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Numerical Analysis of the Optimal Conditions of the Selected Tests for Bending Rigidity of Textiles

Szablewski, P.[1]

[1]*Department of Mechanical Engineering, Informatics, and Chemistry of Polymer Materials, Lodz University of Technology, Lodz, Poland*

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Abstract
This work concerns numerical analysis two tests for bending length proposed by Peirce. These tests are the alternative for traditional gravitational cantilever Peirce's test. The applied mathematical model treats the textile product as elastic, which is subject to large deflections – the bending moment is proportional to the curvature of the bent axis. The optimal conditions of these tests were also considered in order to obtain the results of measurements most sensitive to changes of the input parameters. The results also show how long the specimen to measure the bending rigidity should be within a certain range of values.

Keywords: *textile mechanics, fabrics, elastic, bending length, bending rigidity, numerical methods.*

I. INTRODUCTION

The bending behavior of textile materials is a significant characteristic that largely determines the ability of fabrics to drape. Furthermore, it also influenced the formability, handle, flexibility, buckling behavior, wrinkle resistance, and crease resistance. The measurement of bending is often characterized by its bending or flexural rigidity and hysteresis. Several testing devices offer the function to test the bending rigidity of the fabrics. Not only that, they come with various principles of measurement such as pure bending principle, folded loop, and cantilever methods [1], [2].

Pure bending tests are the most complex as compared to the others. Kawabata Evaluation System (KES) uses this principle in its bending measurement where the sample is mounted vertically on the device and the bending moment is measured as the sample is bent, enabled to obtain moment versus curvature results during the bending cycle [3], [4]. Using the same principle, the bending curvature could also be measured using Instron, as reported by Kocik et al. [5]. This pure bending method was also proposed and explained by other researchers in their publications [6]–[8]. The folded loop is another principle used to measure bending behavior. By folding a fabric back to itself, a loop-like structure is obtained, and the height of the folded loop is measured as bending length is found to be proportional to it [1], [9]. A commonly used principle, cantilever deformation, is a globally accepted principle that was originally initiated by Peirce [10]. During the experiment, one edge of the fabric strip is fixed on a platform, glided with a ruler, and deflected from the platform under its own weight as a cantilever. Then, the cantilever length is measured once it reaches a pre-determined deflection angle [11], [12]. Many researchers applied this principle and developed their own test method for the determination of bending. Sun introduced a tester that uses a cross-shaped specimen with a fixed strip length at the central part. The specimens are hanging on their own weight, and their x and y coordinates are determined, thus drape angle could be measured [13]. The drape angle describes fabric drape, and the bending length and flexural rigidity can thus be calculated from it. The commercial testing instruments from Commonwealth Scientific and Industrial Research Organization (CSIRO), Fabric Assurance by Simple Testing (FAST), also uses the cantilever principle in its bending module with an optical device to detect the bending angle [14].

The cantilever method is applied in the standards which are commonly referred to today, such as EN ISO 9073-7 (European standard) and BS 3356-1990 (British standard). These standards use a manual bending tester to measure the bending length of the fabric samples. The whole procedure starts from placing the sample on the pathway, then sliding the fabric until the 41.5° line, and lastly taking the reading of the overhang length from the scale. All these are done by the operator. As this is prone to inaccuracies, an automated bending tester was developed, which is still based on the cantilever principle. Still, in the prototype version, this equipment which was developed at Ghent University, Belgium, introduces some automated approaches for the measurement [15]. Unlike the manual (standard) testing where the fabric strip is bent by the operator until it reaches 41.5° from the plane platform, this movement was automated by using actuators. Also, the human eyesight, which is used in determining the fabric when it reaches the angle, is replaced by sensors. Hence, a constant movement can be achieved, and inaccuracies of the measurement can be reduced.

The authors, who deal with the bending problem of textiles in most cases, take advantage of Peirce's theory presented in classical work [10]. In this work, there are theoretical fundamentals on which base most of today's statical measurement methods of bending rigidity of textiles.

II. MATERIALS AND METHODS

A. Tests

Peirce has proposed a simple test for describing bending rigidity ([16], [17]). As an alternative for Peirce's test, new tests also based on gravitational methods are proposed. These tests are presented in Fig. 1. The name of the test derives from the shape of the loop: „heart" and „pear." It should be pointed out that the height of the loop is taken into consideration (loop height) as the parameter characterizing the bending rigidity of the sample.

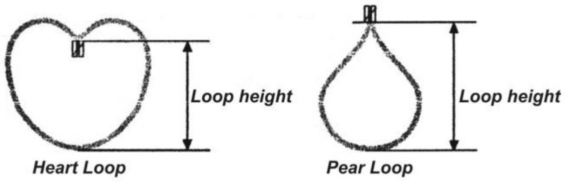

Figure 1: The gravitational tests: „heart" and „pear."

