

Original Article

Trajectory Tracking Control for Autonomous Vehicles based on Gaussian Process Regression and Adaptive Control Increment Constraint Adjustment

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Abstract - Under high-speed and extreme conditions, model mismatch of autonomous vehicles leads to degraded tracking accuracy and reduced lateral stability performance. To solve this problem, a Model Predictive Control (MPC) method using Gaussian Process Regression (GPR) is proposed in this paper. Based on this, an adaptive strategy for adjusting control increment constraints based on steering angle variation is developed using the prediction variance of GPR. This method employs GPR to learn the single-step residuals between the nominal 3-Degree-of-Freedom (3-DOF) vehicle model and its high-fidelity counterpart in Carsim, and then compensates for these residuals recursively within the MPC prediction horizon. Meanwhile, this method uses the prediction variance of GPR as the uncertainty indicator of the GPR model, and establishes an adaptive mapping relationship between the uncertainty parameters and the control increment constraints. When the uncertainty of the GPR model is high, the system actively reduces the control increment constraint. On the Carsim/Simulink platform, the proposed framework is validated under double lane change maneuvers. Simulation results show that, for high-speed dry-road scenarios, the GPR-MPC method lowers the lateral error peak by 16.2% and the heading error peak by 12.7%, compared to the baseline MPC. The A-GPR-MPC method with an adaptive control increment constraint adjustment strategy can, while maintaining the tracking accuracy, further reduce the peak yaw rate by 9.2% and the peak sideslip angle by 8.4%. Under the medium-speed wet road conditions, the A-GPR-MPC can further reduce the peak yaw rate by 9.0%, the peak sideslip angle by 19.0%, and the peak front wheel steering angle by 24.1%. These results demonstrate the proposed method's ability to enhance both tracking accuracy and lateral stability.

Keywords - Autonomous vehicles, Gaussian process regression, Lateral stability, Model predictive control, Trajectory tracking.

1. Introduction

Trajectory tracking for autonomous vehicles is a typical multi-constraint optimization problem. Model Predictive Control (MPC) provides an effective approach to multi-constraint optimization problems and has found extensive application in trajectory tracking [1-3]. However, the predictive performance of MPC depends heavily on the fidelity of the vehicle dynamics model employed. High-precision vehicle models would increase the computational load. Therefore, in engineering practice, simplified 3-DOF vehicle models are often adopted to balance accuracy against computational burden. However, this model significantly deviates from the actual vehicle dynamic response in high-speed or sharp-curvature driving scenarios. Specifically, it manifests as the tire cornering stiffness entering the nonlinear region, yaw rate response lag, and a sharp increase in the centroid slip angle, etc., which leads to model mismatch [4, 5].

The research carried out by Zhou et al. [6] also confirmed that when using a simplified vehicle model for MPC trajectory tracking control, the tracking accuracy would significantly decrease under high-speed conditions, and tracking error increases with vehicle speed.

To mitigate the problem of model mismatch resulting from model simplification, researchers have proposed various improved MPC methods. Wang et al. [7] used Recursive Least Squares (RLS) to identify parameters such as tire lateral stiffness, aiming to improve the flexibility of the vehicle model against time-variant characteristics. Zou et al. [8] utilized the Extended State Observer (ESO) for estimating the lumped disturbance and compensated for it within the framework of incremental linear MPC. Zhang et al. [9] combined fuzzy control with MPC, using a fuzzy logic scheme to adaptively tune both prediction and control horizons, along



with correcting the front steering angle. Zhang et al. [10] developed an MPC scheme with adaptive learning capabilities. This scheme uses RLS for online identification of system parameters, thereby improving the prediction accuracy of MPC. Moreover, by designing robustness constraints, the robustness of the system is enhanced. MPC methods based on nonlinear models and intelligent algorithms such as particle swarm optimization have also found extensive application in trajectory tracking for autonomous vehicles and mobile robots [11]. Although these improved MPC methods have achieved certain results, they are highly model-dependent and have limited compensation effects in non-linear conditions. They also struggle to maintain tracking accuracy and lateral stability in extreme driving conditions.

In parallel with these model-based developments, data-driven methods have gained increasing attention in autonomous vehicle control. Hamadeh et al. [12] employed deep reinforcement learning to automate MPC weight tuning, reducing reliance on manual calibration. Neural-network-based approaches have also been integrated with MPC to learn vehicle lateral dynamics and compensate for model mismatch [13]. In a related direction, probabilistic dynamics models learned from real data have been used to construct controller tuning frameworks that enable rapid parameter adjustment [14]. However, these data-driven methods face their own challenges, including data efficiency and generalization to unseen scenarios.

Among these data-driven methods, GPR is especially appropriate for model error compensation, as it can not only predict residuals but also quantify the uncertainty of its predictions. As a non-parametric probabilistic model, GPR can learn the uncertainty of residuals from the data and provide prediction confidence intervals [15].

Several studies have applied GPR to compensate for model uncertainties in MPC frameworks. Kabzan et al. [16] demonstrated its effectiveness in autonomous racing by combining GPR with MPC to handle unmodeled dynamics. Yu et al. [17] extended this approach with a deep GPR structure to further improve closed-loop performance. For standard trajectory tracking, Li et al. [18] used GPR residuals to compensate for vehicle state prediction errors, reporting reductions of 33.3% in lateral error and 27.9% in heading error.

Han et al. [19] applied similar compensation to a monorail vehicle model, achieving notable tracking error reduction. The predictive variance of GPR has also attracted attention in the design of controllers. Hewing et al. [20] quantified the prediction variance of GPR as a probability constraint, which was used for cautious MPC, thereby improving overall system performance. Wu et al. [21] utilized the prediction variance of GPR to drive active exploration, aiming to maximize uncertainty to obtain more information and further improve the control performance. These studies demonstrate that the

prediction variance holds great potential in adaptive control. However, how to utilize it to balance tracking accuracy and lateral stability remains to be explored.

A GPR-based MPC framework is developed in this work to address the issue. An adaptive strategy is embedded in the framework, which uses the prediction variance of GPR—an indicator of the model's confidence—to adjust control increment constraints online. When the variance is low, the controller trusts the GPR compensation and retains nominal steering authority to preserve tracking accuracy. When the variance increases, indicating higher model uncertainty, the controller proactively reduces the steering rate to enhance lateral stability. This strategy maintains the accuracy of trajectory tracking when the model uncertainty remains small; when it becomes large, it actively reduces the steering rate to enhance lateral stability. The strategy is assessed through double lane change maneuvers covering high-speed dry-road conditions as well as medium-speed wet road conditions. The results show that this strategy offers both tracking accuracy and enhanced lateral stability.

2. Nominal Vehicle Dynamics Model and MPC Design

2.1. 3-DOF Vehicle Dynamics Modeling

Selecting a vehicle model inherently involves balancing computational efficiency against predictive accuracy, which is a common issue in vehicle dynamics optimization. This study employs the widely adopted 3-DOF vehicle dynamics model for autonomous driving applications. This model incorporates its longitudinal, lateral, and yaw motions, while neglecting suspension movement and aerodynamic lateral force, as illustrated in Figure 1. In this figure, OXY denotes the global frame, while oxy represents the vehicle-fixed frame.

