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Original Article

Design, Kinematic and Dynamic Modelling, Control and Path Tracking of 3 Wheels Mobile Robot Vehicle

Yagnik Nikunj Sanjaykumar[1,2], Mehulkumar M. Gor[3]

[1]*Gujarat Technological University, Gujarat, India.*
[2]*Automobile Engineering Department, A.D. Patel Institute of Technology, Gujarat, India.*
[3]*PIET, Parul University, Gujarat, India.*

[1]*Corresponding Author : yagniknikunj09@gmail.com*

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Abstract - *The design of a 3-wheeled mobile robot with a front wheel steer for path tracking is presented in this article, along with generic kinematic and dynamics modelling and control strategies. Even though three-wheeled vehicles with front wheel steers are frequently used in public transportation, their benefits for navigation and vehicle localization are rarely used. The brushless DC hub motor is being used as a front active wheel for mobility, and the rear two wheels are passive. The first part of this paper covers three-wheel robotic vehicle dynamic calculations for the selection of motor, battery, and controller. The second section of the article includes kinematic and dynamic mathematical modelling for the specific proposed configuration. The 3D CAD model of the proposed robotic vehicle is developed in solid modelling software. Finally, the kinematic control proposed is applied to the vehicle by explicitly using kinematic governing equations and BLDC hub motor controller modelling by MATLAB/Simulink, which follow path trajectories in simulation.*

Keywords - *Kinematic model, Dynamic model, 3 Wheels, Robot, Vehicle.*

1. Introduction

There are many applications for mobile robots in today's world, including industrial (cooperative mobile robots in factories) [1-3], military (unmanned vehicles to destroy enemy targets), and recreational (robot soccer players). Such robots construction, autonomy, mobility, location, and control are crucial factors to take into account. To regulate the trajectory of the WMR, kinematic and dynamic models would first be taken into account.

Many studies have examined modelling kinematics and dynamic difficulties for wheeled mobile robots using the Lagrangian framework to derive dynamic equations of motion. However, apart from traditional kinematic and dynamic modelling, the proposed three wheels robotic vehicles where the front steer wheel as active wheel and two rear wheels act as passive wheel design is also being developed with automotive dynamic considerations. The said consideration is useful in selecting a motor, battery and feasible controller. After that, the required 3D CAD model is developed with proper dimensions, and based on that, 3 Wheels robotic vehicle configuration based on kinematic and dynamic mathematical modelling is done. Finally, the Simulink model based on the derived available equations has been developed and validated with the path tracing simulation.

2. 3-Wheels Robotic Vehicle Dynamics Calculation

The proposed 3-wheel Robotic Vehicle is modelled with a BLDC hub motor, which is being fixed in front, and 2 other passive wheels are affixed at the rear side. The selection of the proper motor based on the torque required, as well as battery selection based on the power required, can be calculated with the help of dynamics equations. The procedure for the same is given below [2, 6, 7].

m = mass of the Vehicle=20kg
μ_d= Rolling Resistance Co-efficient =0.03

For Gradeability, as per SAE standard θ is being taken out 14^0

A = Frontal area=1 m^2
cd = Drag Co-efficient =0.75

2.1. Calculation of Tractive Force and Effort

1. Rolling resistance= μ_d×m× g
   = (0.03) × (20) × (9.81) = 5.886 N
2. Acceleration force = m × a
   = (20) × (0.556)
   = 11.12 N

3. Gradeability = m×g×sin θ
   = (20) × (9.81) × sin 14
   = 47.46 N
4. Aerodynamic force = 0.5* $\rho*A*v^2$ *c
   = (0.5) × (1.12) × (0.03) × (5.56)2 × (0.75)
   = 0.2596 N
5. Total tractive force = 5.886 + 11.12 + 47.48 + 0.26
   F = 64.746 N