The goal of the study was examination which of these tests is more appropriate for an estimate the bending rigidity of textiles, especially due to changes in the stiffness.

B. Numerical analysis

In this paper is assumed that the flat strip of fabric will be represented as its longitudinal section. The mathematical model will be described by a flat deflection curve which will be treated as a heavy elastic, as shown in Fig. 2. Therefore, instead of studying strips of fabric, the numerical analysis will be concerned with deflections of heavy elastica with given bending rigidity C and appropriate linear weight q. Besides will be assumed that the elastica is inextensible. Each point of centreline of elastica defined by curvilinear coordinates measured along elastica passes to point $x(s)$, $y(s)$ in a fixed coordinate system. Internal forces occurring at any cross-section within elastica reduce to components: horizontal force $F_x(s)$, vertical force $F_y(s)$, and bending moment $M(s)$.

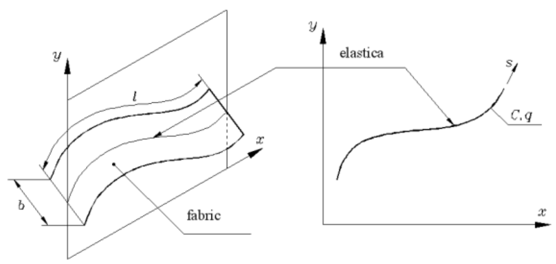

Figure 2: The model of fabric approximated by the elastica

Equilibrium of infinitesimal section of elastica *ds* (Fig. 3) and physical law lead to the following system of nonlinear first-order Differential Equations (1) that describing the elastica bending behavior. The unknowns are the functions of variable s: F_x, F_y, M, α, x, y.

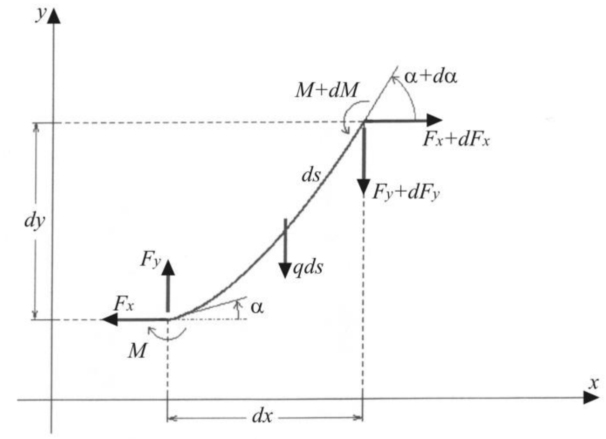

Figure 3: The infinitesimal section of elastica

Therefore, to solve the above-mentioned problem, five Differential Equations (1) of heavy elastica will be used with five boundary conditions (condition $F_y^B=0$ has been used. B is the free endpoint of elastica).

Due to $F_y^B=0$ and $dF_y/ds=-q$ we can write $F_y=q(l-s)$.

Therefore

$$\frac{d\alpha}{ds}=\frac{M}{C}\,,\quad \frac{dM}{ds}=F_x\sin\alpha+q(l-s)\cos\alpha\,,$$
$$\frac{dF_x}{ds}=0\,,\quad \frac{dx}{ds}=\cos\alpha\,,\quad \frac{dy}{ds}=\sin\alpha\,. \qquad (1)$$

The system of equations is completed by boundary conditions that are usually connected with two ends of elastica. Due to the geometrical and mechanical symmetry of the problem, only one-half of the loop of the length l=0.5L will be studied, where L is the length of the whole loop (see Fig. 4).

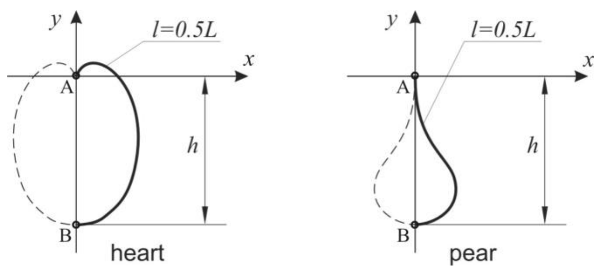

Figure 4: One-half of the loops „heart" and „pear." with marked loop height *h*

The loop height h is an absolute value of the coordinate y of the final point B. The boundary conditions for starting point A and for final point B are presented below.

Hart loop

- Fixed point A (s = 0):
$x^A=0$, $y^A=0$, $\alpha^A=\pi/2$.

- Free point B - free movement along the y-axis ($s = l$):
$x^B = 0$, $F_y^B = 0$, $\alpha^B = -\pi$.

