Table 2. Specifications of the mill and milling conditions

| Item | Description | Value |
| :---: | :---: | :---: |
| Mill | Internal diameter, mm | 365 |
|  | Length, mm | 630 |
|  | Volume $V, m^{3}$ | 0.066 |
|  | Critical speed, rpm | 70 |
|  | Operational speed ( $75 \%$ of critical speed) | 52 |
| Lifters | Number | 4 |
|  | Shape | Rectangular |
|  | Dimensions (length x width x height), mm | $630 \times 30 \times 25$ |
| Grinding Media | Material | Steel balls |
|  | Steel density ( $\rho$ ), kg/m ${ }^{3}$ | 7850 |
|  | Bulk density ( $\rho_{\text {Bulk }}$ ) $\mathrm{kg} / \mathrm{m}^{3}$ | 4675 |
|  | Volume fraction of balls ( $J$ ) | 0.4 |
|  | Ball filling volume ( $V_{B}=V * J$ ), $\mathrm{m}^{3}$ | 0.0264 |
|  | Balls total mass ( $M=\rho_{\text {Bulk }} * V_{B}$ ), kg | 123 |
|  | Ball diameters, mm | 50 (60\%), 35 (20\%), 25 (20\%) |
| Feed | Material | Gold ore + water |
|  | Gold ore density ( $\rho_{\text {ore }}$ ) $\mathrm{kg} / \mathrm{m}^{3}$ | 2600 |
|  | Feed size distribution, mm | -10+5, -5+2.36, -2.36+1.25 |
|  | Ore bulk density per size, $\mathrm{kg} / \mathrm{m}^{3}$ | 1374, 1262, 1192 |
|  | Slurry solid content (\% of solid by weight) | 60, 70, 80 |
|  | Slurry feed rate, m3/h | $0.0073,0.0163$ and 0.025 |

## 3. Results and Discussion

Taguchi's method was employed to reduce the number of experimental runs down to 9 from the 27 runs required in the full factorial design, aiming to save on resources and time. However, to efficiently investigate the interactions between factors, Response Surface Methodology (RSM) was employed. RSM is another statistical technique commonly used for optimization. While Taguchi is effective for identifying the optimal factor settings efficiently, RSM is often employed for a more detailed exploration of the response surface, allowing for a thorough understanding of the interactions between factors and the shape of the response.

The combination of Taguchi's method for initial screening and RSM for a more detailed investigation is a common and powerful approach in experimental design. This strategy allows to achieve a balance between efficiency and a comprehensive understanding of the experimental system. Table 3 presents the responses, specifically the percentage of material retained in the sieve range of $150-75 \mu \mathrm{~m}$ for each experiment set generated through Taguchi's DOE. Among the experiments, numbers 1 and 2 exhibit the most favourable responses, whereas experiments 8 and 9 result in comparatively poorer outcomes.

Table 3. Taguchi's L9 for PSD (-150 + $\mathbf{7 5} \boldsymbol{\mu m}$ ) and responses

| S. <br> No. | Average <br> Feed <br> Size <br> $(\mathbf{m m})$ | Solid <br> Content <br> $(\%)$ | Feed <br> Rate <br> $\left(\mathbf{m}^{\mathbf{3} / \mathbf{h})}\right.$ | PSD within <br> $\mathbf{- 1 5 0 + 7 5}$ <br> $\boldsymbol{\mu m}(\%)$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 7.500 | 0.6 | 0.0073 | 52 |
| 2 | 7.500 | 0.7 | 0.0163 | 56 |
| 3 | 7.500 | 0.8 | 0.0250 | 56 |
| 4 | 3.680 | 0.6 | 0.0163 | 52 |
| 5 | 3.680 | 0.7 | 0.0250 | 48 |
| 6 | 3.680 | 0.8 | 0.0073 | 40 |
| 7 | 1.805 | 0.6 | 0.0250 | 52 |
| 8 | 1.805 | 0.7 | 0.0073 | 32 |
| 9 | 1.805 | 0.8 | 0.0163 | 32 |

### 3.1. Effects of Feed Size, Solid Content and Feed Rate on Particle Size Distribution

### 3.1.1. Main Effects Plot and Interaction Plot

The main effects plot (Figure 4) shows the individual effects of each factor on the response variable while keeping other factors constant. The interaction plot (Figure 5) illustrates the interaction effects between two factors on the response variable. These plots help identify whether the effect of one factor on the response depends on the level of another factor.