2.1.1. Tractive Effort

1. T= F × r (Radius of Wheel; Wheel Diameter =6 inch)
   = 64.746 × 0.0762
   T= 4.9 N = 5N
2. Effort on normal condition = (Rolling Resistance + Aerodynamic Force)* r = 0.4682 Nm
3. On slope condition = (Rolling Resistance + Aero + Gradient) * r = 4.0847Nm
4. Maximum torque = (RR + Aero + Acceleration) * r = 1.31 Nm
5. Power required on normal condition = torque normal * Angular velocity = 57.64 W
6. On slope = torque slope × Angular velocity = 176 W
7. On accelerating = torque acceleration × Angular velocity = 117.267 W

Now, based on the power and torque calculation, the following type of BLDC Hub motor and lithium-ion battery has been selected. Moreover, the 3D CAD model of the robotic vehicle is developed. The details of all dimensions and other parameters are selected in the section below.

3. Selected Battery, BLDC Motor and 3-Wheel Robotic Vehicle Specification

As calculated in the previous section, that is, the torque and power requirement of the robotic vehicle, the following selection is being made to look at the availability and feasibility in the market.

3.1. BLDC Motor and Lithium-ion Battery Specification

Table 1. BLDC motor and lithium-ion battery specification

Rated Voltage	24V
Rated Power	250W
Weight	4 Kg
Rated Efficiency	>80%
Application	Electric Scooter / Robotic Vehicle
Wheel Size	8 Inch
RPM at No Load	60V - 425 RPM
Tyre: F/R	3.00-10
Min Torque	24V - 10 NM
Max Torque	24V - 26 NM
Max Carrying Capacity	200 Kg
Battery Voltage	24 V Lithium-ion Battery
Battery Capacity	07Ah
Battery Efficiency	89%
Nominal Cell Voltage	3.7 V 1.5Ah NMC cell Battery
Dimensions	286 x 76 x 176 mm
No. of Life Cycles	1200
Charge Cut Off Voltage	54.9 V
Discharge Cut Off Voltage	42.9 V

The proposed robotic vehicle 3D CAD model is being prepared using Solid model software. The 3D model of the same is given below.

Fig. 1 Solid model 3-wheels robotic vehicle

Table 2. Robotic vehicle specification

Parameters	Details
Overall Length	1250 mm
Overall Width	535 mm
Overall Height	590 mm
Wheel Base	730 mm
Frame (lbh)	820*452*140
Wheel to Wheel Height	360 mm
Name	**Dimension**
Actuator Travelling	100 mm
Battery Mounting Height on the Wheel	400 mm
Battery Mounting Height on the Leg	350 mm
Weight without Battery	13.5 kg
Weight with Battery	19.6 Kg
C.G. with Battery (x, y ,z)	(390.84, - 163.14,0.58) mm
C.G. without Battery (x, y, z)	(389.84, - 220.14,0.58) mm

4. Mathematical Modelling of 3-Wheel Robotic Vehicle

Understanding the robot's mechanical behaviors is necessary to construct control software for a specific case of mobile robot hardware and to design mobile robots that are suitable for the tasks at hand. Examples of the various design components for wheeled mobile robots include the placement of the robot model in its surroundings, an analysis of maneuverability considering kinematic constraints, a generalized control of the created Kinematic and Dynamic model, and the creation of the control law following the resolution of the trajectory tracking problem using an integral backstepping algorithm based on a single Lyapunov function. Initially, Kinematic modelling was done for the configuration and then, after dynamic modelling with actuator modelling been formulated.

4.1. Kinematic Modelling

The 3-wheel robot wheel parameters are given below Table 3.

Table 3. 3-wheel robot wheel parameters

Parameters	Denotation
Wheel Radius	r
Wheel Linear Velocity	v
Wheel Angular Velocity	ω
Centre Distance between Front and Rear Wheel	d
The Instantaneous Curvature Radius of the Robot Path Relative to the Mid-Point Axis	R