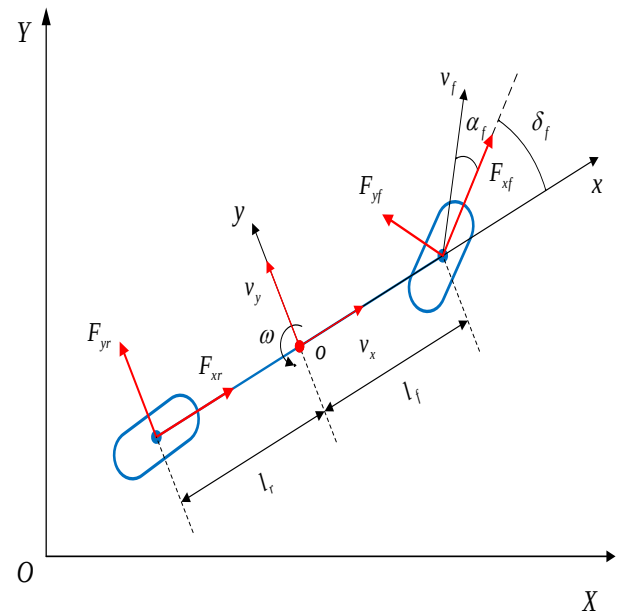


Fig. 1 3-DOF vehicle dynamics model

The governing equations for the vehicle dynamics take the following form:

$$\begin{cases} m(\dot{v}_x - v_y\omega) = F_{xf} \cos \delta_f - F_{yf} \sin \delta_f + F_{xr} \\ m(\dot{v}_y + v_x\omega) = F_{xf} \sin \delta_f + F_{yf} \cos \delta_f + F_{yr} \\ I_z \dot{\omega} = l_f(F_{xf} \sin \delta_f + F_{yf} \cos \delta_f) - l_r F_{yr} \end{cases} \quad (1)$$

Where m denotes the vehicle mass (kg), I_z represents the yaw moment of inertia about the z-axis (kg·m²), l_f and l_r are the distances from the Center of Gravity (CG) to the front and rear axles (m), F_{xf} and F_{xr} denote the longitudinal forces on the front and rear tires (N), F_{yf} and F_{yr} are the lateral forces (N), δ_f is the front steering angle (rad), v_x and v_y represent the longitudinal and lateral velocities at the vehicle CG (m/s), and ω indicates the yaw rate (rad/s).

Using the linear tire formulation, the lateral forces F_{yf} and F_{yr} in Equation (1) are derived. With the longitudinal force effect on the front steering angle neglected, the front and rear axle lateral forces are expressed as:

$$F_{yf} = C_f \alpha_f, \quad F_{yr} = C_r \alpha_r \quad (2)$$

with C_f and C_r being the cornering stiffness values for the front and rear axles, while α_f and α_r denote the slip angles of the front and rear tires. Under the small-angle assumption, the slip angles are determined as:

$$\alpha_f = \delta_f - \frac{v_y + l_f \omega}{v_x}, \quad \alpha_r = \frac{l_r \omega - v_y}{v_x} \quad (3)$$

To facilitate controller design, the above nonlinear model is linearized. Solving the nonlinear model directly would impose a heavy computational burden. Linearization around the reference trajectory transforms the problem into a quadratic programming form that can be solved efficiently. Equations (1)-(3) are rearranged into the state-space form $\dot{x} = f(x, u)$, with the state vector $x = [v_y, \omega, \varphi, Y, X]^T$ and the control input $u = \delta_f$. Performing a Taylor expansion at the reference point (x_0, u_0) and keeping only the first-order terms gives the following linear time-varying system:

$$\dot{x} = A_c x + B_c u + d_c \quad (4)$$

where $A_c = \left. \frac{\partial f}{\partial x} \right|_0$, $B_c = \left. \frac{\partial f}{\partial u} \right|_0$, $d_c = f(x_0, u_0) - A_c x_0 - B_c u_0$. A_c and B_c are the linearization coefficient matrices, and d_c is the constant term.

Discretization at $T_s = 0.02$ s yields the following discrete model:

$$x(k+1) = A_d x(k) + B_d u(k) + d_d \quad (5)$$

where $A_d = I + T_s A_c$, $B_d = T_s B_c$, and $d_d = T_s d_c$. A_d , B_d , and d_d are the corresponding matrices after discretization.

2.2. Fixed-Parameter MPC Design

For comparison, a conventional fixed-parameter MPC, denoted as Baseline MPC, is first designed. The control increment $\Delta u(k) = u(k) - u(k-1)$ is introduced, and Equation (5) is rewritten into the augmented form:

$$\xi(k+1) = \tilde{A} \xi(k) + \tilde{B} \Delta u(k) + \tilde{d} \quad (6)$$

where $\xi(k) = [x^T(k), u(k-1)]^T$, and $\tilde{A}$, $\tilde{B}$ are the associated augmented matrices.

The prediction and control horizons are set as $N_p = 16$ and $N_c = 5$, respectively. The objective function takes the form:

$$J = \sum_{i=1}^{N_p} \|y(k+i|k) - y_{ref}(k+i)\|_Q^2 + \sum_{i=0}^{N_c-1} \|\Delta u(k+i) + i\|_R^2 + \rho \varepsilon^2 \quad (7)$$

where $y = [\varphi, Y]^T$ is the output vector, φ is the yaw angle, Y is the lateral position, $Q = \text{diag}(1.0, 100.0)$ is the output weight matrix, $R = 1.0 \times 10^3$ is the control increment weight coefficient, with ε as the slack variable, and $\rho = 1 \times 10^4$ as its penalty weight. The control input constraint is $|\delta_f| \leq 0.1744$ rad, and the control increment constraint is $|\Delta \delta_f| \leq 0.0148$ rad. The quadratic programming problem is addressed online with the active-set algorithm.

3. Recursive Residual Compensation MPC based on GPR

3.1. Residual Definition and GPR Principle

The nominal model inevitably deviates from the true vehicle dynamics due to model simplification. The deviation of the nominal prediction from the actual response is defined as the one-step residual. The one-step prediction residual $e(k)$ for the nominal model (Equation (5)) is given by:

$$e(k) = x_{true}(k+1) - x_{nom}(k+1) \quad (8)$$

where x_{true} is derived from the Carsim high-fidelity vehicle, and x_{nom} is calculated using Equation (5).