From the main effects plot in Figure 4, it is evident that the percentage of material within the range $-150+75 \mu \mathrm{~m}$ increases sharply with an increase in feed size from low level to high level, i.e., $1.805-7.5 \mathrm{~mm}$. The direct relationship between feed particle size and product size distribution can be attributed to the fact that the presence of small feed particles increases viscosity and thus produces an increased product size. A high viscosity decreases the cascading motion of the grinding balls on the material, which directly affects the mill's product. He et al. [17], as well as Chen and Zhang [18], reported the same observation that at a fixed slurry concentration, the mineral particle size has a significant effect on the rheological behaviour of the slurry, and its apparent viscosity increases exponentially with the decrease of particle size. It was also reported that the relative viscosity variation can be almost $40 \%$ either upwards or downwards by only altering the feed particle size [9].

Figure 4 further indicates that an increase in feed rate leads to an increase in the proportion of fine particles (150$75 \mu \mathrm{~m}$ ). This may be attributed to an increased collision frequency. Higher feed rates often lead to an increased collision frequency between particles and between particles and the drum surface. This increased frequency of collision may enhance the milling action, resulting in a greater proportion of finer particles. This finding is consistent with previous studies [24, 25].

Additionally, Figure 4 represents an indirect relationship between the product PSD and the solid concentration. The observed decrease in milling efficiency with an increased solid content is consistent with the findings of Tangsathitkulchai [21], who observed that particle breakage rate was diminished at a higher solid loading to the total volume of slurry. First of all, higher solid content leads to reduced dispersibility of the particles in the system. Secondly, it is evident that an increase in solid content leads to an increase in the viscosity of the slurry, and this can be attributed to increased particle-particle interactions in the fluid [10]. Higher viscosity can affect the movement and separation of particles, potentially leading to the formation of
larger agglomerates and decreasing the ball's impact force due to increased drag force. A high viscosity leads to coarse particles in the product size because the slurry acts as a damper, creating resistance to the ball movement as the slurry concentration increases [15].

Figure 5 depicts the impact of the interaction between solid content and feed rate on PSD. At a feed rate of $0.0073 \mathrm{~m} 3 / \mathrm{h}$ with an increasing solid content $(60-80 \%)$, the PSD within the specified range remains constant (43\%). This implies that the solid content exerts negligible influence on PSD when the flow rate is low $(0.0073 \mathrm{~m} 3 / \mathrm{h})$.

Contrastingly, at a feed rate of $0.0161 \mathrm{~m} 3 / \mathrm{h}$ and $60 \%$ solid, the PSD reaches $52.5 \%$ but drops down to $43.5 \%$ as solid content increases. Similarly, at $0.0250 \mathrm{~m} 3 / \mathrm{h}$ and a solid content of $60 \%$, the PSD achieves a peak of $62.5 \%$, gradually declining to $44.5 \%$ as the solid content escalates from $60 \%$ to $80 \%$. This observation indicates that a lower solid content coupled with a higher feed rate yields better results in the considered system.

### 3.1.2. Contour Plots

The contour plots (Figure 6) visually represent the response surface, offering a clear understanding of how changes in multiple factors concurrently impact the outcome. The contour plots help easily identify optimal regions and understand interaction effects on PSD. As seen in Figure 6, the PSD in the range $150-75 \mu \mathrm{~m}$ was less than $35 \%$ of the total particle size distribution corresponding to the dark blue colour contours. The PSD gradually increased to more than $60 \%$ corresponding to the boldest green areas. The larger the PSD within the range of $150-75 \mu \mathrm{~m}$, the better the input variables. The combination of a low solid content with a large average feed size results in a maximum PSD in the specified range. This may be due to the mitigation of particle agglomeration or clustering. Larger particles, when present in lower concentrations, may experience fewer agglomeration effects. Reduced agglomeration can enhance the effectiveness of individual particle breakage and contribute to a diverse PSD.


