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Original Article

Determination of the Theoretical Digging Force of Hydraulic Excavator Using Transfer Functions of Manipulator Drive Mechanisms

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Abstract - *Determining the theoretical digging force of the manipulator of the hydraulic excavator is a very important issue since it is a main characteristic of these machines. The arm and bucket drive mechanisms mainly determine the theoretical digging force. The hydraulic cylinders are included in the manipulator drive mechanisms of hydraulic excavators, i.e. they form the input force of these mechanisms. Thus, the driving mechanisms of the boom and the arm are of the type of Inverted Slider-Crank Mechanism (ISCM), and the driving mechanism of the bucket is the six-link mechanism of Watt type, composed of two sequentially connected mechanisms - ISCM and articulated Four-Bar Linkage (FBL). The proposed method uses transfer functions of the driving mechanisms to relate the input force to the output driving moment. The output moment accordingly drives the arm and the bucket, which forms the digging force of the excavator. Thus, the driving moment, resp. The type of transfer functions of the driving mechanism in dimensionless form completely describes the digging force. In the work, the transfer functions of the considered mechanisms are described, and typical diagrams of the driving moment of these mechanisms are attached, respectively of the theoretical digging force.*

***Keywords* -** *Digging force, Driving mechanism, Hydraulic excavator, Manipulator, Transfer function.*

1. Introduction

The hydraulic cylinders form the driving mechanisms of the manipulator of hydraulic excavators. The drive of the first two units of the excavator (boom and arm) is carried out by a simple mechanism - a hydraulic cylinder hinged to the two units of each pivot of the excavator's kinematic chain. This forms a planar mechanism of the type Inverted Slider-Crank Mechanism (ISCM). The drive of the third unit (bucket) is formed by two sequentially connected mechanisms. The first is the already described ISCM, and the second is the articulated Four-Bar Linkage (FBL). In total, the two mechanisms form a six-link mechanism of the Watt type. So, the described mechanisms drive the bucket of the excavator and form the digging force. It is extremely important, given the hydraulic cylinders and dimensions, to determine what force they form on the lip of the bucket, i.e. the theoretical digging force.

Determining the theoretical digging force of the excavator is a very important issue since it is a main characteristic of these machines. It is determined by the arm and bucket actuators of the excavator, which create the driving moment on the arm and bucket, respectively. The tangential digging force [1] is determined by dividing this moment by the distance from the main pivot of the arm or bucket to the lip of the bucket. That is, we have actuators with input force (cylinder force) and output moment or determining the output moment of mechanisms gives us the driving moment of the arm or bucket. Here is the place of the transfer function of the actuators, with which a connection is made between the input force and the output moment. Thus, the transfer functions are an important element in the process of determining digging forces.

In the design of hydraulic excavators, the theoretical digging force is determined by the power of the hydraulic cylinders and the dimensions of the manipulator. An important criterion for the safety of the design is that the theoretical digging force must be greater than the resistance force offered by the soil, respectively the terrain to be excavated. Therefore, it is very important at the design stage to predict the resistance force of the soil, taking into account all soil parameters [2,3]. In other words - the theoretical digging force is the maximum digging resistance that an excavator can overcome, which is an important measure of its digging capacity. [4] Of course, it should be noted that digging force is not the only target parameter in the design and optimization of hydraulic excavators. These processes also take into account other important parameters, such as kinematic, tribological, time, weight, dynamic, etc. [5].

There is a lot of research on the topic of excavator digging forces. In [6], a method is proposed for predicting the digging forces of a mini hydraulic excavator in the excavation task, using the basics of excavator kinematics. A similar task is solved in [7], where the digging force is calculated by analyzing the static forces of a mini hydraulic excavator. In [8], an analysis of the theoretical digging forces (normal and tangential and resistance moment) is carried out when the bucket moves along a normal digging trajectory. Another study [9] provides a theoretical method for calculating the digging force and analyzing the structural design and optimization of the excavator. Other authors determine bucket digging forces by considering soil resistance forces as well as the forces in the bucket joints [10,11]. In [12], a new method based on a three-dimensional dynamic convex polytope is proposed to comprehensively quantify and analyse the theoretical mining performance of an excavator. The proposed method can quantitatively analyse the dynamic digging performance of an excavator in the entire working space and effectively evaluate the characteristics of the manipulator mechanisms. In [13], a method was developed for determining the dynamic digging force of excavators along a certain theoretical trajectory in comparison with measured resistances. It should be noted that the dynamic models of the excavator, determining the dynamic force of digging, more fully describe the operation of the excavator.