This paper only compensates on the lateral velocity and yaw rate residuals that have the greatest impact on trajectory tracking, namely $e = [\Delta v_y, \Delta \omega]^T$. Since these residuals are related to the current state and control inputs, the input feature $z(k)$ selected is:

$$z(k) = [v_x(k), v_y(k), \omega(k), \delta_f(k)] \quad (9)$$

The residual function is modeled as a zero-mean Gaussian process using a squared-exponential kernel:

$$g(\mathbf{z}) \sim \mathcal{GP}(m(\mathbf{z}), k(\mathbf{z}, \mathbf{z}')) \quad (10)$$

$$k(\mathbf{z}, \mathbf{z}') = \sigma_f^2 \exp\left(-\frac{1}{2}(\mathbf{z} - \mathbf{z}')^T \mathbf{L}^{-1}(\mathbf{z} - \mathbf{z}')\right) \quad (11)$$

with σ_f^2 as the signal variance and $\mathbf{L}$ as the length-scale matrix.

Given the training set $\{\mathbf{Z}, \mathbf{E}\}$, where $\mathbf{E}$ is the corresponding residual matrix, for the test input $\mathbf{z}^*$, the predicted mean $\mu(\mathbf{z}^*)$ and variance $\Sigma(\mathbf{z}^*)$ by GPR are:

$$\mu(\mathbf{z}^*) = \mathbf{K}_*(\mathbf{K} + \sigma_n^2 \mathbf{I})^{-1} \mathbf{E} \quad (12)$$

$$\Sigma(\mathbf{z}^*) = \mathbf{K}_{**} - \mathbf{K}_*(\mathbf{K} + \sigma_n^2 \mathbf{I})^{-1} \mathbf{K}_*^T \quad (13)$$

Where σ_n^2 is the noise variance, $\mathbf{K}$ is the training kernel matrix, $\mathbf{K}_*$ is the test-training kernel vector, and $\mathbf{K}_{**}$ is the test auto-covariance.

3.2. Training Data Acquisition and GPR Model Training

To cover the nonlinear areas that vehicles may enter, various driving scenarios are designed in Carsim. The vehicle speed range is from 10 to 35 m/s, with the road adhesion coefficient ranging from 0.35 to 0.85. This includes conditions such as step acceleration, fixed-radius turning, sinusoidal sweep-frequency steering, double lane change, and serpentine motion. The sampling frequency is 100 Hz, the total duration is approximately 150 seconds, and a total of about 15,000 sets of data were obtained. 80% of it is used for training and 20% for validation. The input features are normalized to have zero mean and unity variance, and the output residuals are also subjected to the same normalization process. The specific parameters for each working condition are shown in Table 1.

Table 1. Data collection conditions

Condition	Vehicle speed (m/s)	Control method	Duration (s)	Purpose
Step acceleration	10→35	Straight driving	30	Cover longitudinal speed range
Constant-radius cornering	15, 20, 25, 30	Fixed steering angle (lateral acceleration 0.3g)	80	Steady-state nonlinear characteristics
Sine sweep steering	20, 30	Sine sweep (0.5–4 Hz, amplitude 0.03 rad)	60	Dynamic response
Double lane change	20, 25, 30, 35	Closed-loop path following	60	Target condition pre-coverage
Slalom	20, 30	Sine steering (0.5 Hz, amplitude 0.12 rad)	40	High-curvature continuous curves

Using the fitrgp function in the MATLAB Statistics and Machine Learning Toolbox, train two independent GPR models to predict Δv_y and $\Delta \omega$, with the kernel function being the squared exponential. On the validation set, the predicted residuals by the GPR model were in good agreement with the actual residuals, indicating that the trained model has a high level of prediction accuracy.

3.3. Recursive Residual Compensation in the Prediction Horizon

As shown in Figure 2, the overall control architecture of GPR-MPC consists of modules such as reference trajectory generation, recursive prediction, MPC optimization, control input update, GPR compensation, and vehicle state feedback.

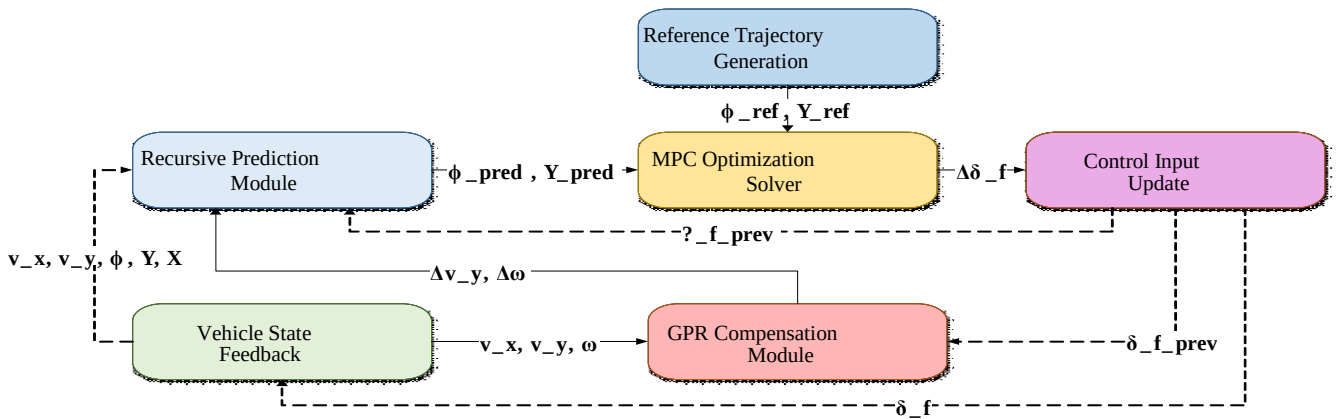


Fig. 2 GPR-MPC control architecture

Specifically, during each control cycle, the controller, using the current vehicle state and the previous control input, applies the GPR model to compensate for the first-step prediction error of the nominal model. Then, within the prediction horizon, the future state sequence is recursively obtained. The detailed procedure for the recursive process is outlined below. Let the current sampling instant be k . At this instant, the actual state $\mathbf{x}(k) = [v_y(k), \omega(k), \phi(k), Y(k), X(k)]^T$ and the previous control input $u(k-1)$ are known. With these known quantities, the initial values for the recursion are set as:

$$\begin{aligned} \hat{v}_y(1) &= v_y(k), \quad \hat{\omega}(1) = \omega(k), \quad \hat{\phi}(1) = \phi(k), \\ \hat{Y}(1) &= Y(k), \quad \hat{X}(1) = X(k) \end{aligned}$$

$$\begin{cases} \hat{v}_y(i+1) = \hat{v}_y(i) + T_s \left(-v_x(k)\hat{\omega}(i) + \frac{F_{yf}^{nom}(i) + F_{yr}^{nom}(i)}{m} + \Delta v_y(i) \right) \\ \hat{\omega}(i+1) = \hat{\omega}(i) + T_s(\ddot{\phi}^{nom}(i) + \Delta\omega(i)) \\ \hat{\phi}(i+1) = \hat{\phi}(i) + T_s\hat{\omega}(i) \\ \hat{Y}(i+1) = \hat{Y}(i) + T_s(v_x(k)\widehat{\sin\phi}(i) + \hat{v}_y(i)\widehat{\cos\phi}(i)) \\ \hat{X}(i+1) = \hat{X}(i) + T_s(v_x(k)\widehat{\cos\phi}(i) - \hat{v}_y(i)\widehat{\sin\phi}(i)) \end{cases} \quad (15)$$

After the recursion is complete, the predicted output sequence is obtained: $\mathbf{Y}_{pred} = [\hat{\phi}(2:N_p+1), \hat{Y}(2:N_p+1)]^T$. Use this sequence as the prediction output of the MPC, substitute it into the optimization problem for solution, and obtain the current optimal control increment $\Delta u(k)$. Then update the front wheel steering angle: $u(k) = u(k-1) + \Delta u(k)$.