Fig. 5 Interaction plot for PSD ( 150 - $\mathbf{7 5} \boldsymbol{\mu m}$ )


Fig. 7 Pareto chart of the effects of the input variables on product size distribution

The combination of a high feed rate with a large average feed size gives a high amount of PSD within the specified range.

Larger particles entering the milling chamber experience enhanced attrition due to increased contact surfaces, while the high feed rate ensures a higher frequency of particle collisions within the milling chamber. Optimized attrition processes lead to a more diverse and well-distributed PSD.

The combination of a high feed rate with a low solid content maximizes the PSD within the specified range. A high feed rate introduces a larger quantity of material into the milling system per unit of time, while the low solid content results in a more dispersed arrangement of particles in the milling chamber. Together, these conditions can lead to a higher frequency of collisions, both attrition and impact, between particles due to the increased number of particles in motion and their good distribution.

Table 3. Analysis of Variance for product size distribution

| Source | DF | Adj SS | Adj MS | F-Value | P-Value | Remarks |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Model | 4 | 714.42 | 178.606 | 33.11 | 0.003 | Significant |
| Linear | 3 | 653.11 | 217.705 | 40.36 | 0.002 |  |
| Average feed size | 1 | 350.55 | 350.550 | 64.98 | 0.001 |  |
| Solid content | 1 | 129.26 | 129.259 | 23.96 | 0.008 |  |
| Feed rate | 1 | 170.65 | 170.650 | 31.63 | 0.005 |  |
| 2-Way Interaction | 1 | 43.48 | 43.477 | 8.06 | 0.047 |  |
| Solid content*Feed rate | 1 | 43.48 | 43.477 | 8.06 | 0.047 |  |
| Error | 4 | 21.58 | 5.394 |  |  |  |
| Total | 8 | 736.00 |  |  |  |  |

### 3.1.3. Analysis of Variance

For a mathematical model to be considered statistically significant, the P -value has to be less than 0.05 ( $95 \%$ confidence) and the F-value greater than 3.55 [2]. The Analysis of Variance (ANOVA) was used to investigate the relative importance of control parameters that affect the PSD. The ANOVA (Table 4) reveals that there is only a $0.3 \%$ chance that the model F -value this large could occur due to noise.

The P-values of the input variables are $0.001,0.005$ and 0.008 , respectively, for the average feed size, the feed rate and the solid content. This means that the three parameters are critical for the product size distribution. The interaction of solid content and feed rate was also found to be significant.

The pareto chart (Figure 7) reveals that the average particle size is the most significant parameter, followed by the feed rate and solid content, respectively. This finding is consistent with previous studies, which confirmed that feed size is the predominant factor affecting the behaviour of particulate material [4, 14, 23].

The model summary (Table 5) presents R-squared, adjusted R -squared and predicted R -squared. The R -sq ( $97.07 \%$ ) indicates a better fit between the response model and the actual data from the experimental domain. R-sq (adj) ( $94.14 \%$ ) suggests a better balance between model complexity and explanatory power, while R-sq (pred) ( $82.37 \%$ ) indicates a better predictive ability on new, unseen data, respectively. The regression Equation (1) generated by RSM for prediction is:
$\operatorname{PSD}(150-75 \mu \mathrm{~m})=-2.4+3.535 \mathrm{~A}+34.3 \mathrm{~B}+4102 \mathrm{C}-$ 4999 B*C

Where A is the average particle size, B is the solid content, and C is the feed rate.

Table 4. Model summary

| $\mathbf{S}$ | R-sq | R-sq(adj) | R-sq(pred) |
| :---: | :---: | :---: | :---: |
| 2.32258 | $97.07 \%$ | $94.14 \%$ | $82.37 \%$ |

### 3.2. Process Parameter Optimization

The goal here is to determine conditions that maximize the product particle contained within the size range of $-150+$ $75 \mu \mathrm{~m}$.