<u>Pear loop</u>

- Fixed point A ($s = 0$):
$x^A = 0$, $y^A = 0$, $\alpha^A = -\pi/2$.
- Free point B - free movement along the y-axis ($s = l$):
$x^B = 0$, $F_y^B = 0$, $\alpha^B = -\pi$.

We can get the solution of Equations (1) in the complete form only in one case if $q = 0$. The solution is then expressed by elliptic integrals. Generally, solving this problem is more complicated, and numerical methods will be applied. In order to solve the system of Equations (1), the iterative shooting method for the boundary problem was used.

To solve the problem, the Mathematica program was used with the appropriate function to solve the boundary problem using the shooting method. I this task, the Newton-Raphson method to find unknown initial values of the corresponding functions were used.

III. RESULTS OF CALCULATIONS

In order to illustrate the presented method, both tests, „heart" and „pear," have been examined. In this way, a shape of the formed loop and its height h for a wide range of bending lengths have been obtained.
The example shapes of the loop for „heart" and „pear" tests are presented in Fig. 5.
The parameters are as follows:

„heart" test	„pear" test
$l = 0{,}20$ m	$l = 0{,}20$ m
$D_1 = 0{,}200$ m	$D_1 = 0{,}200$ m
$D_2 = 0{,}123$ m	$D_2 = 0{,}121$ m
$D_3 = 0{,}045$ m	$D_3 = 0{,}041$ m

For both tests: $q = 0{,}005$ N/m and the length of specimen $L = 2l$.

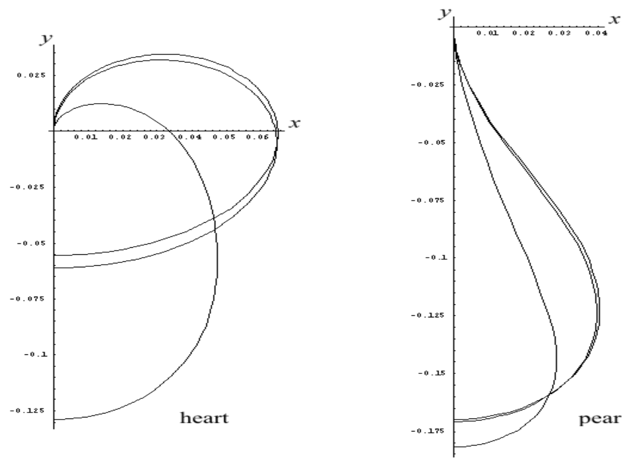

Figure 5: The example shapes of the loop for „heart" and "pear" tests

For each specimen, the quotient h/l never exceeds 1. Therefore the graph of the quotient h/l as a function of bending length has been prepared. This graph for q = 0,005 N/m is shown in Fig. 6.

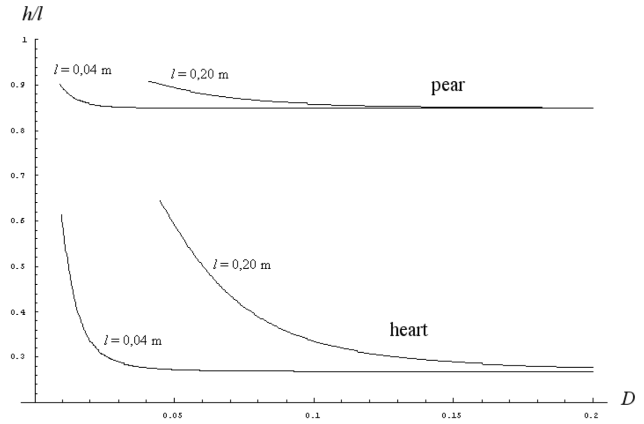

Figure 6: The graph of the relative loop height *h/l* as a function bending length *D*

The estimation of bending rigidity measured by the loop height is better for the „heart" test. In this case, the loop height is more sensitive to changes in bending rigidity.
Further considerations concern the „heart" test.

In Fig. 7, the graph of the relative loop height h as a function bending length D for different length l of specimens from 0,24 m to 0,02 m with step 0,02 m is presented ($l = 0{,}5L$, where L is the full length of the specimen).