4. Adaptive Control Increment Constraint Adjustment based on GPR Variance

4.1. Theoretical Basis for Uncertainty Quantification

A distinctive feature of GPR, compared to deterministic learning methods, is its ability to return not only a predicted mean but also a predictive variance. This variance acts as a quantitative indicator of the model's confidence in its prediction: when the current vehicle state falls within the training data distribution, the variance remains low; when the vehicle enters regimes poorly represented by the training data, the variance rises accordingly. Therefore, based on the magnitude of the variance, corresponding control strategies can be designed. When the variance is small, the model has high confidence, and the controller can trust the GPR residual compensation. A faster response control is adopted to pursue tracking accuracy. When the variance is large, the model's

confidence level is low. The controller should actively tighten the control increment constraints and adopt a conservative strategy to maintain lateral stability.

$$\begin{cases} \Delta v_y(i) = \mu_{GPR, v_y}([v_x(k), \hat{v}_y(i), \hat{\omega}(i), u(k-1)]) \\ \Delta\omega(i) = \mu_{GPR, \omega}([v_x(k), \hat{v}_y(i), \hat{\omega}(i), u(k-1)]) \end{cases} \quad (14)$$

Finally, the predicted state is updated via Euler integration:

4.2. Definition of Comprehensive Uncertainty Parameters

During each MPC control cycle, the current state $\mathbf{z}_{now} = [v_x, v_y, \omega, \delta_f^{prev}]$ is used as the input feature to call the GPR model, where δ_f^{prev} is the steering angle at the preceding step. The predicted variances $\Sigma_{v_y}(\mathbf{z}_{now})$ and $\Sigma_{\omega}(\mathbf{z}_{now})$ are obtained.

$$\sigma_{v_y} = \sqrt{\Sigma_{v_y}(\mathbf{z}_{now})}, \quad \sigma_{\omega} = \sqrt{\Sigma_{\omega}(\mathbf{z}_{now})} \quad (16)$$

$$U_c = 0.5\sigma_{v_y} + 0.5\sigma_{\omega} \quad (17)$$

4.3. Adaptive Control Increment Constraint Adjustment Strategy

The control increment constraint du_{max} directly limits the maximum variation of the front steering angle between adjacent control cycles of the MPC. Based on the comprehensive uncertainty U_c defined in the previous section, the following piecewise linear adjustment law is designed:

$$du_{max} = \begin{cases} du_{max}^{nominal}, & U_c \leq u_{low} \\ du_{max}^{nominal} - (du_{max}^{nominal} - du_{max}^{safe}) \frac{U_c - u_{low}}{u_{high} - u_{low}}, & u_{low} < U_c < u_{high} \\ du_{max}^{safe}, & U_c \geq u_{high} \end{cases} \quad (18)$$

Where the nominal reference constraint value is $du_{\max}^{\text{nominal}}=0.0148$ rad/step, the safe constraint value is $du_{\max}^{\text{safe}}=0.008$ rad/step, u_{low} and u_{high} are the lower and upper thresholds of uncertainty.

The role of this adjustment law is as follows: when U_c is small, the controller adopts the nominal constraint value $du_{\max}^{\text{nominal}}$; when U_c is large, the controller adopts the safety constraint value $du_{\max}^{\text{safe}}$; the intermediate area transitions smoothly through linear interpolation.

To determine the specific values of u_{low} and u_{high} , under the condition of high-speed dry-road double lane change, based on the simulation residual data of the fixed GPR-MPC, the comprehensive uncertainty U_c of each sampling point is calculated, and its distribution is analyzed. The 20th percentile is taken as the lower threshold $u_{\text{low}}=0.008$, and the 80th percentile as the upper threshold $u_{\text{high}}=0.025$.

5. Simulation Verification and Results Analysis

5.1. Experimental Setup and Comparative Controllers

The proposed controller was evaluated on the Carsim/Simulink platform. The control algorithm was developed in Matlab/Simulink, and Carsim provided the high-fidelity vehicle dynamics model. Two representative operating conditions were chosen: a high-speed dry-road scenario with $\mu = 0.85$ at 100 km/h, and a medium-speed wet-road scenario with $\mu = 0.35$ at 80 km/h.

The baseline MPC already performs satisfactorily at low speeds because the nominal linear model remains reasonably accurate. The proposed compensation is therefore most beneficial in the high-speed and low-adhesion roads. The reference path was defined by a double lane change maneuver. The sampling period of MPC is $T_s = 0.02$ s. The main vehicle parameters used in the simulation are listed in Table 2.

Table 2. Vehicle parameters

Parameter	Symbol	Value	Unit
Vehicle mass	m	1412	kg
Yaw moment of inertia	I_z	1536	kg·m ²
Distance from CG to front axle	l_f	1.015	m
Distance from CG to rear axle	l_r	1.895	m
Total cornering stiffness of front axle	C_f	148,970	N/rad
Total cornering stiffness of rear axle	C_r	82,204	N/rad

In order to analyze and verify the contributions of each core module of the proposed control strategy, three comparison controllers were set up. The Baseline MPC is a

standard fixed-parameter MPC controller that does not include any GPR compensation or adaptive adjustment modules. It serves as the baseline for the evaluation. GPR-MPC is a controller that incorporates GPR recursive residual compensation on the basis of Baseline MPC.

However, the control increment constraint $du_{\max}$ is fixed at the baseline value of 0.0148 rad/step, to isolate the contribution of the GPR compensation module to tracking accuracy. A-GPR-MPC is a controller based on GPR-MPC, which incorporates the adaptive adjustment of GPR variance to control the increment constraint $du_{\max}$. It is used to evaluate the improvement effect of the adaptive module on lateral stability while maintaining tracking accuracy. The MPC parameters common to all three controllers are listed in Table 3.

Table 3. MPC controller parameters

Parameter	Symbol	Value
Prediction horizon	N_p	16
Control horizon	N_c	5
Sampling period	T_s	0.02 s
Output weight matrix	Q	diag(1.0,100)
Control increment weight	R	1.0×10^3
Steering angle limit	δ_f	≤ 0.1744 rad
Steering rate limit	$\Delta\delta_f$	≤ 0.0148 rad/step

The per-cycle computation time was measured on a PC equipped with an Intel Core i5-1240P processor and 16 GB of memory. The Baseline MPC required about 8.5 ms per cycle. The GPR-MPC and A-GPR-MPC, which involve additional GPR prediction calls for lateral velocity and yaw rate residuals, increased the computation time to about 12.5 ms and 12.8 ms per cycle, respectively.