The theoretical digging force is needed to determine the performance of the hydraulic excavator, the power characteristics in the working zone and the maximum digging force at different positions. The theoretical digging force is the most important performance parameter used by the designer in the design stage of the excavator [9,14]. It is also a good idea to conduct an experimental study on real excavators, in which the theoretical digging force is compared with the measured digging force in different soils [15]. This will provide evidence for the adequacy of used theoretical mechano-mathematical model.

2. Mechano-Mathematical Model for Determining the Theoretical Digging Force of the Excavator

2.1. Mechano-Mathematical Model of Digging

In the process of digging, the teeth of the bucket must overcome the resistance of the soil. The hydraulic cylinders of the driving mechanisms of the excavator create a tangential force at the bucket lip, which is the theoretical digging force that must overcome the resistive force of the soil. Digging is primarily done by arm and bucket motion, so the arm and bucket hydraulic cylinders create the theoretical digging forces. Figure 1 shows the scheme of the studied model, including the hydraulic cylinders of the arm and bucket in the corresponding driving mechanisms considered in a number of studies [6-10]. The task is, starting with the forces of the two input cylinders (F_a and F_b), to determine the arm and bucket driving moment they produce (M_a and M_b), which form the theoretical digging forces (P_a and P_b). That is, here, the power capabilities when digging the excavator are determined, taking into account only the forces of the hydraulic cylinders, not taking into account the weights of all links and the weight of the earth mass in the bucket.

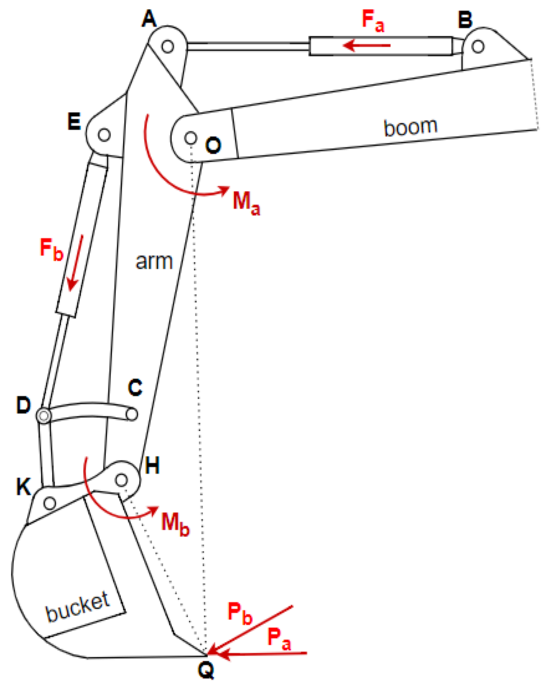

Fig. 1 The scheme of the mechano-mathematical model for determining the theoretical digging force of the excavator.

The determination of the theoretical digging force is carried out under the condition that the boom unit of the excavator is stationary, its corresponding drive cylinder is inactive, and the manipulator is in equilibrium under the action of the applied forces.

2.2. Mechano-Mathematical Model of Arm Digging

Here, we determine the force characteristics and the digging force created by the arm cylinder alone, assuming that the other two cylinders - the boom and the bucket - are inactive. Figure 2 shows the kinematic diagram of the actuator. The cylinder of the arm AB is hinged at points A and B to the boom and the arm, respectively, and the boom and the arm have a common pivot at point O. This is how a mechanism of the type – Inverted Slider-Crank Mechanism (ISCM) is formed. Its position is defined by the angle φ or by the stroke of the cylinder *s*, resp. The current length of the cylinder is $S=AB$. The figure shows the dimensions $R<L$, which form the small parameter λ – basic characteristic of the mechanism:

$$\lambda = \frac{R}{L} < 1 \tag{1}$$

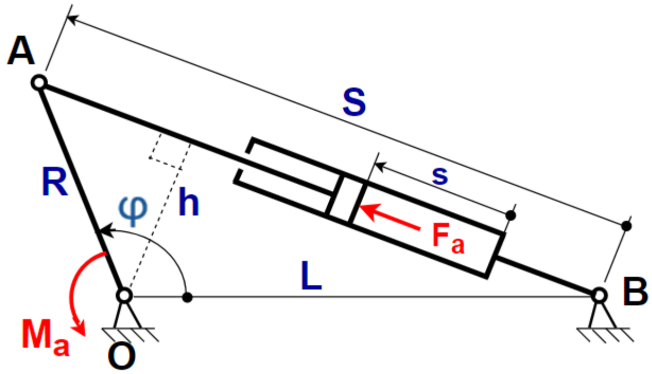

Fig. 2 The scheme of the actuator – ISCM mechanism

The relation between parameters S and s is given by:

$$S = L - R + s, \quad 0 < s < 2R \tag{2}$$

The length of the cylinder $S=AB$ is determined by:

$$S = L\sqrt{1 + \lambda^2 - 2\lambda \cos\varphi} \tag{3}$$

By differentiating the Equation 3, the transfer function of ISCM is obtained:

$$\frac{dS}{d\varphi} = \frac{R \sin\varphi}{\sqrt{1+\lambda^2-2\lambda\cos\varphi}} \tag{4}$$

After that, by expressing the area of the triangle OAB in two ways ($S.h/2=L.R.\sin\varphi/2$) and using Equation 3 for the arm h of the force F_a relative to the point O, it turns out that it is equal to the transfer function of ISCM:

$$h(\varphi) = \frac{dS}{d\varphi} \tag{5}$$

Then the force F_a of the arm drive cylinder creates a driving moment M_a of the arm, next way:

$$M_a(\varphi) = F_a \frac{dS}{d\varphi} = F_a R \frac{\sin\varphi}{\sqrt{1+\lambda^2-2\lambda\cos\varphi}} \tag{6}$$

If Equation 4 is divided by R, the transfer function takes the following dimensionless form:

$$f(\varphi) = \frac{\sin\varphi}{\sqrt{1+\lambda^2-2\lambda\cos\varphi}} \tag{7}$$

The function $f(\varphi)$ completely defines the driving moment of the ISCM in dimensionless form:

$$\widetilde{M}_a(\varphi) = \frac{M_a(\varphi)}{F_a R} = f(\varphi) = \frac{\sin\varphi}{\sqrt{1+\lambda^2-2\lambda\cos\varphi}} \tag{8}$$

From Equation 6, the theoretical digging force is easily determined depending on the transfer function $f(\varphi)$:

$$P_a(\varphi) = \frac{M_a(\varphi)}{L_{OQ}} = \frac{F_a R}{L_{OQ}} \frac{\sin\varphi}{\sqrt{1+\lambda^2-2\lambda\cos\varphi}} \tag{9}$$

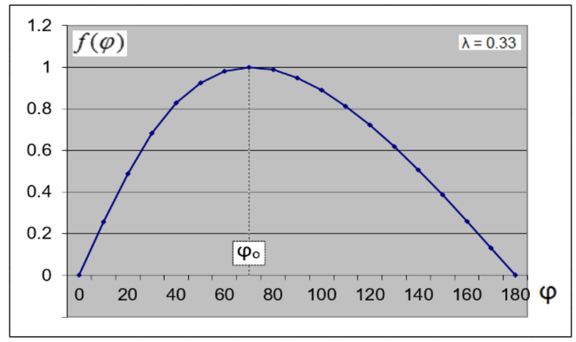

Fig. 3 Graph of the dimensionless transfer function $f(\varphi)$, which defines the driving moment $Ma=C_{Ma}.f(\varphi)$ and the digging force $Pa= C_{Pa}.f(\varphi)$

Where L_{OQ} is the distance from pivot O to the lip of the bucket, which can easily be determined from the arm's geometry.

That is, the moment function M_a of the arm and the digging force F_a are completely determined by the dimensionless transfer function $f(\varphi)$ to within one constant:

$$M_a(\varphi) = C_{Ma}.f(\varphi)$$
$$P_a(\varphi) = C_{Pa}.f(\varphi) \tag{10}$$

Where the constants *"C"* are:

$$C_{Ma} = F_a.R$$
$$C_{Pa} = F_a.R/L_{OQ} \tag{11}$$

Then, the maximum value of the driving moment M_a and of the digging force P_a is determined by the constants *"C"* (Equations11). The investigation of the function $f(\varphi)$ shows that the maximum value at the point of maximum φ_0 is the following:

$$f(\varphi_0)\, max = 1$$
$$\cos\varphi_0 = \lambda \tag{12}$$

Figure 3 shows the graph of the dimensionless transfer function $f(\varphi)$ in the full range for changing the angle φ (0 – 180^O). This diagram defines exactly the driving moment M_a and the digging force P_a with an accuracy of one constant (Equations 11). A similar view of the digging function was obtained in [9].