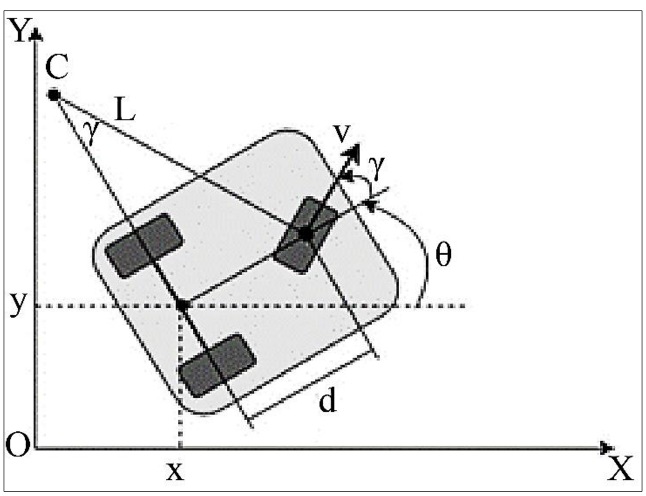

Fig. 2 Schematic diagram of the proposed 3-wheel model with front active wheel and rear passive wheels

The line diagram of the proposed 3-wheel robotic vehicle, with the consideration of the front wheel being active and the two rear wheels being passive, is given in Figure 2. Now, the kinematic mathematical modelling calculation needs Control Variables to be defined for the robotic vehicle. The control variables are given below:

Yt = Steering Angle Direction;
Ws(t) = Angular Velocity of Steering Wheel;

Now, linear velocity of steering wheel Vs(t) = Ws(t) * r;

R(t) = d tan (Π/2 - Υ_t);

Angular velocity of the moving front wheel with reference to the base frame:

W(t) = Ws(t) *r / $\sqrt{d}$2+R (t)^2;
W(t) = Vs(t) Sin Υ_t;

The kinematic model in the robot frame is calculated as follows:

Vx(t) = Vs(t) * Cos Υ_t ;
Vy(t) = 0 (No Slippage);
$\dot{\theta}$ = (vs(t)/d) * Sin Υ_t;
X_t = Vs(t) Cos Υ_t Cosθ(t);
Y_t = Vs(t) Cos Υ_t Sinθ(t);
θ = Vs(t) (Sin ϒ(t) /d);

So we can write linear and angular velocity for the 3-wheel vehicle as below by formulating the above equations:

$$V(t) = Vs(t) * \text{Cos } \Upsilon_t \quad (1)$$

$$\omega = Vs(t) * \text{Sin } \Upsilon_t/d \quad (2)$$

The above two Equations (1) and (2) are kinematic mathematic equations for the 3-wheel robotic system.

4.2. Dynamics of Proposed 3-Wheel Configuration

The following equations of motion can be used to describe non-holonomic wheel configuration with n generalized coordinates (q1, q2,...,qn) and subject to m constraints:

$$(q)\ddot{q} + (q, \dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_d = B(q)\tau - \Lambda^T (q) \quad (3)$$

Where,
Inertia matrix = M(q)
Centripetal and coriolis matrix = V(q,$\dot{q}$)
Surface friction matrix = F($\dot{q}$)
Gravitational vector = G(q)
Vector of bounded unknown disturbances, including unstructured unmodeled dynamics = τ_d
Input matrix = $B(q)$
Input vector = τ
Matrix associated with the kinematic constraints = $\Lambda^T(q)$
Lagrange multipliers = λ

4.2.1 Lagrange Dynamic Approach

The following format can be used to express the Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)+\frac{\partial L}{\partial q_i}=F-\Lambda^T\,(q)^{\lambda}$$

Where
L=T-V is the Lagrangian function
The kinetic energy of the system = T
The potential energy of the system = V
The generalized coordinates = qi
Generalized force vector = F
Constraints matrix = Λ
Vector of Lagrange multipliers = λ

Here, wheels are travelling in the X and Y planes. Hence, the robot's potential energy is assumed to be zero [5, 6].