All three controllers operated well below the 20 ms sampling period, confirming their feasibility for real-time implementation. The additional overhead of the GPR module, about 4.3 ms per cycle, is acceptable given the performance improvements achieved in tracking accuracy and lateral stability.

5.2. High-Speed Dry Road Conditions

Figures 3 and 4 show the curves of the lateral and heading errors for the tracking accuracy. As can be seen from Figure 3, compared with the Baseline MPC, the GPR-MPC that incorporates GPR residual compensation significantly reduces the peak value of the lateral error. The lateral error peak of A-GPR-MPC is basically the same as that of GPR-MPC. From the heading error curve in Figure 4, it can be seen that both GPR-MPC and A-GPR-MPC have significantly improved heading errors compared to Baseline MPC. The A-GPR-MPC maintained a similar heading error to that of the GPR-MPC.

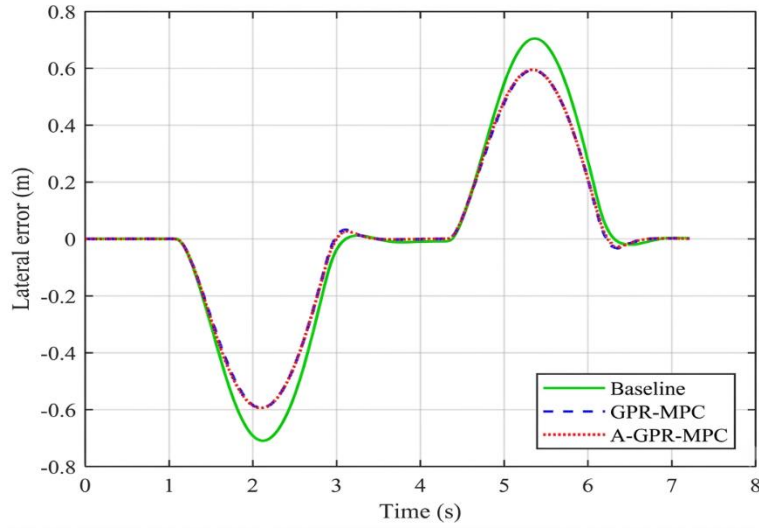


Fig. 3 Lateral Error under High-Speed Dry-Road Conditions

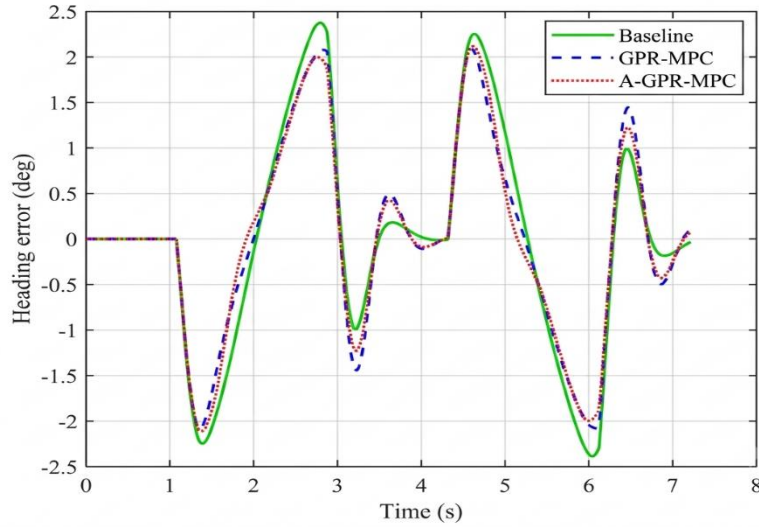


Fig. 4 Heading Error under High-Speed Dry-Road Conditions

Table 4 shows that the GPR-MPC lowers the lateral error peak by 16.2% (from 0.7089 m to 0.5937 m) and the heading error peak by 12.7% (from 2.3866° to 2.0845°), compared with the Baseline MPC. It indicates that the GPR residual compensation is able to considerably enhance the trajectory

tracking accuracy. The yaw rate peak and sideslip angle peak of the GPR-MPC only decreased by 2.9% and 2.1%, respectively. This indicates that the GPR residual compensation helps to improve the tracking accuracy, but has a limited effect on enhancing the lateral stability.

Table 4. Performance comparison under high-speed dry-road conditions

Controller	Lateral error RMSE (m)	Lateral error peak (m)	Heading error RMSE (°)	Heading error peak (°)	Yaw rate peak (rad/s)	Sideslip angle peak (°)	Front wheel steering angle peak (°)
Baseline MPC	0.3450	0.7089	1.1800	2.3866	0.2453	1.2375	2.5072
GPR-MPC	0.2901(15.9%)	0.5937(16.2%)	1.0553(10.6%)	2.0845(12.7%)	0.2381(2.9%)	1.2116(2.1%)	2.5073(0.0%)
A-GPR-MPC	0.2926(15.2%)	0.5949(16.1%)	1.0389(12.0%)	2.1173(11.3%)	0.2162(11.9%)	1.1093(10.4%)	2.2880(8.7%)

RMSE: Root Mean Square Error. Values in parentheses indicate the percentage improvement relative to the Baseline MPC.

Figure 5 compares the yaw rate and sideslip angle characteristics relevant to lateral stability. Figure 6 presents the front wheel steering angle curve. From Table 4, Figure 5 and Figure 6, it can be seen that compared with GPR-MPC, while preserving tracking accuracy, A-GPR-MPC further lowers the yaw rate, sideslip angle, and front wheel steering

angle peaks by 9.2%, 8.4%, and 8.7%, respectively. This indicates that by actively tightening the steering rate constraints through an adaptive control increment constraint adjustment strategy based on GPR variance, A-GPR-MPC can effectively prevent excessive steering actions. Without reducing the tracking accuracy, it enhances the lateral stability.