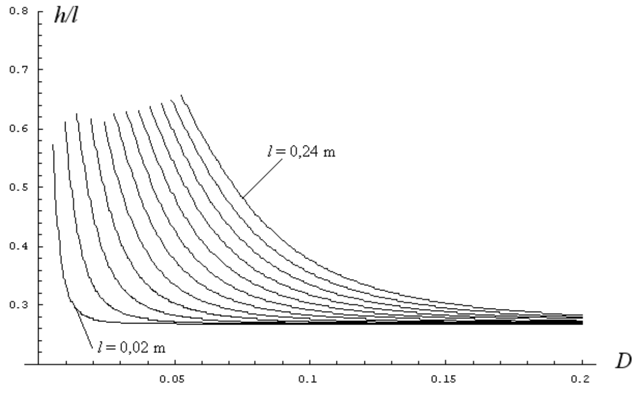

Figure 7: The graph of the relative loop height *h/l* for different lengths of specimens

The relative changes in height and bending rigidity can define as follows:

$$\varepsilon_h = \frac{h_{\text{next}} - h_{\text{prev}}}{h_{\text{prev}}}, \qquad \varepsilon_D = \frac{D_{\text{next}} - D_{\text{prev}}}{D_{\text{prev}}},$$

where the index „next" denotes the next value of the given variable and the index „prev" – previous one. In this way, the graph of the quotient of relative changes of the height to relative changes of bending rigidity $\varepsilon_h/\varepsilon_D = f(D)$ can be created.

Assume that we make the measurements to the moment until the relative changes of the height will be not less than 1/4 (25%) of the relative changes of bending rigidity, i.e.
$\varepsilon_h/\varepsilon_D \geq 0{,}25$. From the graph in Fig. 8, it follows that $\varepsilon_h = 0{,}25\varepsilon_D$ for $D = 0{,}17\,\mathrm{m} \quad \rightarrow \quad C = 2{,}46 \cdot 10^{-5}\,\mathrm{Nm}^2$ (in case of q = 0,005 N/m and l = 0,24 m).
Of course, it is possible to make similar investigations for a wider range of linear weight q and different lengths of specimens.

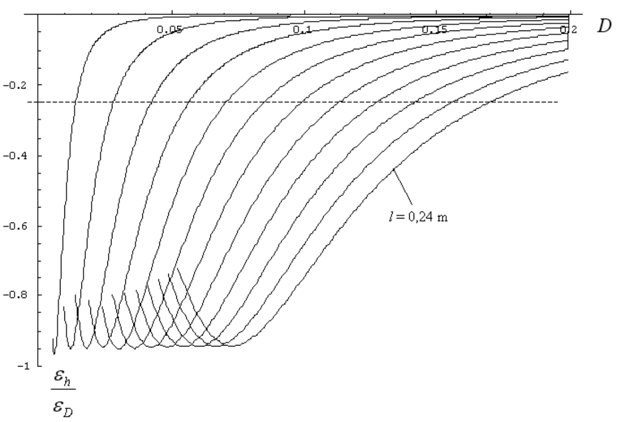

Figure 8: The relative changes of the loop height for different length of the specimen

Fig. 8 are presented the relative changes of the loop height for different lengths of the specimen from the range of 0,24 m to 0,02 m with the step of 0,02 m.
From the graph in Fig. 8, it can be read that for estimating bending rigidity from the range of $D \leq 0{,}17$ m maximum length of the specimen is $L = 2 \cdot 0{,}24 = 0{,}48\,\mathrm{m}$.

The table below shows the limiting values of bending rigidity $D_{\max}$ (or $C_{\max}$) for given lengths of the specimen and for q = 0,005 N/m. The limiting values $D_{\max}$ fulfill the condition $\varepsilon_h = 0{,}25\varepsilon_D$. The measurements for given lengths of specimens should be done for $D \leq D_{\max}$.
Of course, it is possible to make a similar table for a wider range of linear weight q.

TABLE I
Limiting values of bending rigidity Dmax (or Cmax)

L	$D_{\max}$	$C_{\max}$
[m]	**[m]**	**[Nm^2] ×10^{-7}**
0,04	0,014	0,14
0,08	0,028	1,10
0,12	0,042	3,70
0,16	0,057	9,26
0,20	0,071	17,90
0,24	0,084	29,60
0,28	0,098	36,50
0,32	0,113	72,10
0,36	0,127	102,00
0,40	0,141	140,00
0,44	0,155	186,00
0,48	0,170	246,00

IV. CONCLUSIONS

The numerical analysis of the mathematical model of „heart" and „pear" bending test turned out to be effective. The applied iterative shooting method for solving boundary problems has been sufficiently fast and enough stable. The disadvantage of this method is that there is a constraint for using it for very small values of bending rigidity. Below a certain limiting value of bending rigidity, the iterative procedure becomes divergent. This limiting value is different for different lengths of specimens.

The investigated numerical analysis gave the answer concerning the effectiveness of both tests. It turned out that more useful for estimating of bending rigidity of textiles is the „heart" test (loop of type „heart"). It follows that the loop height is more sensitive to changes of bending rigidity for this test. A more accurate analysis of the sensitivity of the loop height has shown how long should be lengths of specimens for the estimating of bending rigidity for a given range of values.

The specific values of lengths of specimens have been presented for the estimating of bending rigidity using the test of type „heart."

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