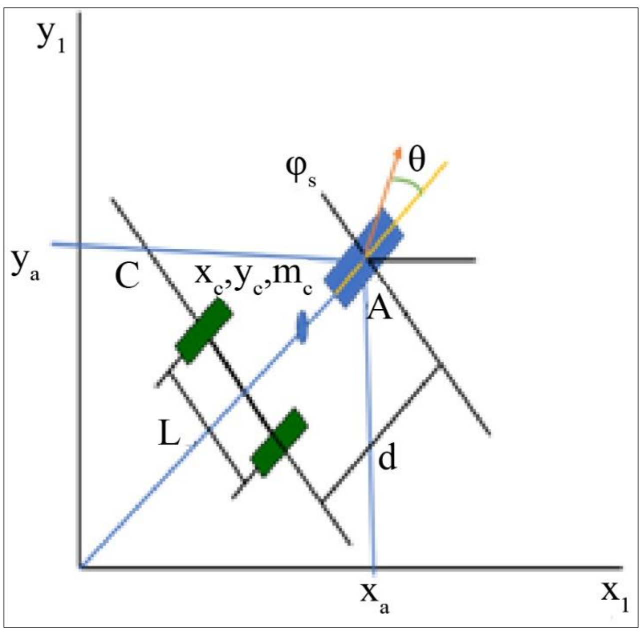

Fig. 3 Line diagram of proposed 3-wheel robotic vehicle

For the proposed configuration, i.e. Only the front active single wheel and two rear passive wheels without slip movement, the chosen generalized coordinates are as follows:

$$q=\left[x_a\ y_a\ \theta\ \varphi_s\right]$$

The total of the kinetic energies of the kinetic energies of wheels, the robot platform without wheels, and the kinetic energies of the actuators make up the kinetic energies of the mobile robot. The robot platform's kinetic energy is,

$$T_p=\frac{1}{2}m_p v_p{}^2+\frac{1}{2}I_p\,\dot{\theta}^2$$

Whereas, the kinetic energy of the steering wheel is,

$$T_{wS}=\frac{1}{2}m_w\,v^2{}_{wS}+\frac{1}{2}I_m\theta^2+\frac{1}{2}I_w\,\varphi_S^2$$

Where,
Mass of the PMR without the driving wheels and actuators = mp;
Mass of each driving wheel (with actuators) = mw;
Moment of inertia of the MR about the vertical axis through The center of mass= Ip;
Moment of inertia of each driving wheel with a motor about The wheel axis = Iw ;
Moment of inertia of each driving wheel with a motor about the wheel diameter= Im;

The following generalized coordinates can be used to calculate the xi and yi components of the centre of mass and wheels:

$$\begin{aligned} xc &= xa + \cos\theta \\ yc &= ya + \sin\theta \\ &= xa + \sin\theta \\ ywS &= ya + L\cos\theta \end{aligned}$$

Using the above all equations, the total Kinetic Energy of the Proposed 3-Wheel Robot,

$$T=\frac{1}{2}m(\dot{x}_a^2+\dot{y}_a^2)-m_p d\,\dot{\theta}\,(\dot{y}_a\cos\theta-\dot{x}_a\sin\theta)+\frac{1}{2}I_w\,(\dot{\varphi}_S^2)+\frac{1}{2}I\dot{\theta}^2$$

Where,
m = mp + mw = total mass of the robot
I = Ic + mp d^2 + 2mw L^2+ 2 Im

Using the standard Lagrangian formula, for the given case L=T, the equation of motion of the 3W Robot is given as follows:

$$m\ddot{x}a-md\,\ddot{\theta}\sin\theta-md\dot{\theta}^2\cos\theta=C1$$

$$m\ddot{y}a-md\,\ddot{\theta}\cos\theta-md\dot{\theta}^2\sin\theta=C2$$

$$I\ddot{\theta}-md\ddot{x}a\sin\theta+md\ddot{y}a\cos\theta=C3$$

$$Iw\ddot{\varphi}S=\tau S+C4$$

Hence, the kinematic constraint coefficients (C1, C2, C3, C4) are expressed in terms of the langrage multipliers vector **λ** & **Λ** as a kinematic constraint matrix. It is now possible to represent the acquired above equation of motion in generic form, and further formulation leads us to the following final equation,

$$\bar{M}(q)\,\acute{\eta}+\bar{V}\,(q,\dot{q})\,\eta=\bar{B}\,(q)\tau$$

Using the kinematic model equation and linear velocities v, angular velocities ω, the dynamic equation of the proposed system is written as below,

$$(m+ 2*Iw/R^2)\dot{V} - mc\, d\, \omega^2 = \tau s /R$$
$$(I+2*Iw/ R^2)\, \dot{\omega} + mc\, d\, \omega\, v = \tau s/R$$

4.2.2. Actuator Modelling

We have selected the BLDC hub motor as the front steer as well as the drive wheel [8, 9].