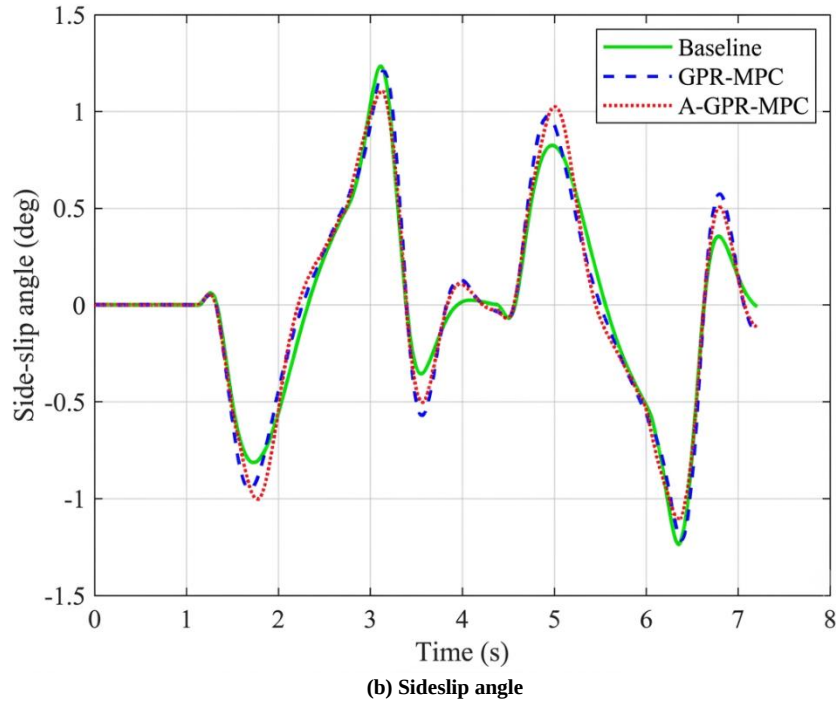
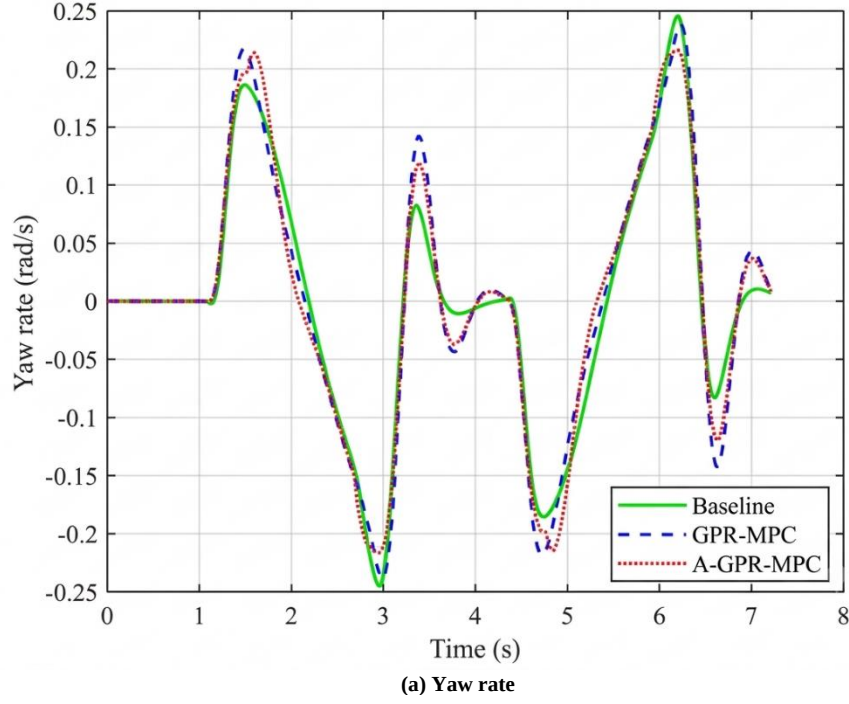


Fig. 5 Yaw Rate and Sideslip Angle under High-Speed Dry-Road Conditions

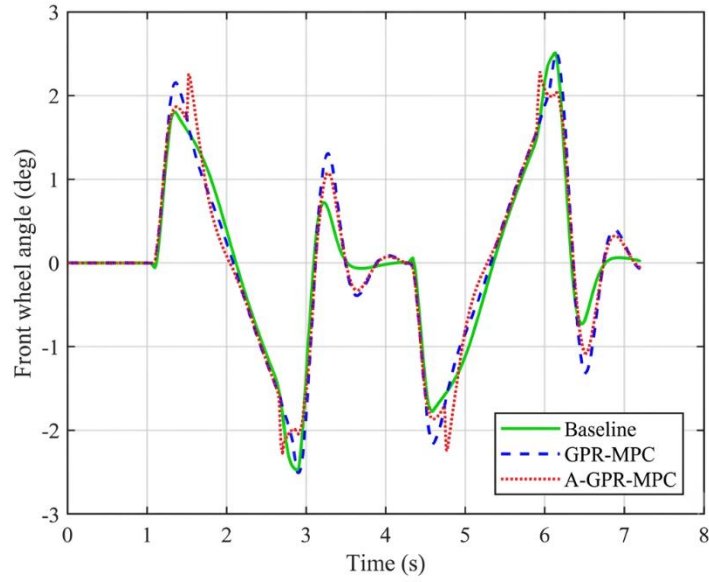


Fig. 6 Front Wheel Steering Angle under High-Speed Dry-Road Conditions

5.3. Medium-Speed Wet Road Conditions

Figure 7 shows that the lateral error peak for the Baseline MPC is relatively large; after adding the GPR residual compensation, the lateral error peak of GPR-MPC significantly decreases. Similar to the high-speed dry-road conditions, the lateral error peak of A-GPR-MPC is close to that of GPR-MPC. Figure 8 shows a similar trend for heading error. GPR-MPC and A-GPR-MPC significantly reduced the heading error. The performance indicators under the medium-speed wet road conditions are summarized in Table 5.

Compared with the Baseline MPC, the GPR-MPC lowers the lateral error peak by 16.7% (from 0.5524 m to 0.4604 m) and the heading error peak by 14.1% (from 2.2111° to 1.8982°). This further demonstrates the improvement effect of GPR residual compensation on tracking accuracy. Meanwhile, the yaw rate peak and sideslip angle peak of GPR-MPC were reduced by only 2.8% and 3.5%, respectively. The results of stability improvement are similar to those in the high-speed dry-road conditions, indicating that residual compensation has a limited effect on lateral stability.