To achieve this, the response optimizer tool within the framework of RSM in Minitab 20 software was chosen for its capability to systematically explore and optimize the interaction of multiple factors, considering both linear and interaction effects. RSM has the ability to model and analyse complex relationships between the input factors (average feed size, solid content, and feed rate) and the response variable (PSD).

The resultant optimal conditions are highlighted in red in Figure 8, revealing that a peak proportion of $72.24 \%$ of particles within the specified size range can be achieved under conditions characterized by an average feed size of 7.5 mm , a solid content of $60 \%$, and a feed rate of $0.025 \mathrm{~m} 3 / \mathrm{h}$. The objective function employed for the optimization process is the PSD, as defined by Equation (1).

### 3.3. Confirmatory Experiment

After obtaining the optimal levels of the input variables using RSM, the next step was to run additional experiments with the predicted levels of variables to assess the model's reliability.

Confirmatory experiments were conducted, implementing the predicted settings comprising an average feed size of 7.5 mm , a solid content of $60 \%$, and a feed rate of $0.025 \mathrm{~m} 3 / \mathrm{h}$, as suggested by the model.

The material proportion within the $-150+75 \mu \mathrm{~m}$ range was determined to be $70 \%$, which compares closely with the anticipated value of $72.24 \%$, as is presented in Table 5. Therefore, the model can be considered valid as the predicted model fits experimental data with an error margin of $3.1 \%$.

Error Margin $=\frac{\text { Predicted Value-Observed Value }}{\text { Predicted Value }}$


Fig. 8 Optimization responses
Table 5. Comparison of the predicted and validation values

| Parameters | Average feed size <br> $(\mathbf{m m})$ | Solid content <br> $(\mathbf{\%})$ | Feed rate <br> $(\mathbf{m 3 / h})$ | PSD (150 - 75 $\boldsymbol{\mu m})$ <br> $(\mathbf{\%})$ |
| :---: | :---: | :---: | :---: | :---: |
| Predicted | 7.5 | 60 | 0.025 | $\mathbf{7 2 . 2 4}$ |
| Confirmatory 1 | 7.5 | 60 | 0.025 | 70 |
| Confirmatory 2 | 7.5 | 60 | 0.025 | 72 |
| Confirmatory 3 | 7.5 | 60 | 0.025 | 68 |
| Mean | 7.5 | 60 | 0.025 | $\mathbf{7 0}$ |

## 4. Conclusion

In this work, a small-scale ball mill was utilized to investigate the effects of average feed size, feed rate and solid content on the mill's efficiency in terms of product size distribution. The Taguchi-based RSM was employed to optimize the input variables. Taguchi ( 9 runs) was used instead of a full factorial ( 27 runs) to minimize the number of trials and save resources while efficiently investigating the interactions between factors using RSM. The pareto graph and the ANOVA table revealed that, of the three factors investigated in this research, the average feed size is the most influential factor, followed by the feed rate and the solid content, respectively. The RSM also helped find that the interaction of solid content and feed rate has a significant effect on the PSD within the specified range. Under the conditions considered in this study, setting the average feed size at its maximum ( 7.5 mm ), the solid content at its minimum ( $60 \%$ ) and the feed rate at its maximum ( 0.025 $\mathrm{m} 3 / \mathrm{h}$ ) yields the maximum product within the specified range of $-150+75 \mu \mathrm{~m}$. The experimental confirmation confirmed the effectiveness of these parameter settings, demonstrating a small error margin of $3.01 \%$.

The optimization results derived from this study hold potential applications in the Artisanal Small-scale Gold Mining (ASGM) sector for an efficient gold recovery. This research focused on optimizing key parameters for wet grinding, and there is room for additional investigations into the impacts of other control variables within a ball mill system. The successful utilization of the Taguchi-based Response Surface Methodology (RSM) in this study suggests its applicability for further exploration of optimal operating conditions. This approach could be extended to industrialscale mills to ascertain improved efficiency in ore recovery.

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