Va is utilized as controller input to regulate the speed of the same armature. (keeping field circuit constant).

For armature circuit,

Va=Raia + La dia/dt + ea;

ia = Armature Current; (Ra ,La)= resistance and inductance of the armature winding

ea= Kb*ωm ; ωm =angular speed of rotor, τm = torque of motor

τm=Kt* ia__(50);

(Kt, Kb)=torque constant, τ = output torque applied to the wheel τ =N* τm__(51); N= gear ratio

Now, as the BLDC hub is directly used or joined to the robot wheels over gears, the motion equations of the actuator are directly linked with the mechanical dynamics of the proposed configuration as follows:

- ωmS = N * φS

The proposed Simulink model for the BLDC hub motor is given below:

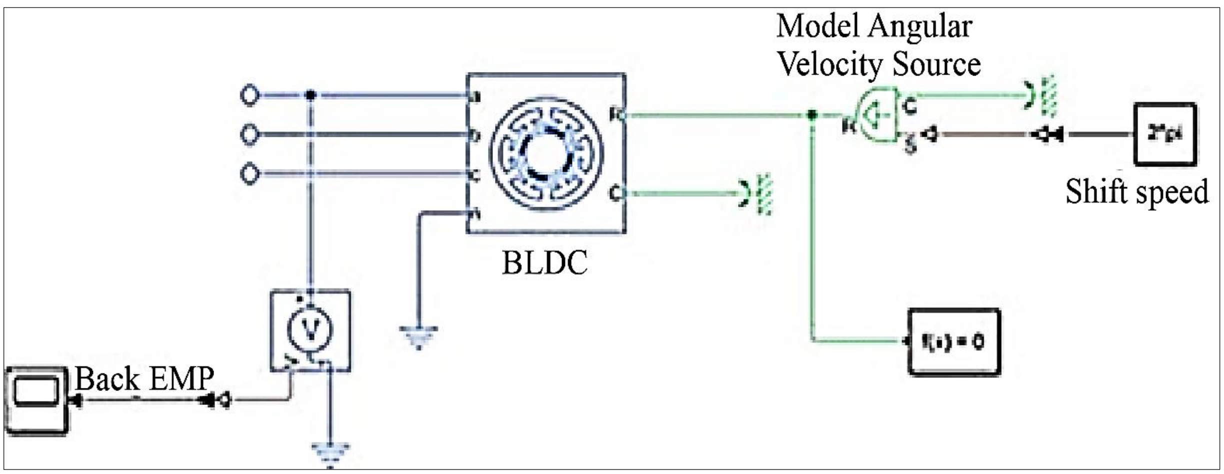

Fig. 4 Simulink model of BLDC hub motor

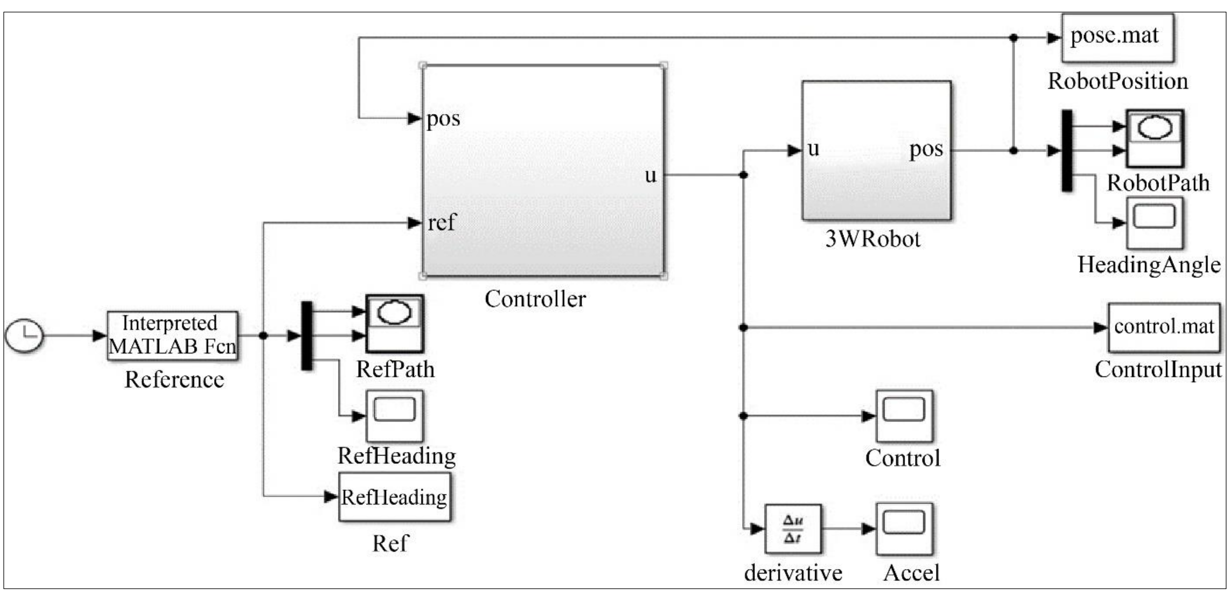

Fig. 5 Simulink control model

4.2.3. Proposed Mobile Robotic Vehicle Simulink Model

We have derived kinematic equations and actuator modelling equations in earlier sections. The Simulink model has been developed based on the derived formulas. The model consists of a trajectory generator block, a kinematic model block, a controller block, and a robot dynamics block. The trajectory generator generates the desired trajectory for the robot to follow, which in this case is a straight line and curved line. The kinematic model block calculates the robot's position and orientation based on the wheel velocities. The controller block calculates the control inputs to the wheels to achieve the desired trajectory, and the robot dynamics block simulates the robot's movement based on the control inputs. The developed model has been applied with assigned trajectories, as shown in Figure 6.