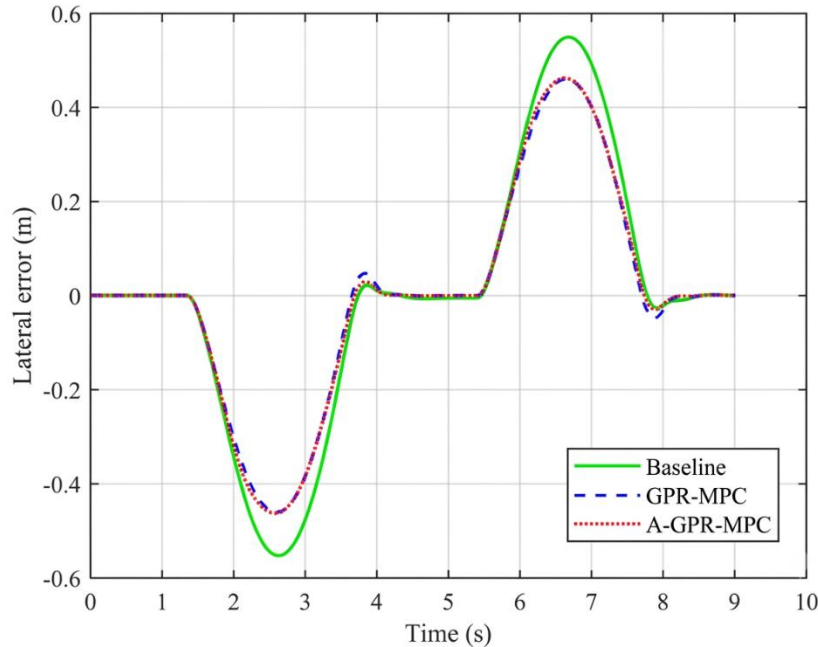


Fig. 7 Lateral Error under Medium-Speed Wet-Road Conditions

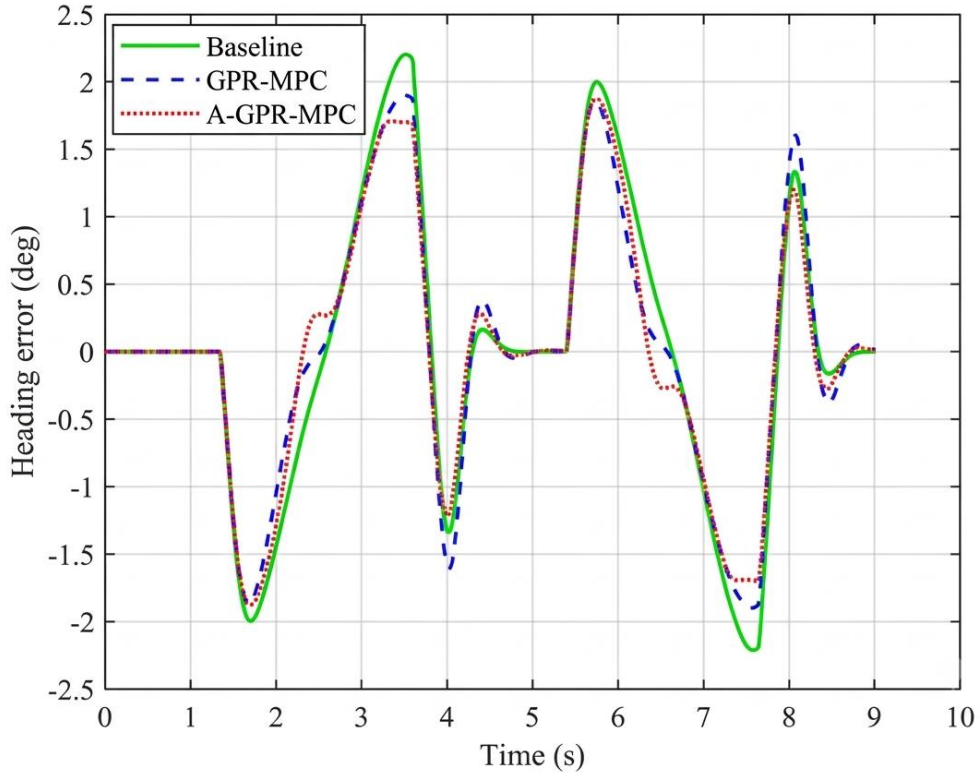


Fig. 8 Heading Error under Medium-Speed Wet-Road Conditions

Table 5. Performance comparison under medium-speed wet road conditions

Controller	Lateral error RMSE (m)	Lateral error peak (m)	Heading error RMSE (°)	Heading error peak (°)	Yaw rate peak (rad/s)	Sideslip angle peak (°)	Front wheel steering angle peak (°)
Baseline MPC	0.2712	0.5524	1.0647	2.2111	0.1780	1.1106	2.8118
GPR-MPC	0.2265(16.5%)	0.4604(16.7%)	0.9601(9.8%)	1.8982(14.1%)	0.1731(2.8%)	1.0718(3.5%)	2.6212(6.8%)
A-GPR-MPC	0.2299(15.2%)	0.4623(16.3%)	0.9248(13.1%)	1.8768(15.1%)	0.1575(11.5%)	0.8682(21.8%)	1.9890(29.3%)

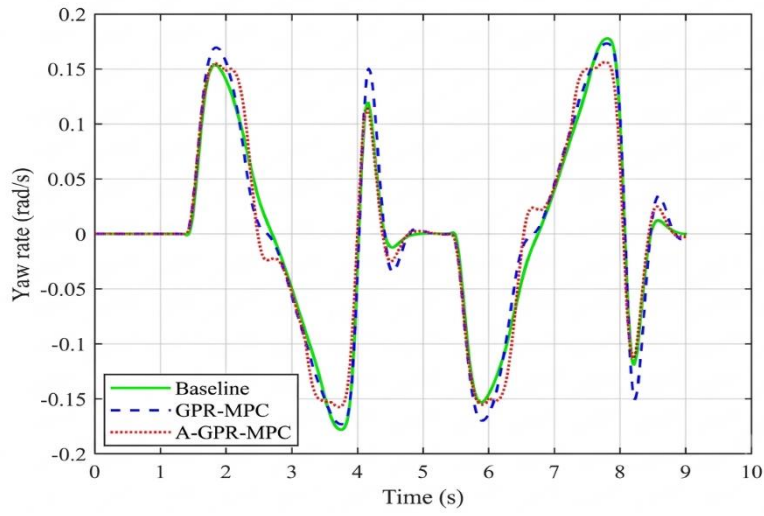
Values in parentheses indicate the percentage improvement relative to the Baseline MPC.

Figure 9 compares the yaw rate and sideslip responses, while Figure 10 presents the front wheel steering angle. Table 5 reveals that A-GPR-MPC, while maintaining a similar tracking accuracy to GPR-MPC, lowers the yaw rate peak by 11.5%, the sideslip angle peak by 21.8%, and the front wheel steering angle peak by 29.3%.

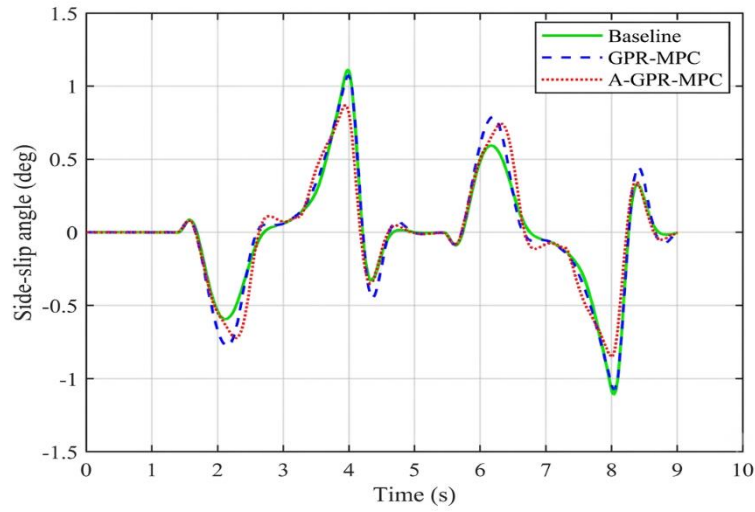
Compared with the high-speed dry-road conditions, the stability improvement in the medium-speed wet road condition is more significant, indicating that the adaptive control increment constraint adjustment strategy has stronger robustness in the wet road condition. The improvement patterns observed across the two conditions are largely consistent. Under both dry and wet roads, the GPR compensation reduces lateral and heading errors by approximately 16%, confirming that the learned residuals

effectively capture the unmodeled dynamics regardless of road adhesion. The adaptive strategy A-GPR-MPC maintains tracking accuracy comparable to GPR-MPC while delivering greater stability benefits on the wet road: sideslip angle reduction reaches 19.0%, compared with 8.4% on the dry road.