2.3. Mechano-Mathematical Model of Bucket Digging

Here, we suppose again that the other two cylinders – of the boom and the arm are inactive. The bucket is driven by the bucket's hydraulic cylinder, with included in the system of two sequentially connected mechanisms. The first one is the already described ISCM (points C, D, E), and the second one is an articulated four-bar linkage (FBL) - points C, D, H, and K (Figure 1). Together, the two mechanisms form a six-link mechanism of the Watt type. Here, we need to determine the theoretical digging force of the bucket P_b by means of the driving moment of the bucket M_b.

Figure 4 shows the diagram of the researched model of bucket Digging with the indicated angles and geometrical dimensions. Here, the same designations of ISCM mechanism parameters are used as in point 2.2., since the Bucket Digging task is solved independently and does not depend on Arm Digging. Of course, these parameters have different values here.

We determine the driving moment of the bucket by equating the powers of the forces on the input and output of the six-link mechanism:

$$M_b \frac{d\psi}{dt} = F_b \frac{ds}{dt} \tag{13}$$

After some transformations, the moment M_b takes the following form:

$$M_b = F_b \frac{dS}{d\varphi}\frac{d\varphi}{d\psi} = F_b.R.f(\varphi).g(\varphi) \tag{14}$$

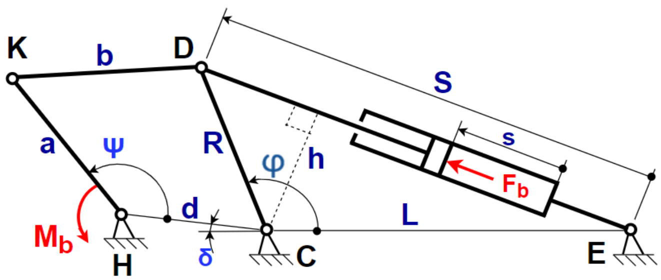

Fig. 4 The scheme of the actuator of bucket digging

Where *f(φ)* is the dimensionless transfer function of the ISCM, and *g(φ)* denotes the transfer function of the FBL mechanism. The function *g(φ)* can be easily obtained from the geometry of FBL in the following form.

$$g(\varphi) = \frac{d\varphi}{d\psi} = \frac{\frac{d}{R}\sin\psi - \sin(\varphi-\psi+\delta)}{\frac{d}{a}\sin\varphi - \sin(\varphi-\psi+\delta)} \tag{15}$$

Formally, the FBL transfer function is a function of two variables – *g(φ,ψ)*. But the FBL has one degree of freedom, and the existing geometric relationship between the angles φ and ψ brings the transfer function into the form *g(φ)*. The angle ψ can be expressed as a function of the angle φ with the equation:

$$\psi = \delta + tan^{-1}\left(\frac{R.\sin\varphi - d.\sin\delta}{R.\cos\varphi + d.\cos\delta}\right) + cos^{-1}\left(\frac{m^2+a^2-b^2}{2.m.a}\right) \tag{16}$$

Where the distance $m=HD$ is expressed as:

$$m = \sqrt{(R.\sin\varphi - d.\sin\delta)^2 + (R.\cos\varphi + d.\cos\delta)^2}$$

From Equation 14 it follows the dimensionless driving moment in the following form:

$$\widetilde{M}_b(\varphi) = \frac{M_b(\varphi)}{F_b.R} = f(\varphi).g(\varphi) \tag{17}$$

That is, the dimensionless driving moment of the bucket is equal to the product of the dimensionless transfer functions of the ISCM and the FBL mechanisms.

From here, the theoretical digging force of the bucket P_b is easily determined depending on the product of transfer functions *f(φ)* and *g(φ)* and the distance L_{HQ}:

$$P_b(\varphi) = \frac{M_b(\varphi)}{L_{HQ}} = \frac{F_b R}{L_{HQ}} f(\varphi).g(\varphi) \tag{18}$$

Figure 5 shows the graph of the dimensionless transfer functions *f(φ)* and *g(φ)*, as well as their product *F(φ)= f(φ).g(φ)* in the full range of angle variation φ (0 – 180^O). Thus, the product of the two transfer functions *F(φ)* determines with an accuracy of one constant *"C"* the driving moment $M_b(\varphi)$ and the digging force $P_b(\varphi)$:

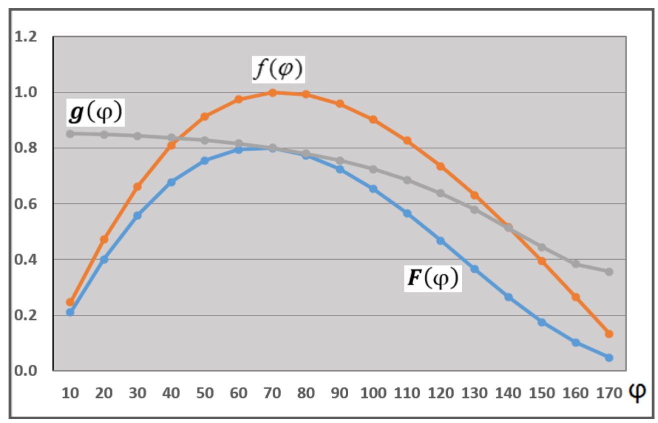

Fig. 5 Graph of the dimensionless transfer functions *f(φ)* and *g(φ)*, as well as their product *F(φ)=f(φ).g(φ)*, which describes exactly the driving moment $M_b=C_{Mb}.F(\varphi)$ and the digging force $P_b= C_{Pb}.F(\varphi)$.

$$M_b(\varphi) = C_{Mb}.F(\varphi)$$
$$P_b(\varphi) = C_{Pb}.F(\varphi) \tag{19}$$

Where is denoted:

$$F(\varphi) = f(\varphi).g(\varphi)$$
$$C_{Mb} = F_b.R$$
$$C_{Pb} = F_b.R/L_{HQ} \tag{20}$$

3. Discussion

Determination of the theoretical digging force of the hydraulic excavator is a very important issue since it is a main characteristic of these machines. This topic is widely researched, and numerous studies obtain the digging force for different points and curves from the workspace. The obtained results depend on the position of the manipulator and the generalized coordinates.

When determining the digging force of the hydraulic excavator using the transfer functions of the drive mechanisms, the power characteristics of the excavator depend on the power of the cylinder and the geometric parameters of the drive mechanism. Studying the type of transfer functions gives us complete information about the power characteristics of the excavator. Two drive mechanisms are used in the excavator manipulator - ISCM and a six-link mechanism of the Watt type.

The transfer function of the ISCM mechanism is strictly defined in the form of a sine function with a shifted maximum. Here, the driving moment *Ma* strictly copies the transfer function $f(\varphi)$ and the only way to influence it in the synthesis process is through the parameter λ and the power of the cylinder naturally. The proportional increase in size R, respectively L only increases the size of the mechanism without changing the function of the driving moment. With the ISCM mechanism, we have no special possibilities to influence the driving moment except for the correct location of the transfer function in relation to the entire operating range.

The transfer function of the six-link mechanism of the Watt type is equal to the product of the transfer functions of the two-component mechanisms - ISCM and FBL. Here, unlike the ISCM, the FBL mechanism has a greater opportunity to influence the type of driving moment by changing the link sizes and angular orientation relative to the ISCM mechanism. But still, the resulting transfer function that defines the driving moment is close to the type of the transfer function of ISCM – $f(\varphi)$, but with some distortion.

In this paper, a theoretical mechano-mathematical model of excavator digging has been built. The proposed theoretical method can be applied as the first stage of the synthesis or optimization of the power characteristics of the manipulator of the hydraulic excavator. The obtained power characteristics of the two drive mechanisms - of the arm and the bucket, can be used to determine the power characteristics for each point of the working area of the excavator. In a subsequent stage of development, the influence of the weights of the units should be added and later the dynamic loads also. Of course, it is good to apply the proposed mechano-mathematical model to a real excavator model. The obtained theoretical results can be validated by comparing them with practically measured forces, diagrams and tabular data of the excavator. In this direction, there is a wide field for practical application of the proposed method for different types of excavators.

4. Conclusion

The present study proposes a method to determine the theoretical digging force of the hydraulic excavator using the transfer functions of the driving mechanisms. It is shown that the obtained theoretical digging force is completely determined by the transfer function of the corresponding mechanism within one force constant. This constant depends on the force of the cylinder and some dimensions of the mechanisms. I.e. the type and characteristic of the transfer function accurately determine the type and characteristic of the theoretical digging force of the hydraulic excavator for the entire working range of the corresponding driving mechanism. Knowing the transfer functions of the driving mechanisms sheds new light on the force characteristics of the excavator.

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