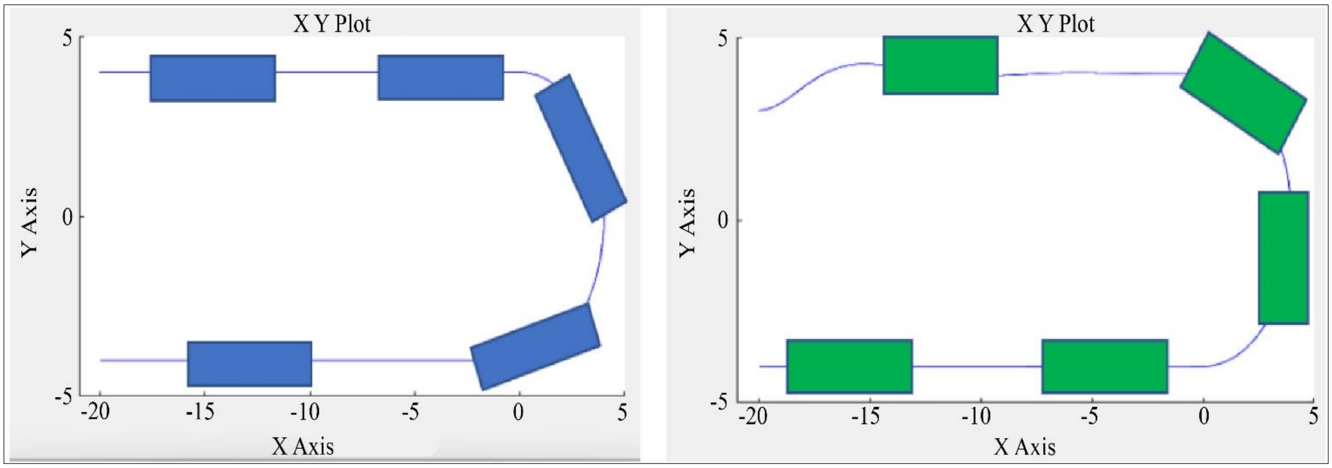

Fig. 6 Trajectory movement

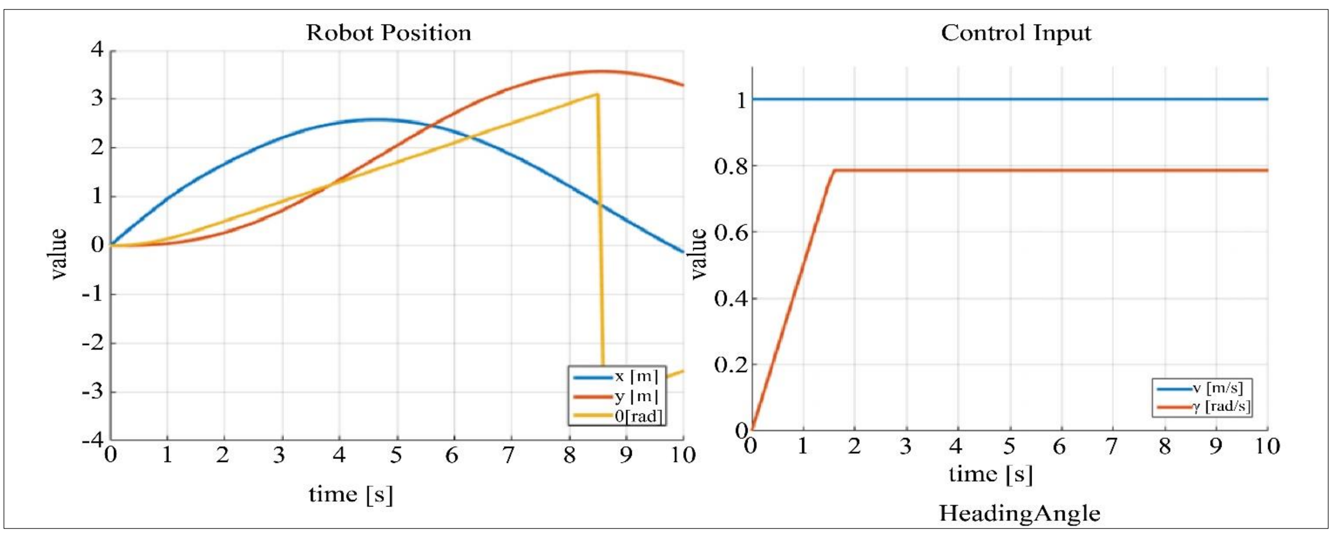

Fig.7 Robot positioning and control

5. Result and Discussion

Trajectory control is an essential part of robot control, allowing the robot to move from one point to another while avoiding obstacles and other obstacles. In the simulation results, the blue line signifies the anticipated trajectory and the green line characterizes the actual trajectory tracked by the robotic vehicle.

The robot appears to successfully follow the specified trajectory, as the tracking error is minimal and can be seen at the end. The control effort is also relatively low, indicating that the control algorithm is efficient in controlling the robot's movement. Finally, the settling time is relatively short, indicating that the robot quickly reaches its final position.

6. Conclusion

Mathematical modelling, that is, the Kinematic model and Langrage Dynamic model for the proposed 3-wheel mobile robotic vehicle, have been developed. Moreover, vehicle dynamic-based findings are being used for the BLDC hub motor and battery selection. The selected hub motor modelling is also being done, and the Simulink model is being developed. Based on the formulas, the 3D CAD model and Simulink model integration have been created, and the simulated robotic vehicle follows reference trajectories without error. The frictional effects created between the robot's wheels and the surface are disregarded because the analysis was conducted under ideal circumstances. Moreover, such conditions can be applied for the development of a perfect control algorithm along with a dynamics modelling based algorithm to be developed as a future scope.

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