This difference is physically expected. On low-adhesion surfaces, the reduced lateral tire force capacity makes the vehicle more susceptible to sideslip under aggressive steering. By actively tightening the steering rate constraint through the adaptive adjustment of control increment limits, the controller yields more pronounced stability improvements precisely where they are most needed. These findings indicate that the adaptive strategy responds effectively to the underlying vehicle dynamics and proves robust across different road conditions.



(a) Yaw rate



(b) Sideslip angle

Fig. 9 Yaw Rate and Sideslip Angle under Medium-Speed Wet-Road Conditions

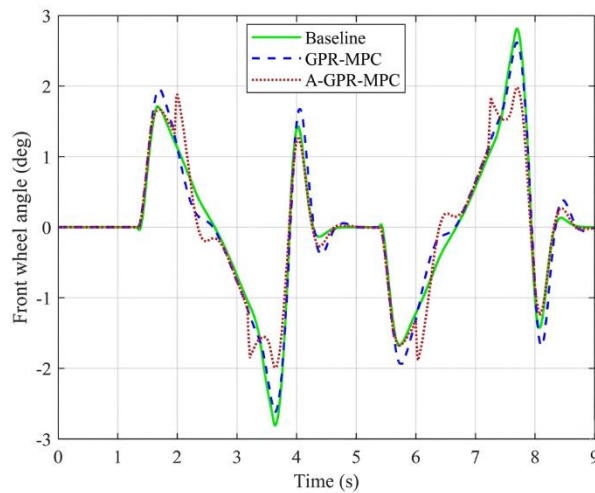


Fig. 10 Front Wheel Steering Angle under Medium-Speed Wet-Road Conditions

5.4. Discussion

The GPR models are trained offline using data collected from a predefined set of maneuvers. While the training data cover speeds from 10 to 35 m/s and road adhesion coefficients from 0.35 to 0.85, the models may not generalize well to scenarios significantly outside this range—such as extreme maneuvers beyond the vehicle's handling limits or surfaces with very low adhesion. Another limitation is the assumption of constant longitudinal velocity over the prediction horizon, a common simplification in lateral control studies that may introduce errors during aggressive acceleration or braking. Furthermore, the current study is simulation-based. While CarSim provides a high-fidelity representation of vehicle dynamics, real-world conditions such as sensor noise, actuator delays, and unmodeled disturbances are not fully represented within the simulation environment.

Regarding practical deployment, the proposed algorithm can be integrated into existing autonomous vehicle control architectures by replacing the standard MPC module. The algorithm relies on standard sensor inputs (vehicle states from IMU and GPS) and actuator commands via CAN bus. Future work will also explore online learning mechanisms to enable the GPR model to update dynamically based on real-time data, further enhancing its adaptability to varying driving conditions. The proposed method is evaluated against recent adaptive MPC studies for trajectory tracking on high-speed, low-adhesion roads. Liu and Zhang [22] designed an adaptive time-domain MPC for low-adhesion roads and demonstrated effective reduction in lateral deviation, with the sideslip angle maintained within 0° - 1.5° . A hybrid MPC scheme for distributed-drive EVs, in which the yaw-rate and sideslip-angle errors were reduced by 6.52% and 10.76%, in that order, at high speeds on low-adhesion roads [23]. A stability margin-based adaptive MPC achieved reductions of 33.96% in yaw rate RMSE and 24.00% in sideslip angle RMSE on slippery roads [24].

The proposed A-GPR-MPC achieves comparable or superior stability improvements. Under medium-speed wet road conditions, the sideslip angle peak drops by 21.8%, which is comparable to the 24.00% sideslip angle RMSE reduction reported in [24] and significantly higher than the 10.76% reduction reported in [23]. In addition, the peak front wheel steering angle is reduced by 29.3%, indicating smoother steering actions. These findings demonstrate that the adaptive constraint adjustment strategy effectively enhances lateral stability in low-adhesion scenarios—an aspect not addressed by existing GPR-MPC studies that focus primarily on tracking accuracy.

6. Conclusion

This paper proposes an MPC method that combines GPR recursive residual compensation with adaptive control increment constraint adjustment. This combination addresses the degradation of tracking accuracy and lateral stability

caused by model mismatches. The key findings of this study are summarized below. First, the GPR recursive residual compensation improves trajectory tracking accuracy. Under high-speed dry-road conditions, GPR-MPC lowers the lateral error peak by 16.2% and the heading error peak by 12.7% compared with the baseline MPC. However, its influence on yaw rate and sideslip angle is marginal. Second, the adaptive control increment constraint adjustment enhances lateral stability without compromising tracking accuracy. Under the same high-speed dry-road conditions, A-GPR-MPC maintains a similar tracking accuracy to GPR-MPC while reducing the peak yaw rate by 9.2% and the peak sideslip angle by 8.4%. Third, under medium-speed wet road conditions, the peak yaw rate, peak sideslip angle, and peak front wheel steering angle are reduced by 9.0%, 19.0%, and 24.1%, respectively. The adaptive strategy thus jointly optimizes tracking accuracy and lateral stability across different road surfaces, with more pronounced stability improvements observed on wet roads. The method proposed in this paper shows consistent performance under different driving speeds and road conditions, verifying its robustness and generalization ability. The handling of complex situations, such as sudden changes in dynamic parameters, still needs to be explored. In the future, an online learning mechanism will be introduced to enable the GPR model to dynamically update based on real-time data.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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Nomenclature

Symbol	Description	Unit
m	Vehicle mass	kg
I_z	Yaw moment of inertia	$\text{kg} \cdot \text{m}^2$
l_f, l_r	Distance from C.G. to front/rear axle	m
C_f, C_r	Cornering stiffness of front/rear axle	N/rad
v_x, v_y	Longitudinal/lateral velocity	m/s
ω	Yaw rate	rad/s
ϕ	Yaw angle	rad
δ_f	Front wheel steering angle	rad
α_f, α_r	Front/rear tire slip angle	rad
F_{xf}, F_{xr}	Front/rear longitudinal tire force	N
F_{yf}, F_{yr}	Front/rear lateral tire force	N
e	One-step prediction residual	-
μ	Predicted mean of GPR	-
Σ	Predicted variance of GPR	-
U_c	Comprehensive uncertainty	-
$du_{\max}$	Control increment constraint	rad/step
N_p, N_c	Prediction/Control horizon	step

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